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Effect of a superconducting defect on the Cooper pairs of a mesoscopic sample

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Abstract

We investigate the vortex state in a very long prism of square cross section with a central square defect in the presence of an external perpendicular magnetic field. We considered that the inner defect edge is in contact with a thin superconducting layer at higher critical temperature and/or with a dielectric material, while the outer edge of the sample is in contact with the vacuum. We have evaluated the superconducting order parameter, magnetization and vorticity as a function of the size of the defect at the first vortex penetration field. Therefore we conclude and we are able to show that circular geometry of the vortices near to the defect is mildly modified by the enhanced superconductivity at the edge of the hole.

Keywords: Ginzburg-Landau; Superconducting; Mesoscopics; Square hole.

Efecto de un defecto superconductor sobre los pares de Cooper de una muestra mesoscópica

Resumen

Se investiga el estado de vórtices en un cilindro largo de sección transversal cuadrada con un defecto cuadrado central, en presencia de un campo magnético externo aplicado perpendicular a su superficie. Consideramos que el borde del defecto está en contacto con una pequeña capa de material superconductor, a mayor temperatura crítica y/o con un material dieléctrico, mientras que el borde externo de la muestra está en contacto con el vacio. Evaluamos el parámetro de orden superconductor, magnetización y vorticidad como función del tamaño del defecto en el campo de penetración del primer vórtice. Mostramos que la geometría circular de los vórtices cerca al defecto es levemente modificada por el aumento de la superconductividad en los bordes del defecto.

Palabras clave: Ginzburg-Landau, Superconductor, Mesoscópicos, hueco cuadrado.

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1. Introduction

It is known that the properties of a mesoscopic superconductor are largely influenced by the boundary conditions, the geometry of the sample and by structural defects, therefore simple and complicated loop structures and networks have been theoretically [1-4] and experimentally studied [5] in some of these works, the authors found that in addition to the conventional vortex structures at the matching fields, a variety of vortex states can be stabilized by decreasing the pinning strength of the antidots, also when an antidot array is present the critical temperature is enhanced compared to a non patterned sample and distinct cusps in the phase boundary are found for different matching fields. Also, several authors report experimental results on the synthesis, the structural characterization, the ferroelectric behavior and the electronic properties of complex high temperature superconductors, the results reveal that the perovskite used, crystallizes in a rhomboidal structure, and has a ferroelectric hysteretic behavior at room temperature [6]. Two band or multi-band mesoscopics superconductors [7-10] and fractional vortices [11], present new and very interesting topics for theoretical and experimental study. In previous works, using the Ginzburg Landau formalism, we studied the effect of trench, holes, barrier and boundary conditions on the vortex configurations in circular and square geometries, we found that the lower and upper critical fields are independent of the geometry of the defect, and depend strongly on the boundary conditions [12-14]. In this paper we analyze the superconducting state in a long mesoscopic square cylinder with a central square defect in presence of an external magnetic field applied perpendicularly to its surface at the first vortex penetration field. We calculate magnetization, supercurrent, order parameter and vorticity for two different internal boundary conditions $b < 0$ (superconducting/superconducting at higher critical temperature interface) and $b \to \infty$ (superconducting/dielectric interface). We found that the first vortex penetration field does not depends on the size of the defects and that circular geometry of the vortices near to the defect is mildly modified by the enhanced superconductivity at the edge of the hole.

2. Theoretical Formalism

We take the order parameter and the local magnetic field invariant along the z-direction. The time evolution was incorporated into the Ginzburg Landau equations in such a manner that their gauge invariance is preserved. Superconducting state is described in the time dependent Ginzburg-Landau theory (TDGL) by the order parameter Ψ that describe the superconducting electron density and the potential vector **A** related to the magnetic induction by $\boldsymbol{B} = \nabla \times \boldsymbol{A}$. Also we take the case for electrical potential zero, the TDGL takes the form [13-15]:

$$\dot{\Psi} = \boldsymbol{D}^2\Psi + (1-T)(|\Psi|^2 - 1)\Psi \quad (1)$$

$$\dot{\mathbf{A}} = -(1-T)Re\{\overline{\Psi}\, \mathbf{D}\, \Psi\} - \boldsymbol{J_n} \quad (2)$$

