DETERMINATION OF VERTICAL AND HORIZONTAL PERMEABILITIES FOR VERTICAL OIL AND GAS WELLS WITH PARTIAL COMPLETION AND PARTIAL PENETRATION USING PRESSURE AND PRESSURE DERIVATIVE PLOTS WITHOUT TYPE-CURVE MATCHING

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It has been long recognized that in some reservoirs the flow does not follow the expected radial cylindrical pattern. Spherical flow may take place in systems with wells completed in thick reservoirs where a short completion interval is open to flow yielding a unique and more complex early-time pressure behavior. Some of the main reasons for partial penetration are to avoid coning of water and minimize sand production. A similar early-time pressure behavior may be due to the presence of plugged perforations. Such well completions are referred to as limited-entry, restricted-entry or partially penetrating wells. A typical case of spherical propagation of pressure transients occurs during the repeat formation tester measurements. Such a test measures spot formation pressures and recovers formation fluid samples for gaining an insight into the reservoir flow mechanics.

The purpose of this study is to identify on the pressure and pressure derivative curves the unique characteristics for different flow regimes resulting from these type of completions and to determine various reservoir parameters, such as vertical, horizontal permeability, and various skin factors. The interpretation is performed using Tiab’s Direct Synthesis (TDS) Technique, introduced by Tiab (1993), which uses analytical equations obtained from characteristic lines and points found on the log-log plot of pressure and pressure derivative to determine permeability, skin and wellbore storage without using type-curve matching. The extension of this methodology for the case under study includes wellbore storage and skin effects. It is applied to both drawdown and buildup tests.

We found that a spherical or hemispherical flow regime occurs prior to the radial flow regime whenever the penetration ratio of about 20%. A half-slope line on the pressure derivative is the unique characteristic identifying the presence of the spherical/hemispherical flow. The typical half-slope line of theses flow regimes is used to estimate spherical permeability and spherical skin values. These parameters are then used to estimate vertical permeability, anisotropy index and skin. Results of TDS technique where successfully compared to those of conventional technique for field and simulated examples.

Keywords: hemispherical flow, spherical flow, radial flow, skin factor, analytical solution, diffusivity equation.

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Desde hace mucho tiempo se sabe que en algunos yacimientos el flujo no sigue la trayectoria cilíndrica esperada. El flujo esférico puede ocurrir en sistemas con pozos completados en formaciones con mucho espesor con una pequeña porción de intervalo perforado abierto al flujo dando lugar a una única y más compleja respuesta de presión temprana. Algunas de las principales razones para acudir a penetración parcial es evitar la conificación de agua y minimizar la producción de arena. Un comportamiento similar de la presión a tiempos tempranos podría deberse al taponamiento en las perforaciones. Tales completamientos se denominan entrada limitada, entrada restringida o pozos con penetración parcial. Un caso típico de propagación esférica del transiente de presión ocurre durante medidas de RFT. Dicho registro mide presiones de formación y muestrea los fluidos de la formación para tener una idea de la mecánica de flujo en el yacimiento.

El propósito de este estudio es identificar características únicas en el gráfico de la presión y la derivada para diversos regímenes de flujo que resultan de este tipo de completamientos y determinar los distintos parámetros del yacimiento. Tales como permeabilidad vertical, permeabilidad horizontal, y diversos factores de daño. La interpretación se lleva a cabo usando la Tiab’s Direct Synthesis (TDS) Technique, introducida por Tiab (1993), la cual usa ecuaciones analíticas obtenidas de líneas y puntos característicos hallados en el gráfico log-log de presión y derivada de presión para determinar permeabilidad, daño y almacenamiento sin emplear curvas tipo. La extensión de esta metodología para el caso en estudio incluye almacenamiento y daño. La técnica se aplica tanto a pruebas de restauración como de declinación de presión.

Encontramos que el flujo hemisférico o esférico toma lugar antes del flujo radial siempre que la relación de penetración sea aproximadamente menor del 20%. Una pendiente negativa de $\frac{1}{2}$ en la curva de la derivada es la característica única para identificar la presencia de flujo hemisférico/esférico. Esta línea típica de pendiente $-\frac{1}{2}$ se usa para determinar la permeabilidad esférica y los daños esféricos, para luego estimar la permeabilidad vertical y el índice de anisotropía. Los resultados de la TDS fueron satisfactoriamente comparados con casos de campo y casos simulados.

**Palabras claves:** flujo hemisférico, flujo esférico, flujo radial, factor de daño, solución analítica, ecuación de difusividad.
DETERMINATION OF VERTICAL AND HORIZONTAL PERMEABILITIES

NOMENCLATURE

\( b \) Fractional penetration ratio, \( h_p/h \), dimensionless
\( B \) Formation volume factor, res vol/stb vol
\( C \) Wellbore storage coefficient
\( C_D \) Dimensionless wellbore storage coefficient
\( c_f \) Rock compressibility, psi-1
\( c_o \) Oil compressibility, psi-1
\( c_t \) Total reservoir compressibility, psi-1
\( G \) Function of the fractional penetration ratio
\( h \) Total formation thickness, ft
\( h_p \) Limited interval open to flow, ft
\( h_D \) Dimensionless total formation thickness
\( I_A \) Anisotropy index, dimensionless
\( k \) Reservoir permeability, md
\( k_H \) Horizontal permeability, md
\( k_V \) Vertical permeability, md
\( k_{sp} \) Spherical permeability, md
\( k_{hs} \) Hemispherical permeability, md
\( k_{r1} \) Horizontal permeability from first radial flow, md
\( k_{r2} \) Horizontal permeability from second radial flow, md
\( m(P) \) Pseudopressure, psi²/cp
\( P \) Pressure, psia
\( P_D \) Dimensionless pressure
\( P_i \) Initial reservoir pressure, psia
\( P_{wf} \) Well flowing pressure, psia
\( q \) Oil rate, STB/D
\( q_g \) Gas rate, Mscf/D
\( r \) Radial distance, ft
\( r_D \) Dimensionless radial distance
\( r_w \) Wellbore radius, ft
\( r_{sw} \) Spherical wellbore radius, ft
\( s_m \) Mechanical or damage skin
\( s_{m\text{c}} \) Mechanical or damage pseudoskin (gas wells)
\( s_c \) Partial penetration-skin
\( s_t \) Total skin
\( t \) Testing time, h
\( T \) Reservoir temperature, °R
\( t_D \) Dimensionless time
**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>$z$</td>
<td>Gas deviation factor</td>
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<tr>
<td>$\rho$</td>
<td>Porosity, fraction</td>
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<td>$\mu$</td>
<td>Oil viscosity, cp</td>
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**SUBSCRIPTS**

