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Artículo de investigación

Combined economic-environmental dispatch using the weighting-based optimization model solved via the vortex search algorithm

Despacho económico-ambiental combinado mediante el modelo de optimización basado en ponderaciones resuelto mediante el algoritmo de búsqueda de vórtices

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Abstract

This article explores the multi-objective dispatch of thermal power plants using a weighting-based optimization approach. The objective functions aim to minimize both the hourly CO_2 emissions and the total power generation costs. The multi-objective problem is formulated as a nonlinear programming problem, considering the impact of turbine valves on both objectives. Moreover, the nonlinear programming model integrates the transmission power loss model into the power balance constraint, with these losses being characterized by classical quadratic coefficients. To tackle this intricate optimization problem, a metaheuristics-based approach is adopted. More precisely, the problem is addressed through the iterative solution of an equivalent single-objective model, which is derived from the weighting-based formulation and employs the vortex search algorithm. Numerical simulations conducted on two power systems, one composed of three-generation plants and the other one of ten-generation plants, validate the effectiveness of the proposed multi-objective approach in comparison with interior-point optimizers and metaheuristics-based optimizers. All numerical analyses were performed using the Julia software, with the `Ipopt` solver within the `JuMP` optimizing tool.

Keywords: Economic-environmental dispatch; multi-objective modeling; vortex search algorithm; weighting-based equivalent formulation; interior point optimization.

Resumen

Este artículo explora el despacho multiobjetivo de centrales térmicas utilizando un enfoque de optimización basado en ponderaciones. Las funciones objetivo buscan minimizar tanto las emisiones de CO_2 por hora como los costos totales de generación de energía. El problema multiobjetivo se formula como un problema de programación no lineal, considerando el impacto de las válvulas de turbina en ambos objetivos. Además, el modelo de programación no lineal incorpora el modelo de pérdidas de potencia de transmisión a la restricción del balance de potencia, donde estas pérdidas son representadas por coeficientes cuadráticos clásicos. Para enfrentar este intrincado problema de optimización, se utiliza un enfoque basado en metaheurística. Concretamente, el problema se aborda mediante la solución iteraiva de un modelo mono-objetivo equivalente que se deriva de la formulación basada en ponderaciones y utiliza el algoritmo de búsqueda de vórtices. Las simulaciones numéricas realizadas en dos sistemas de energía, uno compuesto por plantas de 3 generaciones y el otro por plantas de 10, validan la efectividad del enfoque multiobjetivo propuesto en comparación con optimizadores de puntos interiores y optimizadores basados en metaheurística. Todos los análisis numéricos se realizaron utilizando el *software* Julia, con el solucionador `Ipopt` dentro de la herramienta de optimización `JuMP`.

Palabras Clave: Despacho económico-ambiental; modelado multiobjetivo; algoritmo de búsqueda de vórtices; formulación equivalente basada en ponderaciones; Optimización de puntos interiores.

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Nomenclature

Acronyms

CEED Combined economic-environmental dispatch.

CO_2 Carbon dioxide.

EMOCA Enhanced multi-objective cultural algorithm.

Ipopt Interior point optimizer.

JuMP Julia optimization environment.

MODE Multi-objective differential evolution.

NSGA II Non-dominated sorting genetic algorithm II.

PSO Particle swarm optimization.

VSA Vortex search algorithm.

WbVSA Weighting-baed vortex search algorithm.

Parameters

α_k Economic objective function's independent form (USD/h).

β_k Economic objective function's linear coefficient (USD/MWh).

δ_k Power coefficient regarding turbine valve effect in the environmental objective function (1/MW).

η_k Cost coefficient regarding turbine valve effect in the environmental objective function (lb/h).

γ_k Economic objective function's quadratic coefficient (USD/MW^{2}h).

ω Weighting coefficient regarding multi-objective function analysis.

a_k Environmental objective function's independent form (lb/h).

B_0 Independent power loss coefficient (MW).

B_1 Vector containing the linear power losses effect.

B_2 Square matrix containing the power losses effect associated with sources k and m (1/MW).

b_k Environmental objective function's linear coefficient (lb/MWh).

c_k Environmental objective function's quadratic coefficient (lb/MW^{2}h).

e_k Cost coefficient regarding turbine valve effect in the economic objective function (USD/h).

f_k Power coefficient regarding turbine valve effect in the economic objective function (rad/MW).

$G_c^{\max}$ Maximum generation costs (USD/h).

$G_e^{\max}$ Maximum CO_2 emissions (lb/h).

P_D Total power demand (MW/h).

P_L Power transmission losses (MW).

$p_{g,k}^{\max}$ Maximum power generation bound applicable to the k^{th} generator (MW).

$p_{g,k}^{\min}$ Minimum power generation bound applicable to the k^{th} generator (MW).

Sets and Indices

G Set containing all the available generators.

Variables

G_{costs} Objective function value associated with the total generation costs (USD).

G_c Economic objective function (USD/h).

