HORIZONTAL PERMEABILITY DETERMINATION FROM THE ELLIPTICAL FLOW REGIME OF HORIZONTAL WELLS

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The technological development of the oil industry around the globe has resulted in an increase in drilling horizontal wells due to their great efficiency to produce higher amount of oil per unit pressure drawdown. For this reason, it is fundamental to properly identify, evaluate and model the pressure behavior for this type of wells.

The current techniques for interpretation of pressure transient tests in horizontal wells include conventional methods (semilog analysis and Cartesian plot of pressure vs. the square root of time) and semilog and log-log type-curve matching analysis. Defining the accurate starting and ending times of the different flow regimes is a drawback of the conventional technique. Type-curve matching requires all flow regimes to be present. Otherwise, type-curve matching will provide non-unique answers. An additional difficulty in conducting horizontal well interpretation may be due to the absence of some of the flow regimes.

A technique to interpret horizontal well pressure data eliminating type-curve matching was introduced in 1996 by Engler and Tiab. In that study, however, the elliptical-flow regime was not included. This flow regime has been recognized as an important aspect in horizontal well testing and some research has been devoted to that issue 2,8. It is characterized by a 0,36-slope line on the pressure derivative log-log plot and its governing equation has been already presented in the literature. In this paper, the elliptical-flow regime is used to develop analytical equations to obtain horizontal permeability anisotropy. The intersection points of the elliptical-flow regime with early-linear, early-radial, late-linear and/or late-linear flow regimes have also been used to find new analytical expressions to verify the horizontal permeability or to find the permeability in the y-direction. The proposed methodology was verified successfully by means of the analysis of two examples reported in the literature.

Keywords: *anisotropy, radial flow, pseudoradial flow, linear flow, permeability, intersection points, characteristic lines.*

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El desarrollo tecnológico de la industria petrolera alrededor del mundo ha resultado en un aumento en la perforación de pozos horizontales debido a su gran eficiencia para producir mayor cantidad de petróleo por presión por unidad hasta el agotamiento. Por esta razón es fundamenta identificar, evaluar y modelar correctamente el comportamiento de la presión en este tipo de pozos.

Las técnicas actuales de la interpretación de la prueba de presión transitoria en pozos horizontales incluyen métodos convencionales (análisis gráficos de presión semilogaritmicos y cartesianos versus la raíz cuadrada del tiempo) y curvas tipos semilogaritmicas y logarítmicas para análisis de coincidencia. El tiempo exacto de inicio y finalización de los diferentes regimenes de flujo es una desventaja de la técnica convencional. Las curvas tipos requieren que todos los regimenes de flujo estén presentes, de lo contrario, las curvas tipos no suministran una respuesta única. Una dificultad adicional en la realización de la interpretación en un pozo horizontal es la falta de algunos de los regimenes de flujo.

En 1996 Engler y Tiab introdujeron una técnica para interpretar la información de los datos de presión en los pozos horizontales que elimina el cotejo de las curvas tipo. Sin embargo, en ese estudio no se incluyó el regimen de flujo elíptico. Este regimen de flujo ha sido reconocido como un aspecto importante en las pruebas de pozos horizontales y ciertas investigaciones se han dedicado al tema 2,8. Se caracteriza por una línea pendiente de 0,36 en el plano logarítmico de presión derivada y su ecuación determinante ha sido presentada en la literatura. En este trabajo, el regimen de flujo elíptico se utiliza para desarrollar ecuaciones analíticas y obtener anisotropía con permeabilidad horizontal. Los puntos de intersección del regimen de flujo elíptico con regímenes de flujo lineal-temprano, radial-temprano, lineal-tardío y/o flujo lineal-tardío también han sido utilizados para encontrar nuevas expresiones analíticas para verificar la permeabilidad horizontal o para encontrar permeabilidad en la dirección-y. La metodología propuesta fue verificada con éxito por medio del análisis de dos ejemplos reportados en la literatura.

Palabras claves: *anisotropía, flujo radial, flujo pseudoradial, flujo lineal, permeabilidad, puntos de intersección, líneas características.*

NOMENCLATURE

B	Oil formation factor, bbl/STB
c_t	Compressibility, 1/psi
h	Formation thickness, ft
k	Permeability, md
L_w	Horizontal well length, ft
$m(P)$	Pseudopressure function, psi^2/cp
P	Pressure, psi
P_D'	Dimensionless pressure derivative
P_D	Dimensionless pressure
q	Flow rate, bbl/D. For gas reservoirs the units are Mscf/D
r_w	Well radius, ft
S	Skin factor
S_m	Mechanical skin factor
S_x	x-direction pseudoskin factor
S_z	z-direction pseudoskin factor
T	Reservoir temperature, °R
t	Time, hr
$t*\Delta P'$	Pressure derivative function, psi
$t*\Delta m(P)'$	Pseudopressure derivative function, psi^2/cp
t_D	Dimensionless time
Δ	Change, drop
Δt	Flow time, hr
ϕ	Porosity, fraction
μ	Viscosity, cp
1 hr	Time of 1 hr
D	Dimensionless
el	Early linear flow period
ell	Elliptical flow period
er	Early radial flow period
g	Gas
h	Horizontal
i	Intersection
ll	Late linear flow period
pr	Pseudoradial flow period
o	Oil
sc	Standard conditions
t	Total
w	Well
x	x-direction index
y	y-direction index
z	z-direction index