<table>
<thead>
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<tr>
<td>$h_s$</td>
<td>Hemispherical flow</td>
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<tr>
<td>$V$</td>
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**SI METRIC CONVERSION FACTOR**

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<tr>
<td>psi x 6.894 757</td>
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- $\lambda$: Interporosity flow parameter

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**INTRODUCTION**

The most common practice to analyze pressure transient data is to assume a radial flow profile. However, in wells with partial penetration/completion a hemispherical/spherical flow is more representative of the system.

In reality the formation itself is usually nonuniform or heterogeneous in properties such as porosity and permeability, both areally and vertically resulting from deposition, folding or faulting. The vertical anisotropy is fundamental in describing pressure response around a well partially penetrating a formation unbounded laterally and confined at the top and bottom by impermeable layers.

In this work the effect of partial well completion and partial well penetration on pressure behavior will be analyzed in order to determine vertical permeability, horizontal permeability, skin and wellbore storage.
knowledge of vertical anisotropy would allow a better reservoir development and subsequent secondary recovery programs could be planned more efficiently.

Spherical symmetries are appropriate in well pressure analyses when the well does not fully penetrate the productive horizon, or perhaps if it were selectively completed. Muskat presented the problem of steady state spherical flow in 1932.

Several papers, although not enough, have been written to discuss spherical flow phenomena but mostly in connection with wireline formation testing tools. Moran and Finklea in 1962 were the first to suggest that a pressure buildup equation based on spherical flow was necessary to correctly analyze pressure data obtained from a wireline formation tester (a limited entry test). The first general discussion of unsteady-state spherical flow appears to be a paper published by Chatas in 1966. In 1974, Culham presented equations suitable for pressure buildup analysis but the wellbore storage distortion was not included.

Raghavan and Clark (1975) examined the applicability of the spherical flow equations given by Moran and Finklea (1962) to a well producing from a limited section of a thick anisotropic formation. Later in 1980, Brigham et al. extended the Chatas (1966) study to include wellbore storage effects. However, their solution is valid only for the case of no wellbore damage. Later, Joseph (1984) and Proett (1998) presented solutions including wellbore storage and damage skin effects. Additional references discussing the partial penetration problem are available from the literature (Brons and Marting, 1961; Abbot et al., 1978). However, their main concern was to express the partial penetration and limited entry as a skin factor not to estimate vertical permeability.

In 1993, Tiab introduced the TDS Technique for interpreting log-log pressure and pressure derivative plots by using analytical solutions to determine permeability, skin and wellbore storage without employing type-curve matching. This revolutionary technique, already extended to many other systems (Boussalem, et al., 2002; Tiab and Escobar, 2003), uses such unique features as lines, intersection points, and other “fingerprint” characteristics found on the pressure and pressure derivative log-log plot to develop analytical equations to readily obtain reservoir parameters. The main objective of this paper is to extend TDS Technique for the case of wells with partial penetration and partial completion.

### FLOW GEOMETRIES

Vertical wells can exhibit different flow regimes during their transient behavior. Spherical flow can occur when a well is producing from a limited section of a thick reservoir or producing from a thick reservoir under a variety of conditions such as the presence of shale barriers spherical flow will also develop. In the case of partial completion in thick reservoirs.

Spherical flow can be visualized as flow along the radius of a sphere. Figure 1 shows the ideal geometry of spherical flow and demonstrates the concept of perfect radial flow towards a common point in the sphere: its center. Hemispherical flow, also shown in Figure 1, is identical to spherical flow with the obvious exception that the flow is contained within a hemisphere.

In practice, the flow is not purely spherical or hemispherical because the completion interval is not a true point sink. However, the flow is spherical in a practical sense if the completion interval is very short relative to the net pay. In the case of a thick reservoir between two impermeable confining layers and a short partial-completion interval, the spherical flow regime will occur between two periods of cylindrical-radial flow. In both cases, three flow periods can be identified -additional to wellbore storage-as follows: A period 1 corresponding to an initial radial flow over the completion interval. Dur-
ing this period the reservoir behaves as if the formation thickness were equal to the length of the open zone. A Period 2 corresponds to a transition period during which spherical/hemispherical flow may be identified. And the third period corresponds to a second radial flow but this time over the total formation thickness.

**MATHEMATICAL FORMULATION**

The fundamental partial differential equation (Muskat, 1932) describing the flow of a slightly compressible fluid in a homogeneous and infinite porous medium characterized by a spherical geometry, can be stated as:

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Phi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Phi}{\partial \chi^2} = \frac{\phi \mu_e \partial P}{k \partial t} \]  

(1)

Several authors (Moran and Finklea, 1962) have presented solutions to *Equation 1* for different flowing conditions, fluid types and boundary conditions. For the purpose of this study, besides the basic considerations normally assumed in well test analysis, Joseph (1984) also assumed the three following considerations:

1) The flow is perfectly spherical to a well of radius \( r_w \) in an isotropic medium, then the terms \( \frac{\partial \Phi}{\partial \theta} \) and \( \frac{\partial \Phi}{\partial \chi} \) can in *Equation 1* be dropped. Therefore, *Equation 1* can be stated as:

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial P}{\partial r} \right) = \frac{\phi \mu_e \partial P}{k \partial t} \]  

(2)

2) Even though Joseph (1984) assumed the medium is spherically isotropic; hence \( k \) in *Equation 2* is the constant spherical permeability. The analysis in systems possessing simple anisotropy (i.e., uniform but unequal horizontal and vertical permeability components) can be also done without significantly affecting the radial coordinate (Joseph, 1984). In that case, \( k \) should be replaced by \( k_{sp} \) in *Equation 2*.