Here, $\boldsymbol{D} = i\nabla + \boldsymbol{A}$, $\boldsymbol{J_n} = \kappa^2(\nabla \times \nabla \times \mathbf{A})$ is the normal current, and $\boldsymbol{J_s} = -Re\{\overline{\Psi}\, \mathbf{D}\, \Psi\}$ is the supercurrent. Eqs. (1) and (2) were rescaled as follows: Ψ in units of $\Psi_\infty(T)$ lengths in units of $\xi(T)$, the external applied magnetic field $\boldsymbol{H_e}$ in units of $H_{c2}(T)$, $\boldsymbol{A}$ in units of $H_{c2}(T)\xi(T)$, temperatures in units of T_c. The dynamical equations are complemented with the appropriate boundary conditions for the order parameter:

$$\hat{\boldsymbol{n}} \cdot \boldsymbol{D}\Psi = -i\hbar\Psi/b \quad (3)$$

b is the de Gennes parameter and $\hat{\boldsymbol{n}}$ is the unity vector perpendicular to the surface of the superconductor. In this paper we analyze the superconducting state of a long mesoscopic cylinder of square transverse section of area $Sc = 36\xi(T)X36\xi(T)$ with a central square defect of area Sd in presence of an external magnetic field $\boldsymbol{He}$ applied perpendicular to its surface. We considered $Sd = 18\xi(T)X18\xi(T)$, $12\xi(T)X12\xi(T)$, $6\xi(T)X6\xi(T)$, $3\xi(T)X3\xi(T)$ and $Sd = 0$, for two different internal boundary conditions $b = -2\xi(T)$ and $b \to \infty$. The parameters used in our numerical solution were: grid spacing $a_x = a_y = 0.3 \times 0.3$, $N_x = N_y = S_c / a_{x,y} = 120\xi(T) \times 120\xi(T)$ for the computational mesh, constant temperature $T = 0.53$ and Ginzburg-Landau parameter $\kappa = 5$. We ramp up the applied magnetic field adiabatically, typically in steps of $\Delta H = 10^{-5}$. Also, we use the following criterion to obtain the stationary state: if the highest difference $\| \Psi(t_m) - \Psi(t_n) \|$, for any vertex point in the mesh, is smaller than a certain precision ε, then we go over the next field; usually, this test is made over some thousands of times steps, i.e., $m - n \sim 10^{-3}$. We have worked with a precision $\varepsilon = 10^{-9}$. Although the time dependent Ginzburg-landau equations can provide all the metastable states of a fixed field, in the present work we studied only the stationary state at the first vortex penetration field H_1.

The magnetization, $-4\pi\boldsymbol{M_n} = \boldsymbol{B} - \boldsymbol{H_e}$, where $\boldsymbol{B}$ is the induction (the spatial average of the local magnetic field) is:

$$-4\pi M_n = \frac{1}{N_x N_y}\sum_{i=1}^{N_x}\sum_{j=1}^{N_y} h_{z,i,j}^n - H_e \quad (4)$$

The number of vortices can be found integrating the supercurrent $\boldsymbol{Js}$ along a rectangle containing the superconductor. This leads us to:

$$\oint \frac{1}{|\Psi|^2} \Re[\overline{\Psi}\, \mathbf{D}\, \Psi] \cdot d\boldsymbol{r} = 2\pi \boldsymbol{N} - \Phi \quad (5)$$

Where N is the vorticity or number of vortices and Φ is the total penetration flux.