CONVERTION FACTORS

bbl x 1,5899873	E - 01 = m^3
cp x 1,0*	E - 03 = Pa-s
ft x 3,048	E - 01 = m
ft^2 x 9,290 304*	E - 02 = m^2
psi x 6,894757	E + 00 = Kpa

INTRODUCTION

Due to their capability to increase reservoir production, horizontal wells have become very popular in the oil industry during the recent years. Therefore, it is truly important to appropriately interpret and design pressure tests conducted in reservoirs that are drained by horizontal wells.

In 1985, Daviau *et al.,* introduced a technique to analyze pressure tests in horizontal wells including wellbore storage and skin effects. Clonts and Ramey presented an analytical solution to describe the pressure behavior for horizontal wells in anisotropic reservoirs. In 1987, Goode and Thambynayagam obtained an analytical solution for the three-dimensional diffusivity equation in Cartesian coordinates. Ozkan *et al.,* presented new type curves for horizontal well test interpretation. In 1996, Engler and Tiab extended a modern technique for well test interpretation known as “Tiab’s Direct Synthesis Technique” to horizontal wells in an anisotropic porous medium. This method eliminates the use of type-curve matching by utilizing characteristic lines and intersection points found on the pressure and pressure derivative log-log plot to develop analytical equations to estimate reservoir parameters. Their equations were obtained for four characteristic flow regimes: early radial, early linear, pseudoradial and late linear.

Additionally, Issaka *et al.* and Chacon identified a new flow regime, called "Elliptical Flow", during the pressure behavior of a horizontal well. This flow regime corresponds to the transition period between early linear and pseudoradial flow regimes. It is characterized by a 0,36 slope-line on the log-log plot of the pressure derivative. Figure 1 shows the pressure profile for the elliptical flow regime. Figure 2 illustrates the flow regimes developed during a pressure test of a horizontal well.

In this paper, the coordinates of various intersection points, obtained by extrapolating the straight line of slope 0,36 such that it intersects straight lines corresponding to other observed flow regimes, i.e. early radial, early linear, pseudoradial and late linear (Figure 3), are used to characterize horizontal reservoir anisotropy. Equations corresponding to these intersection points have been derived for the purpose of verifying horizontal permeability and estimating k_x and k_y.

MATHEMATICAL FORMULATION

Engler and Tiab adopted the model initially proposed by Goode and Thambynayagam using the reservoir configuration provided in Figure 4 and based on the following assumptions:

1. Reservoir of thickness h_z limited by top and bottom closed boundaries.
2. The reservoir can be considered to be infinite or semi-infinite along the horizontal plane (limited in the *x*-direction).
3. The well is drilled along the maximum permeability direction.
4. The horizontal wellbore lies parallel to the top and bottom boundaries, although off-centered.
5. A single slightly-compressible fluid (oil case) or compressible fluid (for gas case) flows through the porous medium.

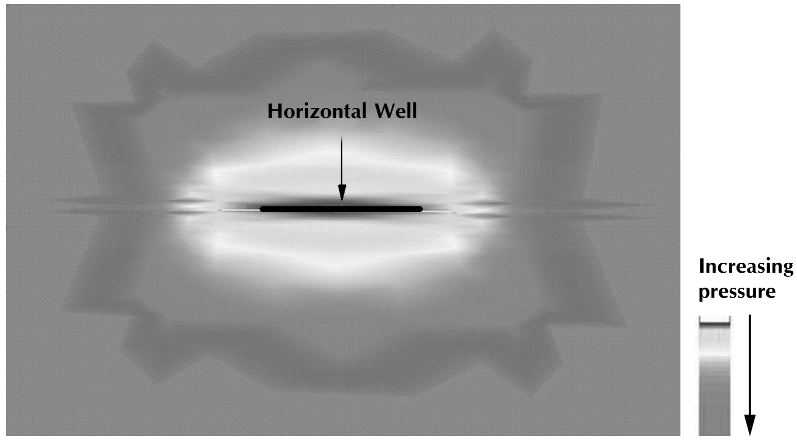

Figure 1. Profile of pressure for a horizontal well during the elliptical flow regime (after Issaka *et al.*)

