Revista Ciencia en Desarrollo. Vol. 16 Num. 2. pp. 53 - 58
julio - diciembre

DOI: 10.19053/uptc.01217488.v16.n2.2025.19300

Artículo de investigación

Alternative way to calculate magnetostatic force

Una forma alternativa de calcular la fuerza magnetostática

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Recepción: 3-mayo-2025 **Aceptado:** 15-junio-2025 **Publicado:** 20-julio-2025
Cómo citar:Díaz, Z. K., Rasero Causil, D. A., & Portacio, A. A. (2025). Una Forma Alternativa de Calcular la Fuerza Magnetostática. *Ciencia En Desarrollo*, 16(2). doi: 10.19053/uptc.01217488.v16.n2.2025.19300

Abstract

In this paper, the traditional expression for the magnetostatic force exerted on a charge q moving at velocity $\vec{v}$ in the presence of a magnetic induction field $\vec{B}$, $\vec{F}_M = q\vec{v} \times \vec{B}$, was rewritten in terms of a conservative component and a not conservative. The conservative component allows us to define a possible magnetostatic potential energy function, in such a way that the resulting expression for the magnetic force contains a term similar to that of the electrostatic force equation, $\vec{F}_E = q\vec{E} = -\nabla U_E$, where $\vec{E}$ is the electric field, ∇ is the gradient operator and U_E is the electrostatic potential energy. Additionally, this expression was applied to some simple systems and the results obtained were compared with the traditional equation.

Keywords: Magnetostatic force, Magnetostatic potential energy, Conservative component and a not conservative.

Resumen

En este trabajo, la expresión tradicional para la fuerza magnetostática ejercida sobre una carga q que se mueve a velocidad $\vec{v}$ en presencia de un campo de inducción magnética $\vec{B}$, $\vec{F}_M = q\vec{v} \times \vec{B}$, se reescribió en términos de una componente conservativa y otra no conservativa. La componente conservativa nos permite definir una posible función de energía potencial magnetostática, de tal forma que la expresión resultante para la fuerza magnética contiene un término similar al de la ecuación de fuerza electrostática, $\vec{F}_E = q\vec{E} = -\nabla U_E$, donde $\vec{E}$ es el campo eléctrico, ∇ es el operador gradiente y U_E es la energía potencial electrostática. Además, esta expresión se aplicó a algunos sistemas sencillos y los resultados obtenidos se compararon con la ecuación tradicional.

Palabras Clave: Fuerza magnetostática, Energía potencial megnetostática, Componente conservativa y no conservativa.

E-ISSN: 2462-7658

1 Introduction

In the context of electromagnetism [1, 2, 3, 4, 5], electric charges that are at rest experience attractive or repulsive electric forces. However, when they are in motion, additional forces perpendicular to the motion emerge, which are referred to as magnetic forces. Consequently, the force exerted on a charge q moving at velocity $\vec{v}$ has an electric component that is independent of its motion, and another magnetic component that depends on its motion. The total force is given by the following equation

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}). \tag{1}$$

where $\vec{E}$ is the electric field and $\vec{B}$ is the magnetic field.
In the electrostatic case, the force is given by the expression $\vec{F} = \vec{F}_E = q\vec{E}$. This expression does not include the magnetic component. The electrostatic fields are conservative because they are associated with an electrostatic potential energy U_E. Consequently, the electrostatic force can be expressed as $\vec{F}_E = -\nabla U_E$, where ∇ represents the gradient operator, and therefore its rotational is null.
In the magnetostatic case, the opposite is true. The magnetic component of the force is given by $\vec{F} = \vec{F}_M = q\vec{v}\times\vec{B}$. However, in contrast to the electrostatic force, the magnetic force cannot be expressed in terms of a potential energy function U_M. In other words, it is not possible to express the magnetic force in the form of a gradient of a magnetic potential energy function, $\vec{F}_M = -\nabla U_M$, as is possible in the case of the electrostatic force [2, 3, 4, 5, 6, 7, 8]. This characteristic of the magnetostatic field is emphasized in undergraduate courses by stating that the magnetic field is associated with a non-conservative force, as its rotational component is not zero.