3. Results

In Fig. 1 we plot (a) Magnetization curve -$4\pi M$ and (b) Vorticity N as a function of the magnetic field, for a square sample with a central hole of area $Sd = 36{,}144{,}324$ in contact with different materials characterized by $b \to \infty$ (top) and $b = -2\xi(T)$ (bottom). We can notice that the presence of the defect causes a noticeable drop of the first penetration field and leads to a qualitative change of the magnetization and vorticity curves. In the Meissner state the magnetization is a linear function of the applied field and in the Abrikosov state it has a series of jumps which indicate the nucleation of one or more vortices.

In Fig. 1(b) (top) we included (a) the superconducting electron density $|\Psi|$ (b) the phase of order parameter $\Delta\phi$ and (c) the supercurrent density Js for a square sample at $H_1 = 0.07$. We found a typical vortex configuration with $N = 4$ vortices, they are arranged symmetrically, but it is not a stationary state, as it is well known, increasing the magnetic field the vortices goes to corners to the sample due to mutual repulsive force, forming configurations to minimize the internal energy of the system. We note also that the magnetic field for the first arrival vortices is $H \approx 0.10$ for all the samples with the defect.

In Fig. 2 we plotted the superconducting electron density $|\Psi|$ for a square sample with the inner defect edge in contact with (top) a superconducting material at higher critical temperature and (bottom) a dielectric material $b \to \infty$ of area (left to right) $Sd = 9\xi^2(T)$,$36\xi^2(T)$,$144\xi^2(T)$, $324\xi^2(T)$ at H_1, N_d represent the vorticity in the hole. For$Sd = 0, 9\xi^2(T), 36\xi^2(T)$, we found that the first entry of vortices occurs for $N = 4$ at $H \approx 0.10$ for all cases. When we analyze for defects with $Sd = 144\xi^2(T)$,$324\xi^2(T)$, we have a square vortex configuration due to the geometry of the sample, with a small increase of the magnetic field, four first vortices are attracted quickly towards to the dielectric defect center forming a giant vortex with vorticity $N = 4$ increasing the magnetic field, then four more vortices enter the sample, four sit in the hole and the other four sit in the superconductor region, although they are not visible in the contour plot of the magnitude of order parameter, also there is a change in the phase around the hole equal to $\Delta\phi = 8\pi$ (Fig. 3 (top)). The vortices inside the hole repel the vortices in the superconductor region, repulsion increases with the increase of Sd for a constant magnetic field H_e, $H_1 = 0.1, 0.098$ for $Sd = 144$ and $Sd = 324$ respectively. It is interesting to note that the presence of the superconducting layer in the defect acts with a repulsion force and repels the vortices deforming its own (top)). It is possible to include a new internal surface energy circular geometry for a distance of ~8% of the sample side for $Sd = 9$ and ~5% for $Sd = 36,144,324$ (green line in Fig. 2 barrier due to the presence of the defect with $b < 0$, this barrier will be greater for smaller values of $|b|$. This small vortex deformation is not present in a sample with a central hole in contact with a dielectric material, even when inside the defect there are vortices (Fig. 2 (bottom)).