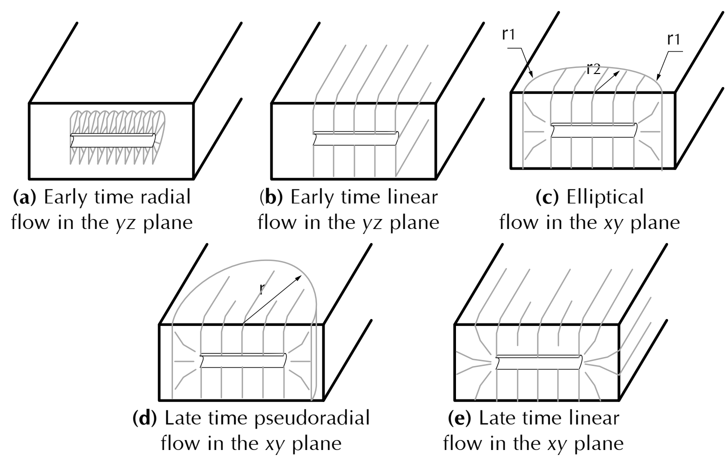

Figure 2. Flow regimes for horizontal wells

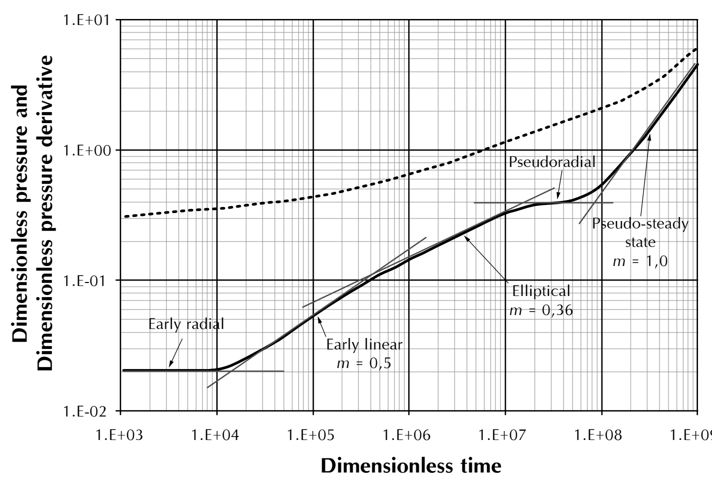

Figure 3. Identification of the early radial, early linear, elliptical and pseudoradial flow regimes on the pressure and pressure derivative plot

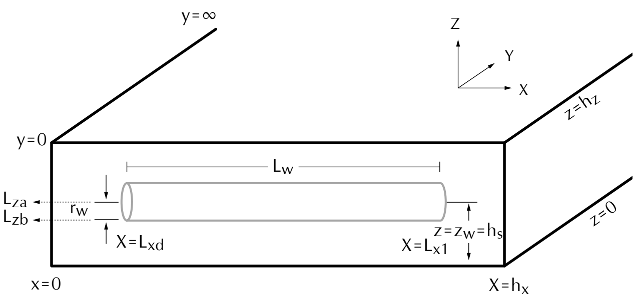

Figure 4. Horizontal well model

Characteristic points and lines

a. Chacon *et. al*., presents the dimensionless pressure and pressure derivative equations for the elliptical flow regime:

$$P_D = \left(\frac{h_x h_z}{L_w r_w}\right)^{0,72} \frac{t_D^{\;0,36}}{985,05 \times \pi^5} + S_{Ell} \qquad (1)$$

$$t_D{*}P_D' = \left(\frac{h_x h_z}{L_w r_w}\right)^{0,72} \frac{t_D^{\;0,36}}{354,62 \times \pi^5} \qquad (2)$$

After substituting the dimensionless quantities into *Equation 2* and solving for the $(k_x k_y)^{0,5}$, it yields:

$$\sqrt{k_x k_y} = \left[\left(\frac{qB\mu^{0,64}}{14930,4\, h_z (\phi c_t)^{0,36} (t{*}\Delta P')_{Ell1hr}}\right) \left(\frac{h_x h_z}{L_w r_w^2}\right)^{0,72}\right]^{\frac{1}{0,64}} \qquad (3)$$

We can also solve for the reservoir length along the x-direction:

$$h_x = \left[\left(\frac{\sqrt{k_x k_y}}{\mu}\right)^{0,64} \left(\frac{14930,4\, h_z (\phi c_t)^{0,36} (t{*}\Delta P')_{Ell1hr}}{qB}\right)\right]^{\frac{1}{0,72}} \frac{L_w r_w^2}{h_z} \qquad (4)$$

Where $(t{*}\Delta P')_{ELL1hr}$ is the pressure derivative value on the elliptical flow line at time of 1 hour.