In this article, we found an expression that, to the best of our knowledge, does not appear in the texts of electromagnetism [2, 3, 6, 7, 8, 5, 9, 4, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. In this expression, the magnetostatic force is written as a combination of two terms: (a) a component that has the structure of a conservative force and (b) another component that would provide the non-conservative character. From the conservative term, one could propose the existence of a potential energy function U_M. Although the magnetostatic force is, in reality, a non-conservative force, it is possible to derive such an expression and provide an appropriate interpretation of the function U_M.

The way of calculating $\vec{F}_M$, proposed in this paper, can be implemented without difficulty in the classroom since it involves known mathematical operations (products of vectors, gradient, divergence, rotational, etc.) by a student of an intermediate course of electromagnetic theory.

The article is organized as follows: in the section 2 the theoretical development is presented, which allows the rewriting of the expression of the magnetostatic force. In section 3, the theoretical results are analyzed and their application to the solution of simple problems is illustrated. Finally, the conclusions are presented in the section 4.

2 Theoretical Methods

According to the potential theory [4, 20], it is established that if a force, $\vec{F}$, can be written as

$$\vec{F} = -\nabla U, \tag{2}$$

where U is a scalar potential function, then,

$$\nabla \times \vec{F} = 0, \tag{3}$$

and, therefore, the force is said to be irrotational.
If, in addition, the work done by the force on a closed path is zero, that is,

$$\oint \vec{F} \cdot d\vec{r} = 0, \tag{4}$$

then this force is conservative.
It should be noted that if (2) is satisfied, so will (3), and vice versa, and as a consequence, (4) will be satisfied. However, the fulfillment of (4) does not imply the other two. For example, the magnetic force satisfies (4) because the force vector is always perpendicular to the velocity vector, but it does not satisfy the other constraints.
Therefore, the condition (2), or equivalently (3), is fundamental. In the theory of differential forms, these are guaranteed when Poincaré's Lemma is applied to 1-forms, as stated in reference [20], pages 329-341.

In electrostatics, the conservative nature of the Coulomb force implies that the force exerted on a point charge q is of the form

$$\vec{F}_E = q\vec{E} = -\nabla U_E, \tag{5}$$

where $\vec{E}$ is an electric field, U_E is the electrostatic potential energy, and ∇ is the gradient operator.
In equation (5), the fact that the electric field can be expressed in terms of the electrostatic potential, as indicated by the equation $\vec{E} = -\nabla\phi_E$, and that the electrostatic potential is equal to the charge times the electric potential, as indicated by the equation $\phi_E = qU_E$, is utilized.

In this article it is proposed that in magnetostatics, starting from the expression of the force exerted on a point charge q moving with constant velocity $\vec{v}$, in the presence of a magnetostatic induction field $\vec{B}$,

$$\vec{F}_M = q\vec{v} \times \vec{B}, \tag{6}$$

an equation similar to (5) can be found, i.e,

$$\vec{F}_M = -\nabla U_M, \tag{7}$$

where U_M could be interpreted as a potential energy function. For this purpose, the expression of $\vec{B}$ in terms of the magnetic vector potential $\vec{A}$ is used,

$$\vec{B} = \nabla \times \vec{A}, \tag{8}$$

and replace it in the equation (6). Therefore

$$\vec{F}_M = q\vec{v} \times \vec{B} = q\vec{v} \times (\nabla \times \vec{A}). \tag{9}$$

Using the vector identity,

$$\nabla(\vec{a}\cdot\vec{b}) = \vec{a}\times(\nabla\times\vec{b}) + \vec{b}\times(\nabla\times\vec{a}) + (\vec{a}\cdot\nabla)\vec{b} + (\vec{b}\cdot\nabla)\vec{a},$$

is obtained,

$$\vec{b}\times(\nabla\times\vec{a}) = \nabla(\vec{a}\cdot\vec{b}) - \vec{a}\times(\nabla\times\vec{b}) - (\vec{a}\cdot\nabla)\vec{b} - (\vec{b}\cdot\nabla)\vec{a}.$$