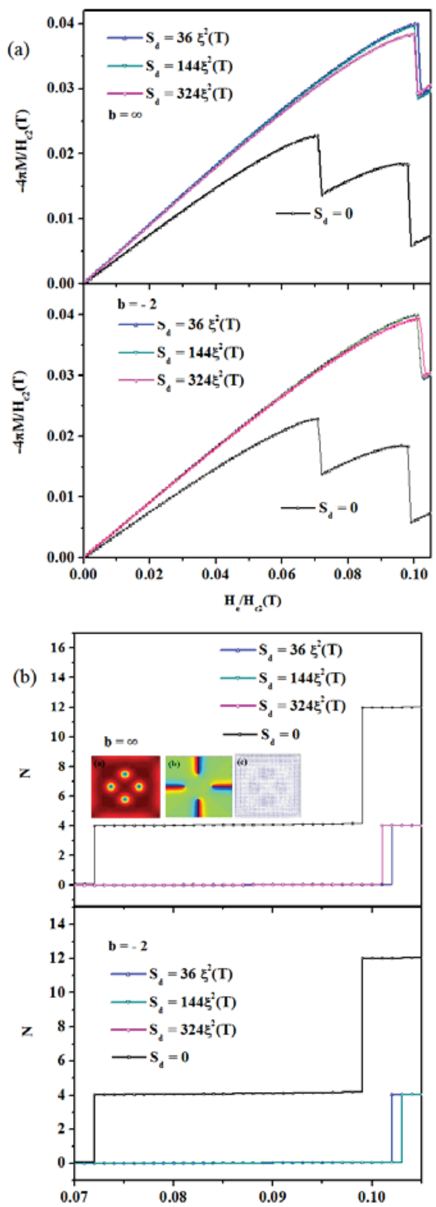

Figure 1. (Color online) (a) Magnetization curve -4πM and (b) Vorticity N as function of the magnetic field, for a square sample with a central hole of area Sd and (top) $b \to \infty$ and (bottom) $b = -2$. Dark and bright regions in the inset represent values of the modulus of the order parameter $|\Psi| = 1(0)$, superconducting (normal) state (as well as $\Delta\phi/2\pi$, from 0 to 1).
Source: The authors

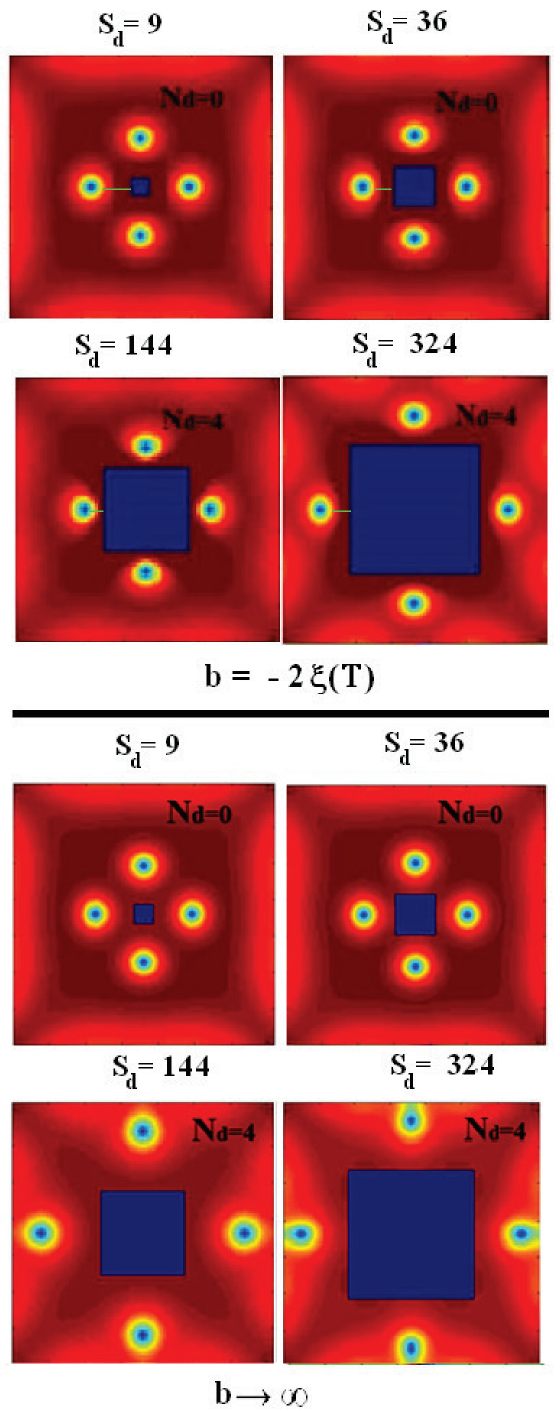

Figure 2. (Color online) Superconducting electron density $|\Psi|$for $b = -2$ case (top) and $b \to \infty$ (bottom for area (left to right) $Sd = 9, 36, 144, 324$ at $H_1 = 0.1$ Nd represent the vorticity in the hole.
Source: The authors