$G_{\text{emissions}}$ Objective function value related to the total CO_2 emissions (lb).

G_e Environmental objective function (lb/h).

$P_{g,k}$ Power output at the k^{th} thermal plant (MW).

1 Introduction

1.1 General context

The combined economic-environmental dispatch (CEED) model for thermal plants in power systems is one of the most classical optimization problems in electrical engineering research and applications, whose main goal is to minimize the expected generation costs of thermal sources while reducing greenhouse gas emissions [1]. This is a particularly important problem for power system operators, as the largest economies in the world continue to produce electricity from fossil generation sources [2].

1.2 Motivation

The primary goal of the CEED problem is to reduce the environmental and operating costs of electrical power systems while meeting the energy demand. Since the CEED model has two objectives, typically quadratic functions, it is necessary to implement multi-objective optimization methods. In the literature, two approaches have been typically implemented to tackle mathematical models with multiple objectives. The first approach involves the use of a multi-objective optimization algorithm, which has the advantage of obtaining Pareto-optimal solutions in a single iteration. The second approach consists of transforming the multi-objective problem into a single-objective one using the weighting-based optimization model. Therefore, it is necessary to execute the optimization model multiple times in order to achieve the Pareto-optimal front. This approach is simpler than the former. It is noteworthy that exploring approaches that allow solving this complex problem (*i.e.*, the CEED problem) in a more straightforward way has become an essential research topic, given its benefits in terms of enhanced accessibility and efficiency.

1.3 Literature review

Several optimization approaches have been proposed in the specialized literature to solve the CEED problem, among which methods based on mathematical optimization and metaheuristic algorithms stand out. In the case of mathematical optimization, strategies such as the Newton–Raphson method [3], lambda iteration [2], Lagrange relaxation [4], and the interior-point method [5] have been used. These methods have some disadvantages: they can easily get stuck in local optima, their performances depend mainly on the initialization point, and the optimization model must be differentiable in all feasible spaces.

As for metaheuristic algorithms, some relevant works are described below.

The authors of [6] used a penalty function integration in order to implement dynamic normalization. This approach outperformed the classical genetic algorithm in finding good solutions for the CEED problem.

The study by [7] proposed a multi-objective differential evolution algorithm to address the CEED problem. This algorithm was based on two stages: the first one transformed the equality constraints of the model into inequality constraints, thus relaxing the model; and, in the second stage, two mutation strategies were implemented to enhance the evolution of the conventional differential evolution algorithm.

In the work by [1], the differential evolution algorithm was enhanced with regard to addressing CEED problems. This enhancement was accomplished by incorporating reinforcement learning, resulting in an adaptive method.

The authors of [8] implemented a quantum-inspired particle swarm optimization (PSO) to find the Pareto-optimal front for the CEED problem. This was tested on three feeders with six, ten, and 11 thermal power plants. This method demonstrated a superior performance when compared to four other techniques documented in the literature.

Other multi-objective metaheuristic algorithms that have been widely used in the specialized literature include barebone PSO [9], the modified exchange market algorithm [10], the interior search algorithm [11], and the squirrel search algorithm [12], among others. These metaheuristic algorithms can efficiently solve the CEED problem by considering nonlinear and non-convex cost functions, making them highly attractive. However, many of these approaches require the tuning of numerous parameters, which has significantly impacted their performance in various test systems. Therefore, it is necessary to use metaheuristic algorithms that are easier to implement and require less tuning. One of the standout metaheuristic algorithms is the vortex search algorithm (VSA), given its simple structure [13]. Additionally, it offers a combination of efficiency, global exploitability, flexibility, and ease of implementation, making it an attractive option for various optimization problems.

1.4 Contributions and scope

Considering the literature review presented above, this research makes the following contributions:

i. The reformulation of the combined economic- environmental dispatch problem for thermal plants, including the power losses of the transmission lines and the effects of turbine valves on the objective functions, via an equivalent single-objective optimization model using the weighting-based modeling.
ii. The application of the vortex search algorithm (VSA) to solve the equivalent optimization model, which enables the construction of the optimal Pareto front by varying the weight coefficient between the objective function extremes. This proposal will also be validated with the exact solution of the optimization problem, using the interior-point optimizer in the Julia software, as well as with metaheuristic algorithms available in the specialized literature.

Regarding the scope of this research, it is essential to mention that (i) the power losses are modeled using the classical quadratic formulation that relates the power generation outputs with the total expected power losses level. These coefficients have been provided by the power system operator, and they are assumed to be external inputs for the optimization model. (ii) The effect of turbine valves introduces nonlinearities in both objective functions, which transforms the classical and convex economic-environmental thermal plant dispatch problem into a nonlinear non-convex optimization model that requires the design of effective solution methodologies to obtain an optimal solution. (iii) Finally, the exact optimization model is implemented in the Julia software, with the help of the `JuMP` optimization package and the interior-point optimizer (*i.e.*, the `Ipopt` solver).