From which, if $\vec{a} = \vec{A}$ and $\vec{b} = q\vec{v}$, the equation (9) can be written as

$$\begin{aligned}\vec{F}_M &= q\vec{v}\times(\nabla\times\vec{A}),\\ &= \nabla(\vec{A}\cdot q\vec{v}) - \vec{A}\times(\nabla\times q\vec{v})\\ &\qquad - (\vec{A}\cdot\nabla)q\vec{v} - (q\vec{v}\cdot\nabla)\vec{A},\\ &= \nabla(\vec{A}\cdot q\vec{v}) - (q\vec{v}\cdot\nabla)\vec{A}.\end{aligned} \tag{10}$$

where $\nabla\times q\vec{v} = 0$ and $(\vec{A}\cdot\nabla)q\vec{v} = 0$ because velocity is a local quantity (evaluated at the position of the particle, at some instant in time), not a vector field in space and when operators like ∇, $\nabla\cdot$, $\nabla\times$, are applied, they act on fields (like $\vec{A}$), not on variables associated with particles (like $\vec{v}$).
Equation (10) exhibits a structural similarity to those presented in the texts of Vanderlinde [7], equation (3-35) on page 59, and Rand [21], first line of equation (C.9) on page 336. In these equations, time dependence and the joint action of electric and magnetic fields are considered.
Additionally, in this paper we have assumed that, to maintain the magnetostatic case, the charge q always moves with the same velocity $\vec{v}$, independent of position and time. This implies that there must be an additional mechanical force on it, which maintains this condition; similar to what is assumed in reference [2], page 43 for the electrostatic case, and page 220 for the magnetostatic case.

Furthermore, using another vector identity,

$$\nabla\times(\vec{a}\times\vec{b}) = (\vec{b}\cdot\nabla)\vec{a} - (\vec{a}\cdot\nabla)\vec{b} + \vec{a}(\nabla\cdot\vec{b}) - \vec{b}(\nabla\cdot\vec{a}),$$

is obtained,

$$(\vec{b}\cdot\nabla)\vec{a} = \nabla\times(\vec{a}\times\vec{b}) + (\vec{a}\cdot\nabla)\vec{b} - \vec{a}(\nabla\cdot\vec{b}) + \vec{b}(\nabla\cdot\vec{a}).$$

Thus, with $\vec{a} = \vec{A}$ and $\vec{b} = q\vec{v}$, we obtain,

$$\begin{aligned}q(\vec{v}\cdot\nabla)\vec{A} &= \nabla\times(\vec{A}\times q\vec{v}) + (\vec{A}\cdot\nabla)q\vec{v}\\ &\qquad - \vec{A}(\nabla\cdot q\vec{v}) + q\vec{v}(\nabla\cdot\vec{A}),\\ &= \nabla\times(\vec{A}\times q\vec{v}).\end{aligned} \tag{11}$$

Where, again, the fact that $\vec{v}$ is constant has been used, and that in magnetostatics, $\nabla\cdot\vec{A} = 0$.
Then, replacing (11) in (10), we obtain,

$$\vec{F}_M = \nabla(\vec{A}\cdot q\vec{v}) - \nabla\times(\vec{A}\times q\vec{v}). \tag{12}$$

Which has a similar structure to Helmholtz's theorem [2, 4, 11, 19], which states that a vector field $\vec{\Phi}$ can be written as

$$\vec{\Phi} = -\nabla\Gamma + \nabla\times\vec{\Omega}, \tag{13}$$

where Γ is a scalar potential and $\vec{\Omega}$ is a vector potential. In this case $\Gamma = -\vec{A}\cdot q\vec{v}$ and $\vec{\Omega} = -\vec{A}\times q\vec{v}$. In most textbooks, as well as in intermediate level electromagnetics courses, a chapter (almost always the first one) on vector calculus is included where this theorem is stated. However, it is rarely mentioned or used in later chapters.

3 Results and Discussion

In the electrostatic case, an electrostatic potential energy function, U_E, can only be associated with a conservative force, $\vec{F}_E = -\nabla U_E$, and this corresponds to the amount of reversible work that must be done by an external agent to produce the given charge configuration [2].

However, this is not the case with the magnetostatic force. The magnetostatic force, $\vec{F}_M$, is a non-conservative force, and therefore cannot be written in terms of a potential energy function, U_M. Consequently, $\vec{F}_M \neq -\nabla U_M$. However, according to equation (12),

$$\vec{F}_M = \nabla(\vec{A}\cdot q\vec{v}) - \nabla\times(\vec{A}\times q\vec{v}),$$

we have found that the magnetic force can be decomposed into two terms: $\nabla(\vec{A}\cdot q\vec{v})$ and $-\nabla\times(\vec{A}\times q\vec{v})$. In this section, we will attempt to provide a physical interpretation of these terms. Let us proceed to examine the matter in greater detail.