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial P}{\partial r} \right) = \frac{\phi \mu_e \partial P}{k_{sp} \partial t} \]  

(3)

where \( k_{sp} \) is set as a geometric average of the permeability components as follows:

\[ k_{sp} = \sqrt[k_{xx} k_{yy} k_{zz}] \]  

(4)

and assuming an areally isotropic system then:

\[ k_{sp} = \sqrt[k_{xx} k_{yy} k_{zz}] \]  

(5)

3) The physical system of interest considers a sphere itself instead of its center. This region of singularity is called a continuous “spherical sink” which corresponds physically to a wellbore, which must be visualized as a sphere. Hence, the cylindrical wellbore of radius \( r_w \) must be represented by a fictitious spherical wellbore of radius \( r_{sw} \) given by an equation originally suggested by Moran and Finklea (1962):

\[ r_{sw} = \frac{h_p}{2 \ln \left( \frac{h_p}{r_w} \right)} \]  

(6)

\( h_p \) has to be greater or equal than \( r_w \). Under the above assumptions the spherical source solution for long time is given then by:

\[ P_{Dsp} \left( t_{Dsp} \right) = 1 - \frac{1}{\sqrt{\pi t_{Dsp}}} + s \]  

(7)

Where the dimensionless variables suggested by Joseph (1984) for spherical flow are defined in a field units as follows:

\[ P_{Dsp} = \frac{4 \pi k_p r_w}{q \mu} (P_c - P) \]  

(8.a)

\[ t_{Dsp} = \frac{k_p r_w^2}{\phi \mu_e r_w^2} \]  

(9.a)

\[ r_{Dsp} = 1 - \frac{r_w}{r} ; r > r_w \]  

(10.a)

\[ C_{Dsp} = \frac{C}{4 \pi \phi c r_w^3} \]  

(11.a)

and the dimensionless variables normally used in radial coordinates are (Earlougher, 1977):

\[ P_{Dr} = \frac{k_h}{2 \pi q \mu B} \Delta p \]  

(8.b)

\[ t_{Dr} = \frac{k_h t}{\phi \mu_e r_w} \]  

(9.b)

\[ r_D = 1 - \frac{r_w}{r} ; r > r_w \]  

(10.b)

\[ C_{D} = \frac{C}{2 \pi \phi c h r_w^3} \]  

(11.b)
Characteristic points and lines

A log-log plot of dimensionless pressure and pressure derivative versus time is shown in Figure 2. Although, wellbore storage may be present, the three dominant flow regimes identified in a well with partial completion in chronological order are: early radial, spherical and late radial flows (Moncada, 2004).

(1a) Wellbore storage. The first characteristic observed in Figure 3 corresponds to pure wellbore storage flow. The dimensionless wellbore pressure approximation for short time governed by spherical geometry (Joseph, 1984) is given by:

\[ P_D(t_D) = \frac{C_D}{C} \quad s \geq 0; C_D > 0 \]  

Equation 12 is the well known formulation for wellbore depletion which is identical to the results obtained for cylindrical and linear flow. This results in the familiar unit-slope portion on the log-log plot at early times. Combination of Equations 9.a and 11.a will lead to:

\[ \frac{t_D}{C_D} = \left( \frac{5.90 \times 10^{-4} r_s k_w}{\mu} \right) \frac{t}{C} \]  

(13)

Substituting Equation 13 and Equation 8.a into Equation 12 and solving explicitly for \( C \) we obtained:

\[ C = \frac{gB}{24} \left( \frac{t}{\Delta P} \right) \]  

(14)

Where \( t_N \) is any convenient time during the unit-slope portion on the log-log plot and \( \Delta P \) is the value of pressure drop corresponding to \( t_N \).

(1b) Early radial flow. This flow period is usually short and masked by the wellbore storage, but if a downhole shut-in tool is used, then the analysis of this flow regime utilizes the normal radial flow equations. Taking into account that during this period the pressure response behaves as if the formation thickness is equal to the length of the open zone, \( h_p \). According to this the following equations will apply (Tiab, 2003):

\[ k_{r1} = \frac{70.6 gB \mu}{h_p (t_1 \Delta P)_{r1}} \]  

(15)

\[ s_{r1} = 0.5 \left[ \frac{(\Delta P)_{r1}}{(t_1 \Delta P)_{r1}} \right] - \ln \left( \frac{k_{r1} t_{r1}}{\phi \mu' r_o^2} \right) + 7.43 \]  

(16)

The subscript \( r1 \) stands for first radial flow line. Where \( t_{r1} \) is any convenient time during the first radial flow regime on the log-log plot. \( \Delta P_{r1} \) and \( (t_1 \Delta P)_{r1} \) are the values corresponding to \( t_{r1} \).

Figure 2. Pressure derivative with wellbore storage and no mechanical skin

Figure 3. Pressure response in a well with partial completion

Figure 4. Pressure derivative type curves for a vertical well with partial completion for different penetration ratios \( b = h_p/h \) in radial and spherical symmetry
**Intersection point.** As shown in Figure 4, for analytical simulation runs using their respective coordinate systems, the \(-\frac{1}{2}\)-slope line corresponding to the spherical flow and the late radial flow line of the dimensionless pressure derivative in spherical symmetry intersect at:

\[
(t_D \ast P_D) = \frac{1}{2\sqrt{\pi} t_{Dp}}
\]

\[
(t_o \ast P_o) = 0.0066
\]

Equating the above two expression will yield:

\[
\frac{1}{2\sqrt{\pi} t_{Dp}^{-1/2}} = 0.0066
\]

Substituting Equation 9.a into the above expression and solving for the intersection time in field units, it yields:

\[
t_i = 6927748.85 \frac{k_i \mu_i r_{sw}^2}{k_p}
\]