1.5 Document structure

The remainder of this paper is structured as follows. Section 2 describes the general mathematical formulation of the economic-environmental thermal plant dispatch problem aimed at minimizing the expected energy production costs and the CO_2 emissions level. Section 3 presents the weighting-based multi-objective reformulation via a single-objective function equivalent. Section 4 defines the proposed solution methodology, which employs the VSA in the weighting-based equivalent optimization model to construct the optimal Pareto front. Section 5 describes the main characteristics of the power systems under study, which have 3- and 10-generation plants. Section 6 presents all numerical validations and analyzes, discusses, and compares our proposal against the exact solution of the optimization problem via the interior-point optimization method and some literature reports based on metaheuristics. Finally, Section 7 lists the main concluding remarks of this research, including some suggested future works.

2 Economic-environmental dispatch problem

The effective operation of thermal power systems involves minimizing the expected power generation costs for an aggregate demand by determining the power injection at each thermal plant. In addition, with the global concern regarding CO_2 levels, some power system operators are interested in minimizing the total greenhouse gas emissions [14]. This objective function is formulated as the minimization of an environmental performance indicator. In the literature, both of the aforementioned aspects are typically modeled as quadratic functions with the following structure [15]:

$$G_{\text{costs}} = \sum_{k\in\mathbf{G}} \gamma_k P_{g,k}^2 + \beta_k P_{g,k} + \alpha_k, \tag{1}$$

$$G_{\text{emissions}} = \sum_{k\in\mathbf{G}} c_k P_{g,k}^2 + b_k P_{g,k} + a_k, \tag{2}$$

where G_{costs} corresponds to the objective function value associated with the total generation costs; $G_{\text{emissions}}$ represents the objective function value related to the total CO_2 emissions, which the thermal plants generate; $P_{g,k}$ denotes the power output at the k^{th} thermal plant; the coefficients γ_k, β_k, and α_k are the economic objective function's quadratic, linear, and independent terms; and c_k, b_k, and a_k represent the quadratic, linear, and independent terms of the environmental objective function. $\mathbf{G}$ denotes the set containing all the available generators.

Note that the formulation of the objective functions in Equations (1) and (2) is convex, as the coefficients γ_k and c_k are positive-definite. This is a clear advantage for ensuring a globally optimal solution, and multiple authors consider that these functions can be used to approximate the real generation effects in thermal power plants. Two components are added to the objective functions in order to consider the effect of valves on the generation process, as defined in (3) and (4), respectively.

$$G_c = \sum_{k \in \mathbf{G}} \begin{bmatrix} \gamma_k P_{g,k}^2 + \beta_k P_{g,k} + \alpha_k + \\ e_k \left| \sin\left(f_k \left(p_{g,k}^{\min} - P_{g,k} \right) \right) \right| \end{bmatrix}, \quad (3)$$

$$G_e = \sum_{k \in \mathbf{G}} \begin{bmatrix} c_k P_{g,k}^2 + b_k P_{g,k} + a_k + \\ \eta_k \exp\left(\delta_k P_{g,k} \right) \end{bmatrix}, \quad (4)$$

where G_c and G_e denote the economic and environmental objective functions, including the effect of the valves. $p_{g,k}^{\min}$ is the minimum power generation bound applicable to the k^{th} generator, and the coefficients e_k and f_k model this effect in the economic function. Meanwhile, the η_k and δ_k coefficients include this effect in the environmental objective function.

Regarding the operating conditions associated with the economic-environmental dispatch problem in thermal systems, the power equilibrium between generation and demand is maintained by observing the capabilities of the generators. The literature also includes the effect of power transmission losses (P_L) in the power balance constraint via the quadratic transmission losses coefficient [16]. The constraints of the economic-environmental dispatch problem are defined from (5) to (7).

$$\sum_{k \in \mathbf{G}} P_{g,k} = P_L + P_D, \quad (5)$$

$$P_L = \sum_{k \in \mathbf{G}} \begin{bmatrix} \left(\sum_{m \in \mathbf{G}} P_{g,k} B_{2,k,m} P_{g,m} \right) + \\ B_{1,k} P_{g,k} + B_0 \end{bmatrix}, \quad (6)$$

$$p_{g,k}^{\min} \leq P_{g,k} \leq p_{g,k}^{\max}, \ \forall k \in \mathbf{G} \quad (7)$$

where B_2 is a square matrix containing the power losses effect associated with sources k and m; B_1 is a vector containing the linear power losses effect; and B_0 is a constant coefficient that accounts for the expected power losses in the no-load condition scenario [16].

Observación 1 *The model* (3)–(7) *is a nonlinear, non-convex optimization problem due to (i) the trigonometric function in the economic objective function and (ii) the quadratic equality constraint associated with the power losses effect in the power balance constraint. In addition, the objective functions in* (3) *and* (4) *conflict with each other, i.e., they both generate a multi-objective optimization problem.*

3 Multi-objective formulation

This work employs the weighting-based multi-objective optimization approach to deal with the combined economic-environmental dispatch problem in thermal systems while considering transmission losses via the $B-$coefficients. This multi-objective optimization approach is defined below.