1. If we define
$$\vec{A}\cdot q\vec{v} = -U_M, \tag{14}$$
then,
$$\vec{F}_M = -\nabla U_M - \nabla\times(\vec{A}\times q\vec{v}). \tag{15}$$
In order for U_M to correspond to a potential energy function, it is necessary to consider the cases in which $\nabla\times(\vec{A}\times q\vec{v}) = 0$,
   - If we consider the case where $\vec{A}\times q\vec{v} = 0$, we can conclude that $\vec{A}$ and $\vec{v}$ must be two parallel vectors. Therefore, $\vec{A}\cdot q\vec{v} = qAv$.
   - If $\vec{A}\times\vec{v} \neq 0$, then the constraint $\nabla\times(\vec{A}\times q\vec{v}) = 0$ implies that there are no vector sources that produce the vector field $\vec{A}\times q\vec{v}$. One could write $\vec{A}\times q\vec{v} = -\nabla\mathcal{U}$, where $\mathcal{U}$ would be a new potential function to be determined. This more complicated case is not considered in this article.

   In these cases, a scalar function U_M can be defined as a possible magnetic potential energy function. It is not, however, a potential energy in the conventional sense, as it is velocity-dependent. Note that it would correspond to an effective magnetic potential energy, which describes the interaction of the particle with the magnetic field. This is evident in the non-relativistic Lagrangian of a charged particle in an electromagnetic field [8, 9, 21]
$$L = \frac{1}{2}mv^2 - q\phi_E + q\vec{v}\cdot\vec{A}. \tag{16}$$

As with the magnetic scalar potential [2, 4, 7, 5], which is defined in the particular cases where $\nabla \times \vec{B} = 0$, the magnetic potential energy function can be considered a useful auxiliary function for solving some magnetostatic problems. However, it should be noted that this potential can only be used for calculations in current-free regions, and in some cases it is not a univalued function, as indicated in problem 5.29 of Griffiths' book [4].

2. The term $-\nabla \times (\vec{A} \times q\vec{v})$ can be interpreted as the component that gives the non-conservative character to the magnetic force.

We emphasize again that, according to classical electromagnetic theory, the physical nature of the magnetic force is non-conservative. Consequently, there must always be a component in the equation (12), namely, $\nabla \times (\vec{A} \times q\vec{v}) \neq 0$. This is demonstrated in the examples 3.1.1 and 3.1.2. However, in the example 3.1.3, we are able to identify a scenario where $\nabla \times (\vec{A} \times q\vec{v}) = 0$ y $\nabla(\vec{A} \cdot q\vec{v}) \neq 0$. This indicates the existence of the scalar function U_M.

3.1 Illustrative Examples

3.1.1 Two Point Charges.

Consider a point charge q' moving at a constant velocity $\vec{v}' = v_0'\hat{z}$ and interacting with another point charge q moving at a constant velocity $\vec{v} = v_0\hat{y}$. The objective is to determine the magnetic force that the charge q' exerts on the charge q. The situation is depicted in figure 1.

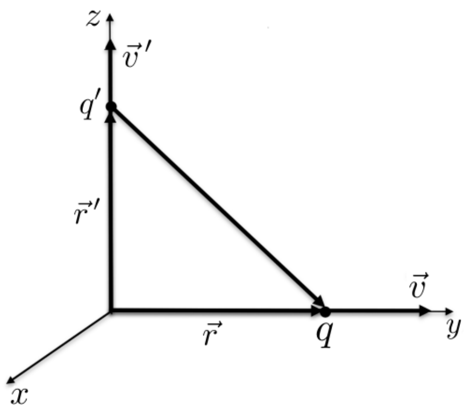

Figure 1: A point charge q is considered to be in motion in the presence of another point charge q'.