b. As shown in Figure 3, the straight lines drawn on the pressure derivative plot for the different flow regimes are intersected to each other at specific points. These characteristic intersecting points provide a tool to estimate or verify reservoir parameters. An equation for horizontal permeability results from the intersection point of the early radial line with the elliptical lines is given as:

$$\sqrt{k_x k_y} = \left[\left(\frac{h_x h_z}{L_w r_w^2}\right)^{0,72} \left(\frac{t_{i,er\text{-}ell}}{\phi c_t \mu}\right)^{0,36} \times \frac{L_w \sqrt{k_x k_y}}{3444,45 \times \pi^5 \times h_z}\right]^{\frac{1}{0,64}} \qquad (5)$$

c. Permeability in the y-direction can be estimated from the intersection point between the early linear and elliptical flow regime:

$$k_y = \left[198,28 \times \pi^5 \left(\frac{t_{i,el\text{-}ell}}{\phi c_t \mu L_w^2}\right)^{0,14} \left(\frac{r_w^2}{h_x h_z}\right)^{0,72} (k_x)^{0,32}\right]^{\frac{1}{0,18}} \qquad (6)$$

d. The intersection point of the elliptical flow regime and the pseudoradial flow regime also provides an expression to estimate horizontal permeability:

$$\sqrt{k_x k_y} = (3444,57 \times \pi^5)^{\frac{1}{0,36}} \left(\frac{\phi c_t \mu}{t_{i,ell\text{-}pr}}\right) \left(\frac{L_w r_w^2}{h_x h_z}\right)^2 \qquad (7)$$

e. The last intersection point corresponds to that of the late linear and elliptical flow regimes. This also provides an equation to estimate the y-direction permeability:

$$k_y = \left[\frac{198,28 \times \pi^5}{h_x} \left(\frac{t_{i,ell\text{-}ll}}{\phi c_t \mu} \right)^{0,14} \left(\frac{L_w r_w^2}{h_x h_z} \right)^{0,72} (k_x)^{0,32} \right]^{\frac{1}{0,18}} \tag{8}$$

f. The skin factor caused by the elliptical flow regime results from dividing the dimensionless pressure equation (*Equation 1*) by the dimensionless pressure derivative. After plugging the dimensionless quantities and solving for the skin factor we obtain:

$$S_{Ell} = \left[\frac{\Delta P'_{Ell}}{(t^*\Delta P')_{Ell}} - 0,36 \right] \frac{1}{6889,12 \times \pi^5} \left(\frac{h_x h_z}{L_w r_w^2} \right)^{0,72} \left(\frac{\sqrt{k_x k_y t_{Ell}}}{\phi c_t \mu} \right)^{0,36} \tag{9}$$

Being ΔP_{Ell} and $(t^*\Delta P')_{Ell}$ are the pressure and pressure derivative values read on the elliptical flow line at any convenient time, t_{Ell}. The total skin factor is the summation of the mechanical skin (S_m), the *x*-direction pseudoskin, the *z*-direction pseudoskin and the skin caused by the elliptical flow regime.

Corresponding equations for gas flow are presented in Appendix A using the same number as for the oil equations, i.e. *Equations 3 - 9*.

Step-by-step procedures

Case 1. All flow regimes are observed (Ideal case)

The diverse combination of line and points provide several ways to estimate unknown parameters:

Step 1 – Plot ΔP and $t^*\Delta P'$ on a log-log paper.

Step 2 – Identify the early linear flow regime, read the pressure derivative at any convenient time on this line and estimate k_y using *Equation 11*.

Step 3 – Use *Equation 3* to estimate the horizontal permeability from the elliptical flow regime.

Step 4 – Read the intersection point of the early radial and early linear lines, $t_{i,er\text{-}el}$, and find k_z using *Equation 22*.

Step 5 – Estimate k_x from the intersection time of the early linear and pseudoradial lines using *Equation 26*.

Step 6 – Verify the horizontal permeability, k_h, by using the intersection of either the early radial flow, $t_{i,er\text{-}ell}$, with the elliptical flow lines (*Equation 5*) or the elliptical with pseudoradial lines, $t_{i,ell\text{-}pr}$, (*Equation 7*).

Step 7 – Estimate k_y using either the intersection point of the early linear with elliptical lines, $t_{i,el\text{-}ell}$, (*Equation 6*) or the intersection point of the elliptical and late linear flow-regime lines, $t_{i,ell\text{-}ell}$, (*Equation 8*).

Step 8 – Determine k_z from the intersection of the early radial and early linear flow regimes, $t_{i,er\text{-}el}$ using *Equation 22*.

Step 9 – Read any point on the late linear flow regime line and estimate the total skin (S_m+S_z+S_x) using *Equation 36*.

Step 10 – Read pressure and pressure derivative values for any convenient point on the elliptical flow regime and find the skin factor caused by elliptical flow using *Equation 9*.

Case 2. Reservoir length in the *x*-direction, h_x, is unkown

The elliptical flow period can be utilized to determine the reservoir length in the *x*-direction even when the pressure transient has not reached this boundary.