Objective function:

$$\min z = \omega \frac{G_c}{G_c^{\max}} + (1-\omega) \frac{G_e}{G_e^{\max}}, \quad (8)$$

Subject to:

$$\text{Equations (5) – (7)}, \\ \omega \in [0,1], \quad (9)$$

where $G_c^{\max}$ is the maximum value of the economic function, obtained when minimizing the environmental objective function; $G_e^{\max}$ denotes the maximum CO_2 emissions reached after minimizing the environmental objective function; and ω is the weighting coefficient, which varies from 0 to 1 in specified steps to build the equivalent Pareto front.

Observación 2 *The solution of the optimization problem* (5)-(7) *requires solving the two underlying single-objective function problems since the minimum CO_2 emissions are obtained by minimizing the corresponding objective function, and the maximum value is obtained when the expected generation costs are minimized.*

4 Solution methodology

It is well-known that optimization problems with complex nonlinearities in their objective function or set of constraints mostly entail a convergence to local optima, especially when the initial solution point is randomly selected [17]. This research proposes the implementation of a metaheuristics-based initialization approach. In the literature, hundreds of metaheuristic optimizers have been used to approximate the solution of complex problems via an initial population (set of solutions) and some evolution rules. This work applies the vortex search algorithm (VSA), considering that reported by [13], who used this technique to solve the economic-environmental dispatch problem in thermal plants, highlighting its efficiency and high solution quality. The main aspects of the VSA are presented herein.

The VSA is a physics-inspired metaheuristic optimization approach initially developed by [18], which is based on the behavior of stirred fluids in pipes, wherein vortices are formed. The main advantage of this optimization algorithm is that a Gaussian distribution is used to explore and exploit the solution space, allowing for high-quality solutions in multiple optimization problems [19], [20].

4.1 Hyper-ellipse center and neighborhood

The VSA starts exploring the solution space by generating an initial set of dispersed solutions. In a given problem, if the upper and lower limits of the decision variables are defined by $x^{\max}$ and $x^{\min}$, the VSA will generate a hyper-ellipse that covers the solution space and is centered at μ_t (with $t = 0$ at the beginning of the optimization process). This is calculated as follows:

$$\mu_t = \frac{x^{\max} + x^{\min}}{2}, \quad (10)$$

From this initial point (*i.e.*, μ_t), a uniformly distributed neighborhood of solutions is generated, which is constructed using a Gaussian probability function $x\left(y|\mu, \Sigma\right)$, with the structure given in (11).

$$x\left(y|\mu,\Sigma\right)=\theta_x \exp^{\left\{-\frac{1}{2}\left(y-\mu\right)^{\top}\Sigma^{-1}\left(y-\mu\right)\right\}}, \quad (11)$$
$$\theta_x=\frac{1}{\sqrt{(2\pi)^{n_v}\left|\Sigma\right|}},$$

where n_v represents the number of variables, or the *dimension of the problem*; y is a vector of random variables; and Σ is the covariance matrix of dimension $n_v \times n_v$.

Observación 3 *Suppose the diagonal elements of the matrix Σ are equal (same variance), and the off-diagonal elements are zero (no covariance). In this case, the shape of the normal distribution will be hyper-spherical. If the elements of the diagonal vary and the covariance values remain zero, the distribution will take on a hyper-elliptical shape, as in this case.*

To construct the initial matrix Σ with the characteristics described above, the initial radius r_t of the system is set as $t = 0$. This value is determined according to (4.1).

$$r_t=\frac{x^{\max}-x^{\min}}{2},$$

Subsequently, the matrix Σ is defined as (12).

$$\Sigma=\sigma_t I_{n_v\times n_v}, \quad (12)$$

where σ_t is defined as r_t, and $I_{n_v\times n_v}$ is an identity matrix with appropriate dimensions [21].

It is necessary to verify whether the neighborhood of solutions generated from the probability distribution $x\left(y|\mu,\Sigma\right)$ is within the permissible range, for which the following rule is applied.

$$x_{k,j}^t=\begin{cases} x_{k,j} & x_j^{\min}\le x_{k,j}\le x_j^{\max} \\ x_j^{\min} & x_{k,j}<x_j^{\min} \\ x_j^{\max} & x_{k,j}>x_j^{\max}\end{cases}, \quad \begin{bmatrix}\forall j=1,2,...,n_v, \\ \forall k=1,2,...,n_s\end{bmatrix}, \quad (13)$$

where n_s is the number of solutions contained in the neighborhood.