The force is initially calculated using the equation

$$\vec{F}_M = q\vec{v} \times \vec{B}, \tag{17}$$

where the field $\vec{B}$ is given by [2]

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{q'v_0'\,y}{(y^2+z'^2)^{\frac{3}{2}}}\hat{x}. \tag{18}$$

According to equation (17), the force exerted by the charge q' on the charge q is

$$\begin{aligned}\vec{F}_M &= q\vec{v} \times \vec{B},\\ &= -\frac{\mu_0}{4\pi}\frac{qq'v_0v_0'\,y}{(y^2+z'^2)^{\frac{3}{2}}}(\hat{y}\times\hat{x}),\\ &= \frac{\mu_0}{4\pi}\frac{qq'v_0v_0'\,y}{(y^2+z'^2)^{\frac{3}{2}}}\hat{z}.\end{aligned} \tag{19}$$

Now the force is calculated using

$$\vec{F}_M = \nabla(\vec{A}\cdot q\vec{v}) - \nabla \times (\vec{A}\times q\vec{v}), \tag{20}$$

where the potential $\vec{A}$ produced by the charge q' is given by [2]

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{q'v_0'}{(y^2+z'^2)^{\frac{1}{2}}}\hat{z}. \tag{21}$$

Consequently,

$$\vec{A}\cdot\vec{v} = \frac{\mu_0}{4\pi}\frac{q'v_0v_0'}{(y^2+z'^2)^{\frac{1}{2}}}(\hat{z}\cdot\hat{y}) = 0, \tag{22}$$

$$\begin{aligned}\vec{A}\times\vec{v} &= \frac{\mu_0}{4\pi}\frac{q'v_0v_0'}{(y^2+z'^2)^{\frac{1}{2}}}(\hat{z}\times\hat{y}),\\ &= -\frac{\mu_0}{4\pi}\frac{q'v_0v_0'}{(y^2+z'^2)^{\frac{1}{2}}}\hat{x},\\ &= \mathcal{G}_x\,\hat{x} + \mathcal{G}_y\,\hat{y} + \mathcal{G}_z\,\hat{z},\end{aligned} \tag{23}$$

where

$$\mathcal{G}_x = -\frac{\mu_0}{4\pi}\frac{q'v_0v_0'}{(y^2+z'^2)^{\frac{1}{2}}}, \quad \mathcal{G}_y = 0, \quad \mathcal{G}_z = 0. \tag{24}$$

Whereupon

$$\begin{aligned}\nabla\times(\vec{A}\times\vec{v}) &= \left(\frac{\partial\mathcal{G}_z}{\partial y} - \frac{\partial\mathcal{G}_y}{\partial z}\right)\hat{x}\\ &\quad + \left(\frac{\partial\mathcal{G}_x}{\partial z} - \frac{\partial\mathcal{G}_z}{\partial y}\right)\hat{y}\\ &\quad + \left(\frac{\partial\mathcal{G}_y}{\partial x} - \frac{\partial\mathcal{G}_x}{\partial y}\right)\hat{z},\\ &= -\frac{\mu_0}{4\pi}\frac{q'v_0v_0'\,y}{(y^2+z'^2)^{\frac{3}{2}}}\hat{z}.\end{aligned} \tag{25}$$

Subsequently, replacing (22) and (25) in equation (20), the following equation is derived:

$$\begin{aligned}\vec{F}_M &= q\,\nabla(\vec{A}\cdot\vec{v}) - q\,\nabla\times(\vec{A}\times\vec{v}),\\ &= \frac{\mu_0}{4\pi}\frac{qq'v_0v_0'\,y}{(y^2+z'^2)^{\frac{3}{2}}}\hat{z}.\end{aligned} \tag{26}$$

This result is in accordance with the outcome of equation (19) derived from the traditional equation.

3.1.2 Point Charge and Current Cylinder – 1.

Consider an infinitely long cylinder with a circular cross-section of radius a and its axis aligned with the z-axis. A constant current I is distributed uniformly throughout the cross-section in the positive z direction. The objective is to determine the magnetic force exerted by this cylinder on a point charge q moving at a constant velocity in the positive x direction, with velocity vector $\vec{v} = v_0\hat{x}$. The situation is depicted in figure 2.

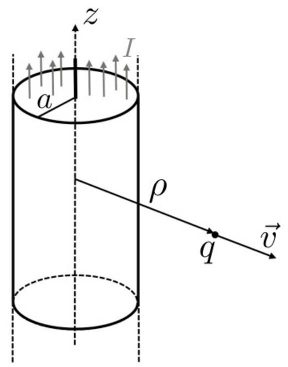

Figure 2: A point charge moving in the presence of an infinitely long current cylinder.