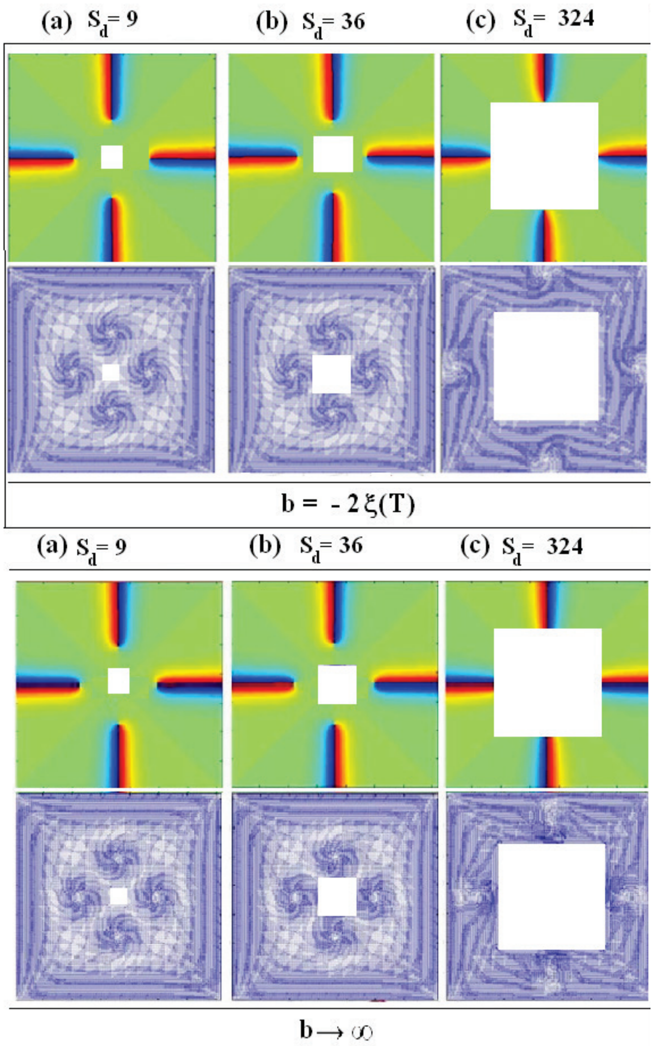

Figure 3. (Color online) Superconducting order parameter phase $\Delta\phi$ and supercurrent density Js, $b = -2$ (up) and $b \to \infty$ (down) and area $Sd = 9, 36, 324$ (left to right) at H_1. Dark and bright regions represent values of the phase $\Delta\phi/2\pi$, from 0 to 1.
Source: The authors

In Fig. 3 we plot the phase of the order parameter $\Delta\phi$ and supercurrent density $\boldsymbol{J_s}$, for two cases: $b = -2$ (up) and $b \to \infty$ (down) with area of the defect $Sd = 9, 36, 324$ (left to right) respectively. For $Sd = 9, 36$ we found $H_1 = 0.99, 0.101$, H_1 increases slowly with the defect size. Four vortices sit in the superconducting area, there is a change in the phase around the sample equal to $\Delta\phi = 8\pi$ and around the hole equal to $\Delta\phi = 0$.

4. Conclusions

We studied the effect of a central square defect on the thermodynamical properties of a mesoscopic superconducting cylinder solving the time dependent Ginzburg-Landau equations. Our results have shown that the lower thermodynamic field H_1 varies slowly depending on the size of the defects, and is independent of the boundary condition. For these samples the presence of the superconducting material inside the defect acts like an antipinning center. The repulsive force of the antipinning center mildly changes the circular geometry of the vortices for a distance of ~8% of the sample side. If the inner defect edge is in contact with a thin superconducting layer at a higher critical temperature, the first critical field H_1 increases with the presence of the defect and the diamagnetism of the sample increases and will be more pronounced for smaller values of the deGennes length $|b|$. In our opinion these findings are important for the groups exploring the superconducting state in nano-engineered materials.

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