4.2 Evolution rules

The VSA defines the new center of the hyper-ellipse based on the best neighborhood content, provided that this new center performs better than the previous one. To determine the best solution in the neighborhood (*i.e.*, x_{best}^t), the entire population X^t is evaluated with regard to the objective function or the adaptation function, with which

$$x_{\text{best}}^t=\left\{x_k^t/x_k^t\to\min\left(f\left(x_k^t\right)\right), \ \forall k=1,2,...,n_s\right\}. \quad (14)$$

Here, x_k^t is the k-th solution contained in the population, and $f\left(x_k^t\right)$ corresponds to the objective function (also known as the *adaptation* or *performance function*) evaluated in the solution x_k^t.

The substitution rule for the new center is given by (15).

$$\mu_t=\begin{cases}\mu_{t-1} & f\left(\mu^t\right)\le f\left(x_{\text{best}}^t\right) \\ x_{\text{best}}^t & f\left(\mu^t\right)>f\left(x_{\text{best}}^t\right)\end{cases}, \quad (15)$$

4.3 Reducing the hyper-ellipse radius

To transition from the exploration stage to the exploitation of promising regions in the solution space, the VSA proposes a gradual radius reduction as the number of iterations increases. Initially, the authors of [18] proposed the use of the incomplete inverse gamma function, but the authors of [21] modified this rule based on a decreasing exponential function:

$$r_t=\left(1-\frac{t}{t_{\max}}\right)\exp\left\{-b\frac{t}{t_{\max}}\right\}\left(\frac{x^{\max}-x^{\min}}{2}\right) \quad (16)$$

where b is the decay parameter of the exponential function, for which a value of 6 is recommended [21].

4.4 Stopping criterion

In the application of metaheuristic optimization techniques, because the solution to the problem is approximated via sequential programming, it is common to find two stopping criteria:

i. The exploration and exploitation stage is terminated when the pre-established maximum number of iterations is reached.

ii. The process is terminated when, upon reaching a given number of iterations $m_{\max}$, the performance function does not exhibit any improvements.

Note that a counter is required to implement the second stopping criterion. To this effect, the value of $m_{\max}$ should be predefined by the user – a value between 10 and 30% of the total iterations is recommended.

5 Test systems

The proposed multi-objective optimization methodology was computationally validated on two power systems composed of three and ten plants.

5.1 Three-generator system

The parametric information of this test feeder was obtained from [17], and it is reported in Table 1.

Table 1: Parametric information of the system with three generation plants

Unit k	γ_k (USD/MW^{2}h)	β_k (USD/MWh)	α_k (USD)	e_k (USD/h)	f_k (rad/MW)	$p_k^{\min}$ (MW)
1	0.03546	38.30553	1243.5311	0	0	35
2	0.02111	36.32782	1658.5696	0	0	130
3	0.01799	38.27041	1356.6592	0	0	125
Unit k	**$p_k^{\max}$ (MW)**	**a_k (lb/MW^{2}h)**	**b_k (lb/MWh)**	**c_k (lb)**	**η_k (lb/h)**	**δ_k (1/MW)**
1	210	0.00683	-0.54551	40.26690	0	0
2	325	0.00461	-0.51160	42.89553	0	0
3	315	0.00461	-0.51160	42.89553	0	0

Table 2: Parametric information of the system with ten generators

Unit k	γ_k (USD/MW^{2}h)	β_k (USD/MWh)	α_k (USD)	e_k (USD/h)	f_k (rad/MW)	$p_k^{\min}$ (MW)
1	0.12951	40.5407	1000.403	33	0.0174	10
2	0.10908	39.5804	950.6060	25	0.0178	20
3	0.12511	36.5104	900.7050	32	0.0162	47
4	0.12111	39.5104	800.7050	30	0.0168	20
5	0.15247	38.5390	756.7990	30	0.0148	50
6	0.10587	46.1592	451.3250	20	0.0163	70
7	0.03546	38.3055	1243.531	20	0.0152	60
8	0.02803	40.3965	1049.998	30	0.0128	70
9	0.02111	36.3278	1658.569	60	0.0136	135
10	0.01799	38.2704	1356.659	40	0.0141	150
Unit k	$p_k^{\max}$ (MW)	a_k (lb/MW^{2}h)	b_k (lb/MWh)	c_k (lb)	η_k (lb/h)	δ_k (1/MW)
1	55	0.04702	-3.9864	360.0012	0.25475	0.01234
2	80	0.04652	-3.9524	350.0056	0.25475	0.01234
3	120	0.04652	-3.9023	330.0056	0.25163	0.01215
4	130	0.04652	-3.9023	330.0056	0.25163	0.01215
5	160	0.00420	0.3277	13.85930	0.24970	0.01200
6	240	0.00420	0.3277	13.85930	0.24970	0.01200
7	300	0.00680	-0.5455	40.26690	0.24800	0.01290
8	340	0.00680	-0.5455	40.26690	0.24990	0.01203
9	470	0.00460	-0.5112	42.89550	0.25470	0.01234
10	470	0.00460	-0.5112	42.89550	0.25470	0.01234

For the three-generator system, the power losses coefficients are defined below [17]:

$$\mathbf{B}_2 = 0.0001 \times \begin{bmatrix} 0.71 & 0.30 & 0.25 \\ 0.30 & 0.69 & 0.32 \\ 0.25 & 0.32 & 0.80 \end{bmatrix}, \tag{17}$$
$$\mathbf{B}_1 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix},$$
$$B_0 = 0.$$

5.2 Ten-generator system

The information of this test system was obtained from [22] and is reported in Table 2.