Equation 17.a can be used to estimate either \(r_{sw}\) or \(k_{sp}\). Another equation to define this intersection time can be found from the \(-\frac{1}{2}\)-slope line corresponding to the spherical flow and the late radial flow line of the dimensionless pressure derivative, but this time in radial symmetry. Knowing from Figure 4 that:

\[
(t_o \ast P_o) = \frac{k^{1/2} h}{4k_p^{1/2} \sqrt{\pi} r_{sw} \sqrt{t_{Dr}}}
\]

\[
(t_o \ast P_o) = 0.5
\]

At the intercept the above equations are equal, then:

\[
\frac{k^{1/2} h}{4k_p^{1/2} \sqrt{\pi} r_{sw} \sqrt{t_{Dr}}} = 0.5
\]

(17.b)

In radial geometry, the dimensionless pressure derivative and time, respectively, defined in field units are:

\[
(t_o \ast P_o) = \frac{k h}{141.2 q B} \left( t \ast \Delta P \right)
\]

(18)

\[
t_{Dr} = \frac{0.002637 \ell}{\phi \mu_r r_c}
\]

(19)

Now, substituting Equation 19 into Equation 17.b and solving for the intersection time, \(t_i\) in field units:

\[
t_i = 301.77 \frac{k_i^2 h^2 \phi \mu_i}{k_p^2}
\]

(20)

Combining Equations 17.a and 20 leads to:

\[
r_{sw} = 0.0066 \frac{k_i h}{k_p}
\]

(21)

**Spherical flow.** From the dimensionless wellbore pressure approximation governed by spherical geometry (Joseph, 1984), Equation 7, the dimensionless pressure derivative is given by:

\[
P_D = \frac{1}{2\sqrt{\pi} t_{Dp}^{1/2}}
\]

(22)

\[
(t_o \ast P_o) = \frac{1}{2\sqrt{\pi} t_{Dp}^{1/2}}
\]

(22.a)

Taking logarithm to both sides of the above expression:

\[
\log(t_o \ast P_o) = -\frac{1}{2} \log t_o + \log \left( \frac{1}{2\sqrt{\pi}} \right)
\]

(23)

The slope of this straight line is \(-\frac{1}{2}\), which is a unique characteristic of spherical flow regime. Substituting the dimensionless terms in Equation 23 and solving for the pressure derivative it results:

\[
(t \ast \Delta P) = \frac{1}{2} m_p t^{1/2}
\]

(24)

being,

\[
m_p = 2453 \frac{q \nu B \left( \frac{\phi \mu_i}{k_p} \right)^{1/2}}
\]

(25)

Taking logarithm to both sides of Equation 24 gives:

\[
\log (t \ast \Delta P) = -\frac{1}{2} \log t + \log \left( \frac{1}{2} m_p \right)
\]

(22.b)

This expression shows that a plot of measured \(t \ast \Delta P\) versus time on a log-log graph will yield a straight line of slope \(-\frac{1}{2}\) when spherical flow is dominant. Combining Equations 24 with 25 and solving for \(k_{sp}\) yields:

\[
k_p = \left[ \frac{1227 q \nu B \left( \frac{\phi \mu_i}{t \ast \Delta P} \right)^{1/3} \left( \frac{1}{2} m_p \right)^{1/3}} \right]
\]

(26)
Substituting the dimensionless groups in Equation 7 will yield:

\[ \Delta P = m_{sp1} - m_p t^{-1/2} + m_{sp} s_{sp} \]  

(27)

where:

\[ m_{sp1} = \frac{70.6 q \mu B}{k_{sp} r_w} \]

\[ m_p = 2453 \frac{q \mu B}{k_p} \left( \frac{\phi \mu c_k}{k_p} \right)^{1/2} \]

An expression relating the spherical flow portion of the pressure and the pressure derivative can be derived by dividing Equation 24 with Equation 27 to yield:

\[ \frac{\Delta P}{(t \Delta P')^*} = \frac{2}{m_{sp1}} \left[ \frac{1}{m_p t^{-1/2}} + \frac{m_{sp} s_{sp}}{m_p t^{-1/2} - 1} \right] \]  

(28)

Solving for the spherical skin, \( s_{sp} \), the following equation is obtained:

\[ s_{sp} = 34.74 \sqrt{\frac{\phi \mu c_k}{k_{sp} t_{sp}}} \left[ \frac{(\Delta P)_{sp}}{2(t \Delta P')_{sp}} + 1 \right]^{-1} \]  

(29)

The subscript \( sp \) stands for spherical flow. \( \Delta P_{sp} \) and \( (t \Delta P')_{sp} \) are the values of pressure and pressure derivative corresponding to an arbitrary value of \( t_{sp} \) on the spherical flow straight line.

**2b) Hemispherical flow.** This is considered as a special case of spherical flow, and it should be realized that all results developed under spherical flow extend over directly for hemispherical flow applications with only minor modifications. The obvious observation is that the flow is now contained within a hemisphere instead of a sphere. Then, Equation 2 also applies to hemispherical flow and the dimensionless pressure derivative will be then equal to the one for spherical flow (Equation 22). It should also be noted that the suffix \( sp \) ought to be changed by \( hs \) indicating hemispherical flow. Figure 5 shows a log-log plot of the pressure and pressure derivative versus dimensionless time for a partial penetrated well. This is then to the one for spherical flow (Equation 22). The dimensionless variables suggested by Joseph (1984) will differ only by replacing the geometric factor \( 4\pi \) by \( 2\pi \). Equation 22.a corresponds to the early-time straight line. The slope of this straight line is also \(-1/2\). Again, substituting the dimensionless terms in Equation 22.a and solving for the pressure derivative, it yields:

\[ \frac{(t \Delta P')^*}{\frac{1}{2} m_{hs} t^{-1/2}} = \frac{1}{2} \]  

(30)

where:

\[ m_{hs} = 4906 \frac{q \mu B}{k_{hs}} \left( \frac{\phi \mu c_k}{k_{hs}} \right)^{1/2} \]  

(31)

Taking logarithm to both sides of Equation 30 gives:

\[ \log(t \Delta P') = -\frac{1}{2} \log t + \log \left( \frac{1}{2} m_{hs} \right) \]  

(32)