Step 1 – Same to step 1 of case 1.

Step 2 – Identify the pseudoradial flow period and find k_h using a pressure derivative value read at a convenient time on this flow regime from *Equation 14*.

Step 3 – From a pressure-derivative value read at any convenient time on the elliptical flow line estimate reservoir length along the *x*-direction, h_x using *Equation 4*.

Step 4 – Find horizontal permeability as done in Step 6 of case 1.

Step 5 – Estimate $k_x k_y$ from the early-radial flow regime using *Equation 5*.

Step 6 – Find k_x, k_y and k_z using *Equation 11*, *Equation 6* and *Equation 22*, respectively.

Step 7 – Identify a convenient point on the early radial flow and estimate the combined skin factor ($S_m + S_z$) using either *Equation 34* or *35*.

Step 8 – Same as Step 10 of case 1.

Step 9 – If any of the linear flow regimes are observed, k_x/k_z can be found from *Equation 30* using the ratio of the pressure derivatives at time of early radial and pseudoradial flow regimes.

APPLICATIONS

Example 1

Goode and Thambynayagam presented an example of a pressure buildup test of a horizontal oil well completed in the center of a semi-infinite anisotropic reservoir. Relevant information is given in Table 1. Determine *x*-, *y*- and *z*-direction permeabilities and skin factor.

Solution

Three well-defined flow regimes are observed in Figure 5: early radial, early linear and elliptical. The pseudoradial flow regime is seen only at the final period of time and it is not well defined. The following information was read from Figure 5:

Table 1. Reservoir and well parameters for examples

Parameter	Value	
	Example 1	Example 2
k_x, md	50	122
k_y, md	100	315
k_z, md	25	12
q (BPD)	3000	5000
μ (cp)	1,5	1,2
B, (bbl/STB)	1,5	1,12
h_s (ft)	30	52
L_w (ft)	1000	2626
ϕ (%)	10	24
c_t (psi^{-1})	3,0x10^{-5}	5,0x10^{-5}
h_z (ft)	60	84
h_x (ft)	+13500	-
r_w (ft)	0,354	0,35
S_m	-1,4	5,0

$(t*\Delta P')_{ell\backslash hr} = 22,99$ psi $t_{i,er-el} = 0,18$ hr

$t_{i,er-ell} = 0,087$ hr $t_{i,el-ell} = 1,44$ hr

$t_{i,el-pr} = 28,0$ hr $t_{i,ell-pr} = 82,07$ hr

Estimate $k_x k_y$ using a pressure derivative at time 1 hour from the elliptical flow period with *Equation 3*.

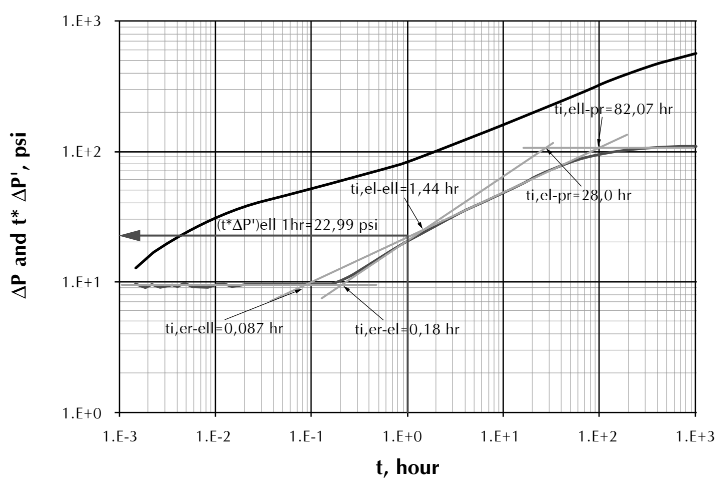

Figure 5. Pressure and pressure derivative log-log plot for example

$$\sqrt{k_x k_y} = \left[\left(\frac{qB\mu^{0,64}}{14930{,}4\, h_z (\phi c_t)^{0,36} (t^*\Delta P')_{Ell1hr}} \right) \left(\frac{h_x h_z}{L_w r_w^2} \right)^{0,72} \right]^{\frac{1}{0,64}}$$

$$k_x k_y = \left[\left(\frac{3000 \times 1{,}5 \times 1{,}5^{\,0,64}}{14930{,}4 \times 60\,(0{,}1 \times (3{,}0 \times 10^{-5}))^{0,36}\,(22{,}99)} \right) \left(\frac{13500 \times 60}{1000 \times (0{,}354^2)} \right)^{0,72} \right]^{\frac{1}{0,32}} = 5009\ md^2$$

Verify k_h from the intersection times of the elliptical line with either early radial line and pseudoradial line, $t_{i,er\text{-}ell}$ (*Equation 5*) and $t_{i,ell\text{-}pr}$ (*Equation 7*).