For the ten-generator system, the power losses coefficients are defined below [22]:

$$\mathbf{B}_2 = 0.0001 \times \begin{bmatrix} 0.49 & 0.14 & 0.15 & 0.15 & 0.16 & 0.17 & 0.17 & 0.18 & 0.19 & 0.20 \\ 0.14 & 0.45 & 0.16 & 0.16 & 0.17 & 0.15 & 0.15 & 0.16 & 0.18 & 0.18 \\ 0.15 & 0.16 & 0.39 & 0.10 & 0.12 & 0.12 & 0.14 & 0.14 & 0.16 & 0.16 \\ 0.15 & 0.16 & 0.10 & 0.40 & 0.14 & 0.10 & 0.11 & 0.12 & 0.14 & 0.15 \\ 0.16 & 0.17 & 0.12 & 0.14 & 0.35 & 0.11 & 0.13 & 0.13 & 0.15 & 0.16 \\ 0.17 & 0.15 & 0.12 & 0.10 & 0.11 & 0.36 & 0.12 & 0.12 & 0.14 & 0.15 \\ 0.17 & 0.15 & 0.14 & 0.11 & 0.13 & 0.12 & 0.38 & 0.16 & 0.16 & 0.18 \\ 0.18 & 0.16 & 0.14 & 0.12 & 0.13 & 0.12 & 0.16 & 0.40 & 0.15 & 0.16 \\ 0.19 & 0.18 & 0.16 & 0.14 & 0.15 & 0.14 & 0.16 & 0.15 & 0.42 & 0.19 \\ 0.20 & 0.18 & 0.16 & 0.15 & 0.16 & 0.15 & 0.18 & 0.16 & 0.19 & 0.44 \end{bmatrix}, \tag{18}$$
$$\mathbf{B}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$B_0 = 0.$$

6 Numerical validations

For our computational implementation, the Julia software, version 1.9.2 [23], was used on a PC with an AMD Ryzen 7 3700 2.3 GHz processor and 16.0 GB RAM running a 64-bit version of Microsoft Windows 10 Single Language.

To demonstrate the effectiveness of the proposed VSA approach in dealing with the studied problem, the following cases were simulated:

i. The construction of the Pareto front construction via the VSA, varying the ω−coefficient from 0 to 1 in steps of about 0.01. The results were compared against those obtained by implementing the exact optimization model in the interior-point optimizer tool `Ipopt` of Julia's `JuMP` environment [23], [24].

ii. A comparative analysis of the best compromise solutions provided by the VSA and some classical multi-objective metaheuristic optimizers, *i.e.*, such as the enhanced multi-objective cultural algorithm (EMOCA) [22], the multi-objective differential evolution (MODE), and the non-dominated sorting genetic algorithm II (NSGA II) [25].

6.1 Multi-objective analysis

This section compares the Pareto fronts obtained with the proposed weighting-based vortex search algorithm (WbVSA) approach and `Ipopt` in both test systems.

6.1.1 Results for the three-generator system

Figure 1 presents a comparison of the Pareto fronts for the three-generator system. Note that both fronts were built using the same weighting-based optimization approach, wherein the maximum values for $G_c^{\max}$ and $G_e^{\max}$ were set as 20844.6841 USD/h and 206.3589 lb/h, respectively.

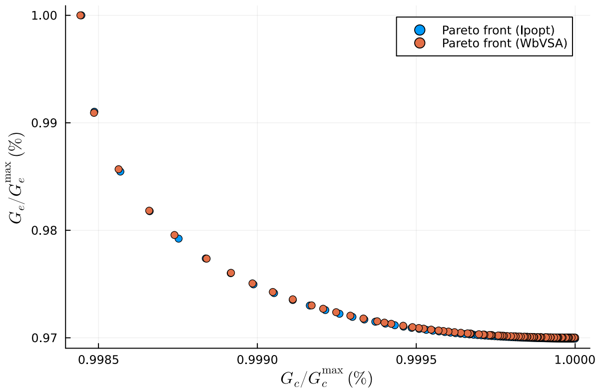

Figure 1: Pareto fronts for the three-generator, generated with the WbVSA and the `Ipopt` solver

As seen in Figure 1, both Pareto fronts are overlapped, which implies that, if the interior-point optimizer is assumed to be the exact model, the proposed VSA approach also finds the exact numerical solution. This is an essential result since, when considering absolute-value functions, there is the possibility that the `Ipopt` optimizer gets stuck in a local optimum.