Also, a plot of \( t \Delta P' \) versus \( t \) on a log-log graph will yield a straight line of slope \(-1/2\) when hemispherical flow is dominant. Combining Equations 30 and 31 and solving for \( k_{hs} \) yields:

\[ k_{hs} = \left( \frac{2453 q \mu B}{(t \Delta P')_{hs} (\frac{\phi \mu c_k}{t_{hs}})^{1/3}} \right) \]  

(33)

where:

\[ k_{hs} = \sqrt{k_t k_r^2} \]  

(34)

Substituting the dimensionless terms in Equation 7 leads to:

\[ \Delta P = m_{hs}(1 + s) - m_{hs} t^{-1/2} \]  

(35)

being:

\[ m_{hs} = \frac{141.2 q \mu B}{k_{hs} r_w} \]  

(36)
An expression relating the hemispherical flow portion of the pressure and the pressure derivative can be obtained by dividing Equation 35 with Equation 30, thus:

\[
\frac{\Delta P}{(\tau \Delta P')} = 2 - \left[ \frac{m_{sw}}{m_{sw} t^{-1/2} + m_{sw} t^{-1/2} - 1} \right]
\]

(38)

Solving for the skin, \(s_{hs}\), the following Equation is obtained:

\[
s_{hs} = 34.74 \left( \frac{\phi \mu_r r^2}{k_{sw} t_{inh}} \right) \left( \frac{1}{2 (\tau \Delta P')} + 1 \right) - 1
\]

(39)

Where \(\Delta P_{hs}\) and \((\tau \Delta P')_{hs}\) are the values corresponding to any arbitrary time, \(t_{inh}\), on the hemispherical flow straight line.

(2c) Intersection point. As shown in Figure 6, for analytical simulation runs using their respective coordinate systems, the negative half-slope line corresponding to the hemispherical flow and the late radial flow line of the dimensionless pressure derivative in hemispherical symmetry intersect at:

\[
(t_0 * P_{D} ') = \frac{1}{2 \sqrt{r} t_{inh}}
\]

(40a)

\[
(t_0 * P_{D} ') = 0.0033
\]

Equating the above two expression will yield:

\[
\frac{1}{2 \sqrt{r} t_{inh}}^{-1/2} = 0.0033
\]

Substituting an expression similar to Equation 9.a in hemispherical coordinates into the above expression and solving for, \(t_i\), the intersection time in field units, it yields:

\[
t_i = 27710995.41 \frac{\phi \mu_r r^2}{k_{sw}}
\]

(40b)

Equation 40.a can be used to estimate either \(r_{sw}\) or \(k_{hs}\). Another equation to define this intersection time can be found from the -½-slope line corresponding to the hemispherical flow and the late radial flow line of the dimensionless pressure derivative but, this time, in radial symmetry. From Figure 6 we know that:

\[
(t_0 * P_{D} ') = 0.5
\]

At the intercept the above equations are equal, then:

\[
\frac{k_{sw}^{1/2} h}{2 \sqrt{r} s_{hin}^2} = 0.5
\]

(41)

Substituting Equation 19 into Equation 40.b and solving for \(t_i\) in field units:

\[
t_i = 1207.09 \frac{k^2 h^2 \phi \mu_r}{k_{sw}^2}
\]

(42)

Combining Equations 40.a and 42 will yield:

\[
r_{sw} = 0.0033 \frac{k^2 h^2}{k_{sw}}
\]

(43)

(3) Late radial flow. For the analysis of this flow regime the normal radial flow equation applies considering the total formation thickness. Permeability and mechanical skin can be calculated according to Tiab (2003):

\[
k_{r2} = \frac{70.6 q B \mu}{h (\tau \Delta P')_{r2}}
\]

(44)

\[
s_{r2} = 0.5 \left[ \left( \frac{\Delta P_{r2}}{(\tau \Delta P')_{r2}} \right) - \ln \left( \frac{k_{r2}}{\phi \mu_r r^2} \right) + 7.43 \right]
\]

(45)

Subscript \(r2\) stands for the second radial flow line. Being \(t_{r2}\) any convenient time during the radial flow line on the log-log plot, \(\Delta P_{r2}\) and \((\tau \Delta P')_{r2}\) are the values corresponding to \(t_{r2}\).
DETERMINATION OF VERTICAL AND HORIZONTAL PERMEABILITIES

Important considerations

One of the purposes of this study was to determine the range of values for the dimensionless wellbore storage, \( C_D \), based upon Equation 11b, that can mislead the interpretation of spherical/hemispherical flow regime. A better understanding can be reached plotting \((t_D^*P_D')\) vs. \( t_D/C_D \) as shown in Figure 2. Notice that for values of \( C_D < 10 \), the \(-\frac{1}{2}\)-slope behavior can be easily identified while for \( C_D \) around 100, the \(-\frac{1}{2}\) slope is more difficult to observe. Definitely, for \( C_D > 100 \), the spherical/hemispherical flow regime is completely masked by wellbore storage effects and this will make impossible to apply the method here established to calculate vertical permeability.

The length of the completed interval or the partial penetration length, \( h_p \), also plays an important role on the definition of the spherical/hemispherical flow as described in Figure 7. The characteristic slope of \(-\frac{1}{2}\) is absent for penetration ratios greater than 40 %.

TDS TECHNIQUE PROCEDURE

The following procedure can be used to calculate and verify values of \( C \), \( k_{sp} \), or \( k_{hs} \), \( k_H \), \( k_V \) and \( s_t \) from a log-log plot of pressure and pressure derivatives versus time without type curve matching (Moncada, 2004).

Step 1. Plot pressure and pressure derivative on a log-log paper. If given the case, draw the early unit-slope line corresponding to the wellbore storage flow regime and take any convenient \( t \) and \( \Delta P \) values on the unit-slope line and estimate \( C \) from (Tiab and Escobar, 2003):

\[
C = \frac{qB}{24} \left( \frac{t}{\Delta P} \right)_n
\] (46)

If no wellbore storage effect is seen, an initial radial flow regime may develop. Then, the normal radial flow equations will apply, taking into account that during this period the pressure response behaves as if the formation thickness is equal to the length of the open interval, \( h_p \). Permeability and skin can be calculated using Equations 15 and 16, respectively.