$$\sqrt{k_x k_y} = \left[\left(\frac{h_x h_z}{L_w r_w^2} \right)^{0,72} \left(\frac{t_{i,er\text{-}ell}}{\phi c_t \mu} \right)^{0,36} \times \frac{L_w \sqrt{k_x k_y}}{3444{,}45 \times \pi^5 \times h_z} \right]^{\frac{1}{0,64}}$$

$$k_x k_y = \left[\left(\frac{13500 \times 60}{1000 \times (0{,}354^2)} \right)^{0,72} \left(\frac{0{,}0867}{0{,}1 \times (3{,}0 \times 10^{-5}) \times 1{,}5} \right)^{0,36} \times \frac{1000\sqrt{(100 \times 25)}}{3444{,}45 \times \pi^5 \times 60} \right]^{\frac{1}{0,32}} = 5013\ md^2$$

$$\sqrt{k_x k_y} = (3444{,}57 \times \pi^5)^{\frac{1}{0,36}} \left(\frac{\phi c_t \mu}{t_{i,ell\text{-}pr}} \right) \left(\frac{L_w r_w^2}{h_x h_z} \right)^2$$

$$k_x k_y = (3444{,}57 \times \pi^5)^{\frac{1}{0,18}} \left(\frac{0{,}1 \times (3{,}0 \times 10^{-5}) \times 1{,}5}{82{,}066} \right)^2 \left(\frac{1000 \times (0{,}354^2)}{13500 \times 60} \right)^4 = 4973\ md^2$$

Determine k_y from *Equation 6*.

$$k_y = \left[198{,}28 \times \pi^5 \left(\frac{t_{i,el\text{-}ell}}{\phi c_t \mu L_w^2} \right)^{0,14} \left(\frac{r_w^2}{h_x h_z} \right)^{0,72} (k_x)^{0,32} \right]^{\frac{1}{0,18}}$$

$$k_y = \left[198{,}28 \times \pi^5 \left(\frac{1{,}441}{0{,}1 \times (3{,}0 \times 10^{-5}) \times 1{,}5 \times 1000^2} \right)^{0,14} \left(\frac{0{,}354^{\,2}}{13500{*}60} \right)^{0,72} (50)^{0,32} \right]^{\frac{1}{0,18}} = 92{,}51\ md$$

Using an average horizontal permeability value of 70,52 md , k_x is solved from $(k_x k_y)^{0,5}$ to be 53,76 md , k_z is found from *Equation 22*.

$$k_z = 301{,}77\, \phi c_t \mu \frac{h_z^2}{t_{i,er\text{-}el}} = (301{,}77)(0{,}1)(1{,}5)(3{,}0 \times 10^{-5}) \times \frac{60^2}{0{,}18} = 27{,}16\ md$$

k_x can also be determined from *Equation 26*.

$$k_x = \frac{301{,}77\, \phi c_t \mu L_w^2}{t_{i,el\text{-}pr}} = \frac{(301{,}77)(0{,}1)(1{,}5)(3{,}0 \times 10^{-5})(1000^2)}{28} = 48{,}49\ md$$

The mechanical skin factor is found from the early radial flow period using *Equation 33*:

$$S_m = \frac{1}{2}\left[\frac{\Delta p_{er}}{(t^*\Delta P')_{er}} - \ln\left(\frac{\sqrt{k_x k_y t_{er}}}{\phi c_t \mu r_w^2}\right) + 7{,}43\right] = \frac{1}{2}\left[\frac{52{,}34}{9{,}56} - \ln\left(\frac{\sqrt{(100 \times 25)} \times 0{,}1}{0{,}1 \times 1{,}5 \times 0{,}354^2 \times (3{,}0 \times 10^{-5})}\right) + 7{,}43\right]$$

$$S_m = -1{,}55$$

The skin factor from linear flow regime is calculated using *Equation 34*.

$$S_m + S_z = \frac{0{,}029}{h_z}\sqrt{\frac{k_z t_{el}}{\phi c_t \mu}}\left[\frac{\Delta P_{el}}{(t^*\Delta P')_{el}} - 2\right] = \frac{0{,}029}{60}\sqrt{\frac{(25)(0{,}5)}{0{,}1 \times 1{,}5 \times (3{,}0 \times 10^{-5})}}\left[\frac{70{,}18}{15{,}035} - 2\right] = 2{,}15$$

The skin factor caused by the elliptical flow regime is found from *Equation 9*:

$$S_{Ell} = \left[\frac{157{,}91}{47{,}92} - 0{,}36\right] \times \frac{1}{6889{,}12 \times \pi^5} \times \left(\frac{13500 \times 60}{1000 \times 0{,}354^2}\right)^{0{,}72} \left(\frac{\sqrt{(50 \times 100) \times 10}}{0{,}1 \times 1{,}5 \times (3{,}0 \times 10^{-5})}\right)^{0{,}36} = 0{,}69$$

All the estimated values of permeability and skin factor are in a good agreement with the information provided in Table 1.