The extremes of the Pareto front (Figure 1) $p_1 = (20844.6841, 200.1534)$ and $p_2 = (20812.2935, 206.3589)$. In addition, note the following:

i. The extreme point p_1 is reached when the ω−coefficient is set as 0. This implies that objective function (8) does minimize the expected CO_2 emissions, finding a value of 200.1534 lb/h. On the other hand, the extreme point p_2 is reached when the ω−coefficient is set as 1, which minimizes the total generation costs, yielding 20812.2935 USD/h.

ii. Comparing the extreme points p_1 and p_2 shows maximum variations of 32.3906 USD/h and 6.2055 lb/h in the objective functions. These values imply that, if the power system operator arbitrarily selects a point of the Pareto front, the expected generation costs and greenhouse emissions will vary between 20844.6841 to 20812.2935 USD/h and between 200.1534 to 206.3589 lb/h.

iii. Figure 1 presents the results in percentage form, showing maximum reductions of about 0.1534% and 3.0071% in the expected generation costs when minimizing CO_2 emissions.

6.1.2 Results for the ten-generator system

Figure 2 presents the Pareto fronts obtained with the WbVSA and Ipopt. It is important to remember that both Pareto fronts were built using the same weighting-based optimization approach, and that the maximum values for $G_c^{\max}$ and $G_e^{\max}$ were set as 116412.4422 USD/h and 4572.1949 lb/h, respectively.

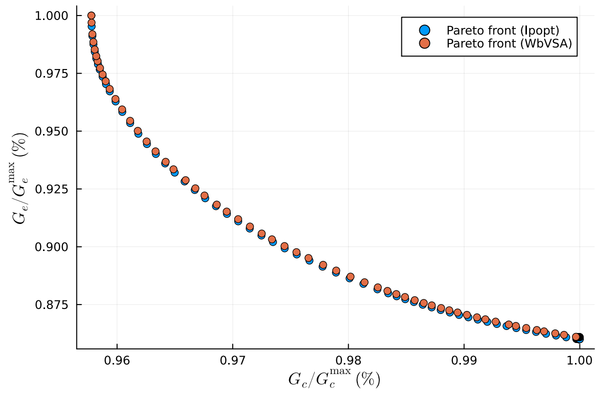

Figure 2: Pareto fronts for the ten-generator system, as obtained with the WbVSA and the Ipopt solver

As with the three-generator system, the WbVSA approach reached a very similar profile compared to the interior-point solution (Figure 2).

Note that the extremes of the Pareto front in Figure 2 $p_1 = (116412.4422, 3932.2432)$ and $p_2 = (111497.6314, 4572.1949)$. Also note that:

i. The extreme point p_1 is reached when the $\omega-$coefficient is set as 0, which implies that objective function (8) does minimize the expected CO_2 emissions, finding a value of 3932.2432 lb/h. On the other hand, the extreme point p_2 is reached when the $\omega-$coefficient is set as 1, which reduces the total generation costs to 111497.6314 USD/h.
ii. By comparing the extreme points p_1 and p_2, it can be seen that the maximum variations in the objective functions correspond to 4914.8107 USD/h and 639.9519 lb/h. These values imply that, if the power system operator arbitrarily selects a point of the Pareto front, the expected generation costs and greenhouse emissions will vary between 116412.442 USD/h and 111497.6314 USD/h and between 4572.1949 lb/h and 3932.2432 lb/h, respectively.
iii. Figure 1 presents the results in percentage form, showing maximum reductions of about 4.2219% and 13.9966% in the expected generation costs when minimizing CO_2 emissions.

6.2 Single-objective analysis

This section presents a comparative analysis with literature reports for the three- and ten-generator systems.

6.2.1 Results for the three-generator system

For the sake of comparison, the numerical results reported by [17] were employed, These authors proposed a modification of the optimization model, which is shown below:

Objective function:

$$\min T_c = G_c + h_{\text{eq}} G_e, \tag{19}$$

Subject to:

Equations (5) – (7),

where h_{eq} is a conversion factor to determine the equivalent costs of greenhouse gas emissions. For the studied test system, this parameter takes a value of 43.55981 USD/lb.

In this comparative analysis, the genetic algorithm (GA) [26], the particle swarm optimizer (PSO) [26], and the flower pollination algorithm (FPA) [17] were considered. The numerical results are listed in Table 3.

Table 3: Comparison of the results obtained for the three-generator system within a single-objective analysis

	Power (MW)			
Plant #	**GA**	**PSO**	**FPA**	**WbVSA**
1	102.6170	102.6120	102.4468	102.4818
2	153.8250	153.8090	153.8341	153.7940
3	151.0110	150.9910	151.1321	151.1370
G_c (USD)	20840.0982	20838.3111	20838.0235	20838.1155
G_e (lb)	200.2572	200.2208	200.2266	200.2245
T_c (USD)	29563.2642	29559.8915	29559.8582	29559.8572
P_{loss}(MW)	7.4132	7.4117	7.4126	7.4128