Step 2. Draw a straight line with slope of \(-\frac{1}{2}\), characteristic of the spherical/hemispherical flow regime. Select any convenient time \( t_{sp} \) during spherical/hemispherical flow and read the corresponding value of \((t^*\Delta P')_{sp}\). Then, \( k_{sp} \) (or \( k_{hs} \)) can be readily calculated using either Equation 26 or 33.

Step 3. Also from the \(-\frac{1}{2}\)-slope straight line the spherical or hemispherical skin, \( s_{sp} \) or \( s_{hs} \), can be obtained using either Equation 29 or 39, respectively.

Step 4. From the infinite-acting line, select any convenient time \( t_r \) and read \( \Delta P_r \) and \((t^*\Delta P')_r\). Then, the value of radial permeability (horizontal permeability) and total skin factor can be calculated with Equations 44 and 45.

Step 5. \( k_V \) can be estimated using either Equation 5 or 34, for spherical or hemispherical flow regime, respectively.

Step 6. Calculate the anisotropy index, \( I_A \), using Equation 47.

\[
I_A = \frac{k_v}{k_H} = \left( \frac{k_v}{k_H} \right)^3
\] (47)

Step 7. Knowing the mechanical and the total skin the partial penetration expressed as a skin factor can be calculated from:

\[
s_i = \frac{s_m}{b} + s_c
\] (48)

Step 8. For verification purposes, it is recommended to use either Equations 17.a, 20 and 21 (spherical flow) or Equations 40.a, 42 and 43 (hemispherical flow).
Note: This step-by-step procedure can also be applied to gas wells (Zamora and Nese, 2005) by changing Equations 46, 15, 16, 26, 33, 29, 33, 44 and 45 (appearing chronologically in the procedure) by Equations A.9 through A.17.

**EXAMPLES**

**Field example 1**

Table 1 contains drawdown test data taken from Abbott et al. (1978). Well No. 20 is partially completed in a massive carbonate reservoir. The well was shut-in for stabilization and then was flowing at 5200 BOPD for 8.5 h. Reservoir and fluid parameters are given in Table 1b.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h), ft</td>
<td>302</td>
</tr>
<tr>
<td>(r_w), ft</td>
<td>0.246</td>
</tr>
<tr>
<td>(hp), ft</td>
<td>20</td>
</tr>
<tr>
<td>(q), bpd</td>
<td>5200</td>
</tr>
<tr>
<td>(\sigma), %</td>
<td>0.2</td>
</tr>
<tr>
<td>(\mu), cp</td>
<td>0.21</td>
</tr>
<tr>
<td>(c_t), psi-1</td>
<td>34.2x10^{-6}</td>
</tr>
<tr>
<td>(P_i), psia</td>
<td>2298</td>
</tr>
<tr>
<td>(Bo), rb/STB</td>
<td>1.7</td>
</tr>
</tbody>
</table>

**Solution**

Step 1. The pressure and pressure derivative plot is given in Figure 6. From the unit-slope line we read \((\Delta P)_{N}=23\) psi and \(t_N=1\) h. Then, \(C\) of 16.01 bbl/psi is calculated using Equation 46.

Step 2. \((t^*\Delta P)_{sp}=56.25\) psi, \((\Delta P)_{sp}=96\) psi and \(t_{sp}=4.5\) are read from the \(-\frac{1}{2}\)-slope line. \(k_{sp}=8.05\) md is found with Equation 26.

Step 3. Also from the \(-\frac{1}{2}\)-slope straight line a spherical mechanical skin, \(s_{sp}=-0.87\) is obtained using Equation 29.

Step 4. From the late radial flow regime, \(t_r=7.5\) h, \(\Delta P_{r2}=120\) psi and \((t^*\Delta P)_{r2}=52.2\) psi are read, a value of the horizontal permeability of 8.26 md is estimated by means of Equation 44 and a total skin factor of -5.33 is found with Equation 45.

Step 5. \(k_V=7.64\) md is estimated using Equation 5.

<table>
<thead>
<tr>
<th>(t), h</th>
<th>(P_{wft}), psia</th>
<th>(\Delta P), psia</th>
<th>(t^*\Delta P'), psia</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2266.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>2255.0</td>
<td>11.0</td>
<td>11.5</td>
</tr>
<tr>
<td>1.0</td>
<td>2243.0</td>
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<td>24.5</td>
</tr>
<tr>
<td>1.6</td>
<td>2228.0</td>
<td>38.0</td>
<td>40.0</td>
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<td>2.0</td>
<td>2218.0</td>
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<td>2.5</td>
<td>2208.0</td>
<td>58.0</td>
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</tr>
<tr>
<td>3.0</td>
<td>2197.0</td>
<td>69.0</td>
<td>69.0</td>
</tr>
<tr>
<td>3.5</td>
<td>2185.0</td>
<td>81.0</td>
<td>66.5</td>
</tr>
<tr>
<td>8.5</td>
<td>2140.0</td>
<td>126.0</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8. Pressure and pressure derivative plot for Well No. 20, field example 1

Table 1b. Reservoir, well and rock data for field example 1
Step 6. An $I_A$ value of 0.92 is estimated with Equation 47. This value is not significantly less than 1, but it still indicates an anisotropic reservoir.

Step 7. Since the value of the mechanical skin is unknown because the initial radial flow is not seen, the partial penetration skin can be calculated using the following expression (Brons and Marting, 1961):

$$s_c = \left( \frac{1}{h_y/h} \right) \left( \ln h_y - G \right)$$

(49)

for spherical flow,

$$h_D = \left( \frac{k_H}{k_H} \right)^{0.5} \left( \frac{h}{2r_w} \right)$$

(50)

For hemispherical flow, the 2 in the denominator of Equation 50 ought to be removed.

$$G = 2.948 - 7.363 \left( \frac{h}{r} \right) + 11.45 \left( \frac{h}{r} \right)^2 - 4.675 \left( \frac{h}{r} \right)^3$$

(51)

then, $h_D = 659.25$; $G = 1.57$ and $s_c = 7.46$ are estimated.