Example 2

This example was initially presented by Ozkan for a horizontal oil well completed at the center of a semi-infinite anisotropic reservoir. Table 1 shows information of reservoir and well. Find *x*-, *y*- and *z*-direction permeabilities and reservoir length in the *x*-direction.

Solution

The pressure and pressure derivative curves of Figure 6 exhibit four well-defined flow regimes as follows: early radial, early linear, elliptical and pseudosteady state. The early linear flow regime is not seen. From Figure 6, the following information is obtained:

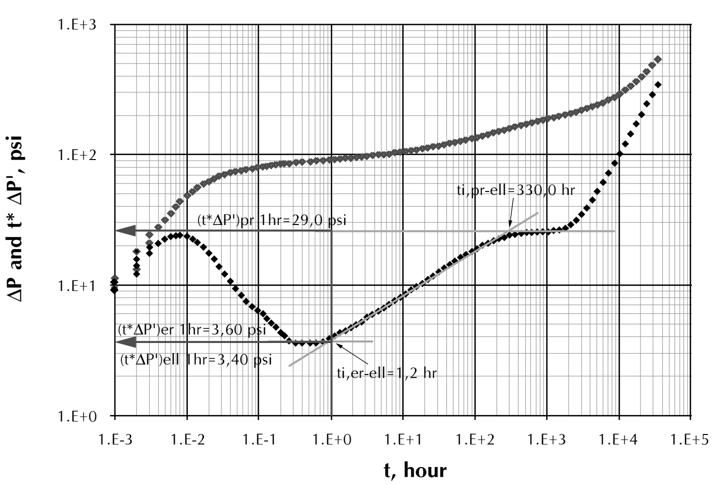

Fig. 6. Pressure and pressure derivative log-log plot for example

$(t^*\Delta P')_{er1h} = 3{,}6$ psi $\quad (t^*\Delta P')_{ell1hr} = 3{,}4$ psi

$(t^*\Delta P')_{pr1hr} = 29{,}0$ psi $\quad t_{i,er\text{-}ell} = 1{,}2$ hr

$t_{i,ell\text{-}pr} = 330{,}0$ hr

k_h is estimated from the pseudoradial flow regime using *Equation 14*.

$$\sqrt{k_x k_y} = \frac{70{,}6\, qB\mu}{h_z\,(t^*\Delta P')_{pr1hr}}$$

$$k_x k_y = \left(\frac{70{,}6 \times 5000 \times 1{,}2 \times 1{,}12}{84 \times 29}\right)^2 = 37931\ md^2$$

h_x can be determined from the elliptical flow using *Equation 4*.

$$h_x = \left[\left(\frac{\sqrt{k_x k_y}}{\mu}\right)^{0,64} \left(\frac{14930{,}4\, h_z(\phi c_t)^{0,36}(t^*\Delta P')_{Ell1hr}}{qB}\right)\right]^{\frac{1}{0,72}} \frac{L_w r_w^2}{h_z}$$

$$h_x = \left[\left(\frac{\sqrt{37931}}{1{,}2}\right)^{0,64} \left(\frac{14930{,}4 \times 84 \times (0{,}24 \times (5 \times 10^{-5})^{0,36} \times 3{,}5}{5000 \times 1{,}12}\right)\right]^{\frac{1}{0,72}} \times \frac{2626 \times 0{,}35^2}{84} = 12801\, ft$$

k_h is verified from the intersection time of the elliptical and pseudoradial lines (*Equation 7*).

$$\sqrt{k_x k_y} = (3444{,}57 \times \pi^5)^{\frac{1}{0,36}} \left(\frac{\phi c_t \mu}{t_{i,ell\text{-}pr}}\right)\left(\frac{L_w r_w^2}{h_x h_z}\right)^2$$

$$k_x k_y = (3444{,}57 \times \pi^5)^{\frac{1}{0,18}} \left(\frac{0{,}24 \times (5 \times 10^{-5}) \times 1{,}2}{330{,}0}\right)^2 \left(\frac{2626 \times (0{,}35^2)}{12801 \times 84}\right)^4 = 44035\ md^2$$

$k_y k_z$ is found from the early radial flow using *Equation 5*.

$$\sqrt{k_x k_y} = \frac{70{,}6\, qB\mu}{L_w(t^*\Delta P')_{er1hr}}$$

$$k_x k_y = \left(\frac{70{,}6 \times 5000 \times 1{,}2 \times 1{,}12}{2626 \times 3{,}6}\right)^2 = 2519\ md^2$$