These results show that:

i. The combined objective function T_c reaches its lowest value with the VSA when compared to the other metaheuristic optimizers. However, the actual improvement with respect to the PSO and the FPA is minimal. Hence, the solution reported by the VSA could be the global optimum.
ii. The objective function values reached by all the algorithms with respect to the expected generation costs are between 20383.0235 USD/h and 20840.0982 USD/h, while the expected equivalent CO_2 emissions are between 200.2208 lb/h and 200.2572 lb/h. Note that all the algorithms find similar solutions, which confirms that the equivalent cost function T_c, as stated by [17], adequately represents the CEED problem using a single-objective formulation.
iii. The numerical solutions reached with the analyzed algorithms imply power losses between 7.4117 MW and 7.4132 MW, *i.e.*, they only differ by a few kilowatts. This is a good result, since it confirms that all the algorithms, in spite of the nonlinearities introduced by the quadratic power loss coefficients, are able to adequately estimate network power losses.

6.2.2 Results for the ten-generator system

To confirm the effectiveness of the proposed multi-objective analysis via the VSA, a comparative study with three different optimization methodologies was conducted, considering the best compromise between the two solutions. The algorithms used for comparison were the MODE, NSGA II, and NSGA, as reported by the authors of [25] and [22].

Table 4 lists the power injection of each analyzed method. It is worth highlighting that the solutions corresponding to both the economic and environmental objective functions were evaluated in order to compare them against those of our proposal.

The numerical results in Table 4 show that:

i. Regarding power losses, all the analyzed algorithms report solutions between 83.5571 MW and 84.3271 MW. This confirms that all the solutions are in a closed loop, with a radius lower than 1 MW in terms of transmission power losses.
ii. The proposed WbVSA obtains the lowest objective function value when evaluating the expected power generation costs, *i.e.*, about 113,462.0840 USD/h, approximately 15.5475 USD/h cheaper than the MODE.

Table 4: Comparison of the results obtained for the ten-generator system

	Power (MW)			
Plant #	EMOCA	MODE	NSGA II	WbVSA
1	55.0000	54.9487	51.9515	54.9993
2	80.0000	74.5821	67.2584	79.9959
3	83.5594	79.4294	73.6879	84.8624
4	84.6031	80.6875	91.3554	83.5182
5	146.5632	136.8551	134.0522	142.9718
6	169.2481	172.6393	174.9504	163.3437
7	300.0000	283.8233	289.4350	299.7449
8	317.3496	316.3407	314.0556	315.1003
9	412.9183	448.5923	455.6978	428.6412
10	434.3133	436.4287	431.8054	430.6671
G_c (USD)	113,713.9982	113,477.6315	113,542.8896	113,462.0840
G_e (lb)	4,088.0577	4,124.8642	4,150.9830	4,110.0153
P_{loss}(MW)	83.5571	84.3271	84.2495	83.8446

iii. The best solution regarding the expected CO_2 emissions was reported by the EMOCA, with a value of about $4,088.0577$ lb/h, followed by the WbVSA with approximately $4,110.0153$. This corresponds to a difference of 21.9576 lb/h.

7 Conclusions and future work

This work studied the combined economic-environmental dispatch problem in thermal power plants, considering the nonlinearities introduced by turbine valves on the objective function values and the network power losses by means of quadratic approximation. The multi-objective optimization problem was transformed into a single-objective one using a weighting-based equivalent formulation, which allowed constructing the Pareto front by varying the $\omega-$coefficient between 0 to 1 in steps predefined the power system analyzer. The solution of the equivalent optimization problem was addressed by applying the vortex search algorithm in two power systems composed of three and ten generation plants. The numerical results allow making the following remarks:

i. The use of the WbVSA to construct the optimal Pareto front and the solution of the exact optimization model via the interior-point optimizer `Ipopt` of the Julia software were compared, confirming the effectiveness of the former in dealing with the CEED problem, with an accurate performance compared to exact optimization algorithms.

ii. A single-objective analysis using the equivalent costs function in the three-generator system demonstrated that the VSA approach is an effective technique to obtain the optimal solution to the CEED problem when compared against the GA, the PSO, and the FPA. In addition, a comparative analysis with additional multi-objective optimizers reported in the literature for the ten-generator system confirmed the compromise between both objective functions: improving one of them worsens the other, and *vice versa*.

In future works, the following aspects could be studied:

- The application of our VSA approach to the multi-objective operation of distributed energy resources in electrical networks, incorporating uncertainties in renewable energy generation and demand data during daily operating scenarios;
- The evaluation of the multi-period economic- environmental dispatch problem while considering renewable energy resources and associated uncertainties;
- The implementation of emerging metaheuristic methods to construct Pareto fronts via the weighting-based approach or the ϵ-constraint, comparing the results against those of our proposal under uncertainty conditions.

Conflict of interest statement

The authors declare no conflict of interest with regard to this contribution.

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Contributions

Conceptualization, methodology, software, and writing (review and editing): O.D.M., W.G.-G., and E.R.-T.. All authors have read and agreed to the published version of the manuscript.

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