Simulated example 1

Using a numerical simulator a pressure build up test for an oil well with partial penetration has been simulated using the same reservoir parameters presented by Joseph and Koederitz (1985), (Table 2) and the pressure data is provided in Table 3.

### Table 2. Reservoir, well and rock data used for simulation examples

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Property</th>
<th>Value</th>
<th>Property</th>
<th>Value</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_H$, md</td>
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<td>$r_w$, ft</td>
<td>0.417</td>
<td>$h$, ft</td>
<td>100</td>
<td>$r_w$, ft</td>
<td>0.3</td>
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<tr>
<td>$k_V$, md</td>
<td>24</td>
<td>$p_{wfi}$, Psia</td>
<td>3850</td>
<td>$h_{pr}$, ft</td>
<td>13</td>
<td>$q_g$, Mscf</td>
<td>2000</td>
</tr>
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<td>$k_V/k_H$</td>
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<td>$q$, bpd</td>
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<td>$\phi$, %</td>
<td>25</td>
<td>$\mu_g$, cp</td>
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</tr>
<tr>
<td>$h$, ft</td>
<td>350</td>
<td>$\mu$, cp</td>
<td>2.18</td>
<td>$c_r$, psi-1</td>
<td>3.0 * 10^-6</td>
<td>$z_i$, 1,003</td>
<td></td>
</tr>
<tr>
<td>$h_{pr}$, ft</td>
<td>28</td>
<td>$B_o$, rb/STB</td>
<td>1.19</td>
<td>$m(P_i)$, psi/2/cp</td>
<td>935369677.7</td>
<td>$T$, °R</td>
<td>671,67</td>
</tr>
<tr>
<td>$\phi$, %</td>
<td>8</td>
<td>$s$, s</td>
<td>5</td>
<td>$kh$, md-ft</td>
<td>3000</td>
<td>$c_r$, psi-1</td>
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</tr>
<tr>
<td>$c_r$, psi-1</td>
<td>1.03x10^-5</td>
<td>$C_D$</td>
<td>0</td>
<td>$C_r$, psi-1</td>
<td>1.7x10^-6</td>
<td>$f_{pr}$, h</td>
<td>200</td>
</tr>
<tr>
<td>$c_r$, psi-1</td>
<td>1.2x10^-5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Step 4. At \( t_2 = 31,05 \text{ hr} \) (late radial flow regime), we read \((t^*\Delta P')_2 = 1.62 \text{ psi}\) and \(\Delta P_2 = 419.28 \text{ psi}\). Then, \(k_r\) of 80.1 md is determined with Equation 44 and \(s_t\) of 121.8 can be calculated using Equation 45.

Step 5. \(k_v\) is found to be 25.79 md using Equation 34.

Step 6. An \(I_A\) of 0.32 is determined with Equation 47.

Step 7. Since \(b = 0.08\), a partial penetration skin of 57.42 is found with Equation 48.

Table 3. Pressure data for simulated example 1

<table>
<thead>
<tr>
<th>(t), h</th>
<th>(P_{wf}), psia</th>
<th>(\Delta P), psia</th>
<th>(t^*\Delta P'), psia</th>
<th>(t), h</th>
<th>(P_{wf}), psia</th>
<th>(\Delta P), psia</th>
<th>(t^*\Delta P'), psia</th>
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<td>0</td>
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<td>304.2</td>
<td>20.2</td>
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<td>48</td>
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</table>

Table 4. Pseudopressure data for simulated example 2

<table>
<thead>
<tr>
<th>(\Delta t), h</th>
<th>(m(P)_p), (psi2/cp)</th>
<th>(\Delta m(P)_p), (psi2/cp)</th>
<th>(t^*\Delta m(P)_p'), (psi2/cp)</th>
<th>(\Delta t), h</th>
<th>(m(P)_p), (psi2/cp)</th>
<th>(\Delta m(P)_p), (psi2/cp)</th>
<th>(t^*\Delta m(P)_p'), (psi2/cp)</th>
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<td>927562604.5</td>
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<td>1986070</td>
<td>0.1600</td>
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</table>
DETERMINATION OF VERTICAL AND HORIZONTAL PERMEABILITIES

Table 5. Comparison of results between conventional and TDS methods

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Field example 1</th>
<th>Simulated example 1 - oil</th>
<th>Simulated example 2 – gas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conventional</td>
<td>TDS</td>
<td>Conventional</td>
</tr>
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<td>16,01</td>
<td>-</td>
</tr>
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<td>7,81</td>
<td>8,05</td>
<td>-</td>
</tr>
<tr>
<td>ssp</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>khs, md</td>
<td>-</td>
<td>-</td>
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</table>

Simulated example 2

Table 4 presents synthetic pseudopressure data for drawdown test in a gas well generated with the data given in Table 2. Estimate permeabilities, pseudoskin and skin factors using TDS technique.

Solution

Step 1. Figure 10 presents the pressure and pressure derivative plot for this simulated example. Values of \((t^*\Delta m(P'))_{r1}=2052510 \text{ psi}^2/\text{cp}\), \((\Delta m(P))_{r1}=10599342,5 \text{ psi}^2/\text{cp}\) and \(t_r=0,0004 \text{ h}\) are read. A radial permeability value of 35,8 md is calculated using Equation A.10 and a skin factor of -4,95 is determined with Equation A.11.

Step 2. From the \(-\frac{1}{2}\)-slope straight line, at \(t_{hs}=0,127128 \text{ hr}\), the following data are obtained: \((t^*\Delta m(P'))_{hs}=517729 \text{ psi}^2/\text{cp}\) and \((\Delta m(P))_{hs}=18813865,1 \text{ psi}^2/\text{cp}\). \(k_r\) of 8,64 md can be calculated using Equation A.13.

Step 3. A value of \(s_{hs}\) of -0,84 is estimated with Equation A.15.