Again, horizontal permeability is verified from the early radial and elliptical flow regimes:

$$\sqrt{k_x k_y} = \left[\left(\frac{h_x h_z}{L_w r_w^2}\right)^{0,72}\left(\frac{t_{i,er\text{-}ell}}{\phi c_t \mu}\right)^{0,36} \times \frac{L_w \sqrt{k_x k_y}}{3444{,}45 \times \pi^5 \times h_z}\right]^{\frac{1}{0,64}}$$

$$k_x k_y = \left[\left(\frac{12801 \times 84}{2626 \times (0{,}35^2)}\right)^{0,72} \left(\frac{1{,}2}{0{,}24 \times (5 \times 10^{-5}) \times 1{,}2}\right)^{0,36} \times \frac{2626\sqrt{2519}}{3444{,}45 \times \pi^5 \times 84}\right]^{\frac{1}{0,32}} = 42658\ md^2$$

Find k_x/k_z from *Equation 30*,

$$\frac{t^*\Delta P'_{er\text{-}1hr}}{t^*\Delta P'_{pr\text{-}1hr}} = \frac{h_z}{L_w}\sqrt{\frac{k_x}{k_z}}$$

then,

$$\frac{3{,}6}{29} = \frac{84}{2626}\sqrt{\frac{k_x}{k_z}}$$

$k_x/k_z = 15{,}0$

Individual permeabilities cannot be estimated since the linear flow regimes are absent. The average horizontal permeability of 37 931 md^2 was found to be in good agreement with the value of 38 430 md^2 obtained from Table 1. The estimated value of h_x also shows good agreement with the one in Table 1.

CONCLUSIONS

- The existence of the elliptical flow regime is characterized by a straight line of slope 0,36 on the pressure derivative curve. This flow regime occurs between the early linear and pseudoradial flow regimes and is caused by the influence of the ratio of horizontal well length and reservoir length in the *x*-direction.

- The intersection points of the elliptical flow regime straight line with the straight lines corresponding to the other flow regimes observed are used to find new expressions for the determination of horizontal permeability and reservoir length in the *x*-direction, even if the test is not long enough to reach the boundary in that direction.

- Permeability in the *y*-direction, k_y, can be estimated from the intersection point of the early linear and elliptical flow regimes. Then, k_x is easily found.

- The new equations were successfully tested with examples provided in the literature.

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APPENDIX A. GAS RESERVOIRS' EQUATIONS

Elliptical flow regime

$$\sqrt{k_x k_y} = \left[\left(\frac{qT}{1482\, h_z\, (\phi \mu\, c_t)^{0,36} (t^* \Delta m(P)')_{Ell1hr}} \right) \left(\frac{h_x h_z}{L_w r_w^2} \right)^{0,72} \right]^{\frac{1}{0,64}} \tag{3}$$

$$h_x = \left[\left(\sqrt{k_x k_y} \right)^{0,64} \left(\frac{1482\, h_z\, (\phi \mu\, c_t)^{0,36} (t^* \Delta m(P)')_{Ell1hr}}{qT} \right) \right]^{\frac{1}{0,72}} \frac{L_w r_w^2}{h_z} \tag{4}$$

Intersection points

$$\sqrt{k_x k_y} = \left[\left(\frac{h_x h_z}{L_w r_w^2} \right)^{0,72} \left(\frac{t_{i,er\text{-}ell}}{\phi c_t \mu} \right)^{0,36} \times \frac{L_w \sqrt{k_y k_z}}{3442{,}5 \times \pi^5 \times h_z} \right]^{\frac{1}{0,64}} \tag{5}$$

$$k_y = \left[198{,}15 \times \pi^5 \left(\frac{t_{i,el\text{-}ell}}{\phi c_t \mu} \right)^{0,14} \left(\frac{L_w r_w^2}{h_x h_z} \right)^{0,72} \frac{k_x^{0,32}}{L_w} \right]^{\frac{1}{0,18}} \tag{6}$$

$$\sqrt{k_x k_y} = (3442{,}5 \times \pi^5)^{\frac{1}{0,36}} \left(\frac{\phi c_t \mu}{t_{i,ell\text{-}pr}} \right) \left(\frac{L_w r_w^2}{h_x h_z} \right)^2 \tag{7}$$

$$k_y = \left[198{,}15 \times \pi^5 \left(\frac{t_{i,ell\text{-}ll}}{\phi c_t \mu} \right)^{0,14} \left(\frac{L_w r_w^2}{h_x h_z} \right)^{0,72} \frac{k_x^{0,32}}{h_x} \right]^{\frac{1}{0,18}} \tag{8}$$

$$S_{Ell} = \left[\frac{\Delta m(P)_{Ell}}{(t^* \Delta m'(P))_{Ell}} - 0{,}36 \right] \frac{1}{6889{,}12 \times \pi^5} \left(\frac{h_x h_z}{L_w r_w^2} \right)^{0,72} \left(\frac{\sqrt{k_x k_y\, t_{Ell}}}{\phi c_t \mu} \right)^{0,36} \tag{9}$$