As in the previous example, the force is first calculated using the equation

$$\vec{F}_{\mathrm{M}} = q\vec{v}\times\vec{B}, \tag{27}$$

where the field $\vec{B}$ can be determined through Ampere's law. For points outside the cylinder, where the charge q is moving, the expression

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{\rho}\hat{\phi}, \quad (\rho > a), \tag{28}$$

is found [2, 4], where the cylindrical coordinates ρ and ϕ are used. As indicated by equation (28), for points outside the infinite current cylinder, the field $\vec{B}$ is solely dependent on the perpendicular distance to the principal axis.
Consequently, substituting the force exerted by the current cylinder on the charge q into equation (27) yields

$$\begin{aligned}\vec{F}_{\mathrm{M}} &= q\vec{v}\times\vec{B},\\ &= \frac{\mu_0}{2\pi}\frac{qv_0I}{\rho}(\hat{x}\times\hat{\phi}),\\ &= \frac{\mu_0}{2\pi}\frac{qv_0I}{\rho}\hat{x}\times(-\sin\phi\,\hat{x}+\cos\phi\,\hat{y}),\\ &= \frac{\mu_0}{2\pi}\frac{qv_0I\cos\phi}{\rho}\hat{z},\end{aligned} \tag{29}$$

where the cylindrical unit vector $\hat{\phi}$ was expressed in terms of the cartesian unit vectors $\hat{x}$ and $\hat{y}$.

The calculation is now made using the following formula:

$$\vec{F}_{\mathrm{M}} = \nabla(\vec{A}\cdot q\vec{v}) - \nabla\times(\vec{A}\times q\vec{v}). \tag{30}$$

The vector potential produced by the cylinder, for $\rho > a$, is given by [2],

$$\vec{A} = \frac{\mu_0}{2\pi}I\ln\left(\frac{\rho_0}{\rho}\right)\hat{z}, \tag{31}$$

where ρ_0 is the position at which $\vec{A}$ cancels out. As indicated by equation (31), for points outside the infinite current cylinder, the vector potential $\vec{A}$ depends solely on the distance perpendicular to the principal axis.
Thus, we have

$$\vec{A}\cdot\vec{v} = \frac{\mu_0}{2\pi}v_0I\ln\left(\frac{\rho_0}{\rho}\right)(\hat{z}\cdot\hat{x}) = 0, \tag{32}$$

$$\begin{aligned}\vec{A}\times\vec{v} &= \frac{\mu_0}{2\pi}v_0I\ln\left(\frac{\rho_0}{\rho}\right)(\hat{z}\times\hat{x}),\\ &= \frac{\mu_0}{2\pi}v_0I\ln\left(\frac{\rho_0}{\rho}\right)\hat{y},\\ &= \frac{\mu_0}{2\pi}v_0I\ln\left(\frac{\rho_0}{\rho}\right)(\sin\phi\,\hat{\rho}+\cos\phi\,\hat{\phi}),\\ &= \mathcal{G}_\rho\,\hat{\rho}+\mathcal{G}_\phi\,\hat{\phi}+\mathcal{G}_z\,\hat{z},\end{aligned} \tag{33}$$

where $\hat{y}$ is written in terms of $\hat{\rho}$ and $\hat{\phi}$, and

$$\mathcal{G}_\rho = \frac{\mu_0}{2\pi}v_0I\ln\left(\frac{\rho_0}{\rho}\right)\sin\phi, \tag{34}$$

$$\mathcal{G}_\phi = \frac{\mu_0}{2\pi}v_0I\ln\left(\frac{\rho_0}{\rho}\right)\cos\phi, \tag{35}$$

$$\mathcal{G}_z = 0. \tag{36}$$

Then, expressing the rotational operator in cylindrical coordinates, we have

$$\begin{aligned}\nabla\times(\vec{A}\times\vec{v}) &= \left(\frac{1}{\rho}\frac{\partial\mathcal{G}_z}{\partial\phi} - \frac{\partial\mathcal{G}_\phi}{\partial z}\right)\hat{\rho}\\ &\quad + \left(\frac{\partial\mathcal{G}_\rho}{\partial z} - \frac{\partial\mathcal{G}_z}{\partial\rho}\right)\hat{\phi}\\ &\quad + \left(\frac{1}{\rho}\frac{\partial(\rho\mathcal{G}_\phi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial\mathcal{G}_\rho}{\partial\phi}\right)\hat{z},\\ &= -\frac{\mu_0}{2\pi}\frac{v_0I}{\rho}\cos\phi\,\hat{z}.\end{aligned} \tag{37}$$

Consequently, by substituting the values of equation (32) and equation (37) into equation (30), the result

$$\begin{aligned}\vec{F}_{\mathrm{M}} &= q\,\nabla(\vec{A}\cdot\vec{v}) - q\,\nabla\times(\vec{A}\times\vec{v}),\\ &= \frac{\mu_0}{2\pi}\frac{qv_0I\cos\phi}{\rho}\hat{z}.\end{aligned} \tag{38}$$

is obtained, which is in agreement with the outcome of equation (29), which was derived using the traditional equation.