Step 4. At \(t_{r2}=2,3 \text{ h}\) (late radial flow regime), we read \((t^*\Delta m(P'))_{r2}=318606 \text{ psi}^2/\text{cp}\) and \((\Delta m(P))_{r2}=19832088,6 \text{ psi}^2/\text{cp}\). Then, \(k_r\) of 29,99 md is found using Equation A.16 and \(s_{r}'\) of 22,5 is determined by means of Equation A.17.

Step 5. \(k_V\) is found to be 0,72 md using Equation 34.

Step 6. \(I_A=0,024\) is determined with Equation 47.

Step 7. Since \(b=0,13\), a partial penetration skin of 36,73 is calculated using Equation 48.

Note: Although not shown here, the examples were also worked using conventional techniques and reported in Table 5 for comparison purposes. Permeabilities may be verified using either Equations 17.a, 20, 21, 40.a, 42, or 43 for the given case. Also, for gas wells, it should be taken into account that pseudoskin factors and rapid flow conditions have to be included to find the true skin factor as explained with great detail in Núñez-Garcia et al. (2003).
CONCLUSIONS

- Analytical equations based upon pressure derivative for a homogeneous reservoir are presented for well test interpretation of vertical gas and oil wells partially completed and partially penetrated including wellbore storage and skin effects.
- The log-log plot of pressure derivative versus time provides much more information about partially completed or partially penetrated vertical wells than conventional methods based on pressure vs. time plots.
- A straight line with slope of negative one-half was identified as the unique characteristic of the pressure derivative plot if spherical/hemispherical flow regime is present.
- The straight line corresponding to the spherical or hemispherical flow regime can be used to calculate the spherical/hemispherical permeability and spherical/hemispherical skin. These parameters are necessary to estimate vertical permeability.
- Using radial coordinates (Equation 11.b), either the spherical or hemispherical flow regime is completely masked for dimensionless wellbore storage values greater than 10 and penetration ratios higher than 40.
- The Tiab’s Direct Synthesis Technique was extended for vertical gas and oil wells with partial completion or partial penetration and its practical step-by-step procedure is presented. It leads to more accurate results compared with conventional methods.

REFERENCES


DETERMINATION OF VERTICAL AND HORIZONTAL PERMEABILITIES

Reservoirs”. SPE Asia Pacific Oil and Gas Conference and Exhibition (APOGCE), Perth, Australia. SPE 88556.


APPENDIX A

Gas Equations

Ah-Hussainy (1966) introduced the pseudopressure defined as:

\[ m(P) = 2 \int_{r_e}^{r} \frac{P}{\mu(P) z(P)} dP \]  \hspace{1cm} (A.1)

The pseudopressure diffusivity equation is:

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial m(P)}{\partial r} \right) = \frac{\phi \mu_g \frac{\partial m(P)}{\partial t}}{k_{sp}} \]  \hspace{1cm} (A.2)

The dimensionless pseudopressure in radial, spherical and hemispherical symmetries, respectively, are:

\[ m(P)_{Dr} = \frac{k_h h}{1422.52 q T} \Delta m(P) \]  \hspace{1cm} (A.3)

\[ m(P)_{Dp} = \frac{k_w r_w}{711.26 q T} \Delta m(P) \]  \hspace{1cm} (A.4)

\[ m(P)_{Dhs} = \frac{k_h r_w}{1422.52 q T} \Delta m(P) \]  \hspace{1cm} (A.5)

The dimensionless pseudopressure derivatives in radial, spherical and hemispherical symmetries, respectively, are:

\[ (t_D \ast m(P)_D)' = \frac{k_h h}{1422.52 q T} \left( t \ast \Delta m(P)' \right) \]  \hspace{1cm} (A.6)

\[ (t_D \ast m(P)_D)'_p = \frac{k_w r_w}{711.26 q T} \left( t \ast \Delta m(P)' \right) \]  \hspace{1cm} (A.7)

\[ (t_D \ast m(P)_D)'_w = \frac{k_h r_w}{1422.52 q T} \left( t \ast \Delta m(P)' \right) \]  \hspace{1cm} (A.8)

\[ C = 0.419 \frac{q T}{\mu e} \left( \frac{t}{\Delta m(P)_n} \right) \]  \hspace{1cm} (A.9)

\[ k_{sr} = k_h = 711.26 \frac{q T}{h \left( t \ast \Delta m(P)' \right)_s} \]  \hspace{1cm} (A.10)

\[ s_{sr} = s'_{sr} = 0.5 \left[ \frac{(\Delta m(P))_s}{(t \ast \Delta m(P)')_s} - \ln \left( \frac{k_{sr} t_s}{\phi \mu r_w^2} \right) + 7.43 \right] \]  \hspace{1cm} (A.11)

\[ k_{sp} = \left( 12355.73 \frac{q T}{(t \ast \Delta m(P))'_{sp}} \right)^{2/3} \]  \hspace{1cm} (A.12)

\[ k_{sh} = \left( 24711.46 \frac{q T}{(t \ast \Delta m(P))'_{sh}} \right)^{2/3} \]  \hspace{1cm} (A.13)

\[ s_{sp} = 34.74 \sqrt{\frac{\phi \mu r_w^2}{k_{sp} t_{sp}}} \left[ \frac{(\Delta m(P))_{sp}}{2(t \ast \Delta m(P)')_{sp}} + 1 \right]^{-1} \]  \hspace{1cm} (A.14)

\[ s_{sh} = 34.74 \sqrt{\frac{\phi \mu r_w^2}{k_{sh} t_{sh}}} \left[ \frac{(\Delta m(P))_{sh}}{2(t \ast \Delta m(P)')_{sh}} + 1 \right]^{-1} \]  \hspace{1cm} (A.15)

\[ k_{r} = 711.26 \frac{q T}{h(t \ast \Delta m(P))'_{r}} \]  \hspace{1cm} (A.16)

\[ s_{s} = 5 \left[ \frac{(\Delta m(P))_{r}}{(t \ast \Delta m(P)')_{r} - \ln \left( \frac{k_{sr} t_{sr}}{\phi \mu r_w^2} \right) + 7.43} \right] \]  \hspace{1cm} (A.17)