3.1.3 Point Charge and Current Cylinder – 2

As a modification of the example in section 3.1.2, let us consider that the charge q moves at a constant velocity in the positive direction of the z-axis, that is, $\vec{v} = v_0\hat{z}$.

In this case, due to the symmetry of the infinite current cylinder, the field $\vec{B}$ is still given by the equation (28), so that the traditional equation for the calculation of the magnetic force leads to

$$\begin{aligned}\vec{F}_M &= q\vec{v}\times\vec{B},\\ &= \frac{\mu_0}{2\pi}\frac{qv_0I}{\rho}(\hat{z}\times\hat{\phi}),\\ &= -\frac{\mu_0}{2\pi}\frac{qv_0I}{\rho}\hat{\rho}.\end{aligned} \tag{39}$$

In addition, to perform the calculation via the equation $\vec{F}_{\mathrm{M}} = \nabla(\vec{A}\cdot q\vec{v}) - \nabla\times(\vec{A}\times q\vec{v})$ is still used (31), so that,

$$\begin{aligned}\vec{A}\cdot\vec{v} &= \frac{\mu_0}{2\pi}v_0 I\ln\left(\frac{\rho_0}{\rho}\right)(\hat{z}\cdot\hat{z}),\\ &= \frac{\mu_0}{2\pi}v_0 I\ln\left(\frac{\rho_0}{\rho}\right),\end{aligned} \tag{40}$$

$$\vec{A}\times\vec{v} = \frac{\mu_0}{2\pi}v_0 I\ln\left(\frac{\rho_0}{\rho}\right)(\hat{z}\times\hat{z}) = 0. \tag{41}$$

Then, expressing the gradient in cylindrical coordinates,

$$\begin{aligned}\nabla(\vec{A}\cdot\vec{v}) &= \left(\hat{\rho}\frac{\partial}{\partial\rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial}{\partial\phi}\right.\\ &\qquad\left. + \hat{z}\frac{\partial}{\partial z}\right)\frac{\mu_0}{2\pi}v_0 I\ln\left(\frac{\rho_0}{\rho}\right)\\ &= -\frac{\mu_0}{2\pi}\frac{v_0 I}{\rho}\hat{\rho}.\end{aligned} \tag{42}$$

Therefore, replacing (41) and (42) in the equation (30), we obtain

$$\begin{aligned}\vec{F}_M &= q\,\nabla(\vec{A}\cdot\vec{v}) - q\,\nabla\times(\vec{A}\times\vec{v})\\ &= -\frac{\mu_0}{2\pi}\frac{q v_0 I}{\rho}\hat{\rho}.\end{aligned} \tag{43}$$

Which coincides with (39) and shows that for this problem, according to (14), the function

$$U_M = -\vec{A}\cdot q\vec{v} = -\frac{\mu_0}{2\pi}q v_0 I\ln\left(\frac{\rho_0}{\rho}\right), \tag{44}$$

can be selected as a potential energy function.

4 Conclusions

In this article, it was demonstrated that a magnetic force can be expressed in terms of two distinct quantities: a conservative component and a non-conservative component. The conservative component can be expressed in terms of a scalar function that acts as potential energy. Furthermore, it is demonstrated that the derived equation is suitable for the resolution of magnetostatic problems.

Acknowledgments

The authors thank their respective institutions (Z. K. Díaz and D. A. Rasero: Universidad Surcolombiana, A. A. Portacio: Universidad de lo Llanos) for the support provided during the execution of this work.

Contributions

Z. K. Díaz: Conceptualization, Formal analysis, Writing – original draft, Writing – review & editing. D. A. Rasero: Conceptualization, Formal analysis, Writing – original draft, Writing – review & editing. A. A. Portacio: Conceptualization, Formal analysis, Writing – original draft, Writing – review & editing.

Conflict of interest statement

The authors declare that they have no conflicts of interest.

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