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Application of Bayesian techniques for the identification of accident-prone road sections

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Abstract
The use of Bayesian techniques for the identification of accident-prone road sections has become very important in recent years. The objective of this investigation consisted of identifying accident-prone road sections in the Municipality of Ocaña (Colombia) using the Bayesian Method (BM); the modeling approach developed involved the creation of a database of accidents that occurred between the years 2007 (January) and 2013 (August) and the application of the methodology on 15 sections of urban road. The final analyses show that the BM is an original and fast tool that is easily implemented, it provides results in which 4 accident-prone or dangerous road sections were identified and ranked them in order of danger, establishing a danger ranking that provides a prioritization for investments and the implementation of preventive and/or corrective policies that will maximize benefits associated with road safety.

Keywords: Bayesian Method, accident-prone sections, hazard ranking, road safety.

Aplicación de técnicas Bayesianas en la identificación de tramos viales propensos a accidentes

Resumen
El uso de técnicas bayesianas para la identificación de tramos de carretera propensos a accidentes ha llegado a ser muy importante en los últimos años. El objetivo de esta investigación consistió en identificar los tramos de carretera propensos a accidentes en el municipio de Ocaña (Colombia), utilizando el método bayesiano (BM); el enfoque de modelación desarrollado consistió en la conformación de una base de datos de accidentes ocurridos entre los años 2007 (enero) y 2013 (agosto) y la aplicación de la metodología en 15 tramos de carreteras urbanas. Los análisis finales muestran que el BM es una herramienta poderosa y rápida de fácil implementación, que proporciona resultados en los que se identificaron 4 tramos de carretera propensos a los accidentes o peligrosos y los clasificó por orden de peligro, el establecimiento de un ranking de peligro proporciona un orden de prioridades para las inversiones y la aplicación de políticas preventivas y / o correctivas que maximicen los beneficios asociados con la seguridad vial.

Palabras clave: Método Bayesiano, tramos propensos a accidentes, ranking de peligrosidad, seguridad vial.

1. Introduction

Accident rates are alarmingly high in Colombia, and this has become a public health problem with great economic impact. Official statistics show that the vulnerable groups are primarily pedestrians and motorcycle riders, which collectively account for 70% of deaths in road accidents. Statistics also show that between the years 2005 and 2010 there was an increase in deaths from traffic accidents, from 5.418 to 5.502, and in 2010 over 39.275 seriously injured persons were registered according to the National Institute of Legal Medicine and Forensic Sciences (INML), with traffic accidents becoming the number-one cause of death for children between five and 14 years of age, and the second leading cause of death for people between 15 and 24 years of age. According to data provided by the INML, 2.044 people under the age of 30 died in traffic accidents in Colombia in the 2010 [1].

The identification of the accident-prone sections is one of the alternatives available to properly address the problem, and tends to be the first step in the investigation and implementation of road safety programs. This is because once these sections have been defined as high-risk, possible factors associated with the frequency of accidents are determined (e.g. volume of vehicles, environmental factors,

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Table 1.
Investment in Road Safety in the Americas 2006-2007

Country	Annual budget to fund the National Strategy for 2008 (USD$)	Population 2007	Expenditure per capita in USD$
Bahamas	10.100,00	331.278	0,03
Canada	14.100.000,00	32.876.047	0,43
Colombia	16.541,20	46.155.958	0,00
Costa Rica	32.980.000,00	4.467.625	7,38
Honduras	1.083.800,00	7.106.001	0,15
México	8.706.240,00	106.534.880	0,08
Nicaragua	1.179.035,20	5.603.190	0,21
United States	838.000.000,00	305.826.246	2,74

Source: Adapted from [14]

geometric characteristics of road infrastructure, and speed), and preventive and/or corrective policies are then proposed to decrease the indicators associated with the accidents. Evidence reports the effect of factors associated with road geometry [2-4], vehicle volumes [5,6], environmental conditions [3,7] and speed [8,9] on the occurrence of accidents, which is estimated with Poisson regression, negative binomial, generalized linear models. Likewise, the assessment of the effectiveness of road safety measures using the Bayesian empirical approach has also been reported in other studies [10-12].

According to [13], an element that is part of road infrastructure (sections of highways, intersections, curves, among others) may experience a high number of accidents due to two conditions: high random variations of traffic accidents during periods of observation and safety problems associated with the surroundings (high vehicular traffic, nature of the site, inappropriate geometric road design). The study and identification of accident-prone sites (also called hotspots, blackspots, sites with promise, high-risk locations, accident-prone locations) can suffer from two types of common effects. The first effect, termed a false negative, corresponds to an unsafe site that does not show high rates of accidents. It is also possible to observe high accident rates in a relatively safe site, which is referred to as a false positive. The two situations above need to be taken into account in determining where to invest in road safety, because, in a bureaucracy such as that of Colombia, investments in road safety are restricted and limited (as shown in Table 1, which shows marked differences in investment in road safety in terms of per capita spending by country). False negatives lead to the loss of opportunities for effective road safety investments. As is to be expected, correct determinations of the safety of a site is essential, including the identification of a safe site as "safe" and an unsafe site as "unsafe." For the purposes of this research, we sought accident-prone sections that produced the lowest proportion of false negatives and false positives using the BM.

In reviewing the available research, it was found that there is evidence of other techniques parallel to the BM with which it is possible to identify accident-prone sections; these include the Classification Method [15,16] and the Confidence Interval Method [17], which use different approaches for the analysis and determination of accident-prone sections. It was concluded that in some cases a large number of false positives are produced using these methods, while in other situations, when dealing with sites with relatively few accidents and low exposure, significant improvements cannot be evaluated and/or experienced [13]. Another method that has been used in the last few years as a reliable method providing pertinent results is the Quantile Regression Method [18]. The BM has greater credibility and better results in the identification of accident-prone sections, for which reason it is the subject of study in this research. Other studies [13, 19-23] have demonstrated that the BM offers a greater capacity to determine highly hazardous or risky sites in terms of safety. With regard to the hazard ranking of the accident-prone sections found, there are studies that report how prioritization for efficient investments can be achieved using the BM [24].

2. Methodological bases

2.1. The Bayesian Method (BM)

The use of the BM in the identification of accident-prone sections is based on relating n random variables (Y_1, ..., Y_n) corresponding to i sections (i = 1,..., n) under study, where a current ratio of accidents (λ_i) occurs during a specific time period. We assume that λ_i is distributed in accordance with a law of probability with a function of density $f(\lambda_i \mid \theta_i)$, where θ_i represents the mean number of accidents in section i (parameter of interest). The Bayesian approach, assuming a distribution with density $\pi(\theta_i)$ in θ_i, allows the incorporation of prior knowledge regarding the behavior of θ_i. This prior information is combined with the information presented by the sample in the subsequent distribution, represented by $p(\theta_i \mid \lambda_i)$. The subsequent distribution of θ_i is a direct application of Bayes' theorem and has the form of the eq. (1) [25]:

$$p(\theta_i|\lambda_i) = \frac{f(\lambda_i|\theta_i)\pi(\theta_i)}{m(\lambda_i)} = \frac{f(\lambda_i|\theta_i)\pi(\theta_i)}{\int f(\lambda_i|\theta_i)\pi(\theta_i)d\theta_i} \tag{1}$$

Where $m(\lambda_i)$ represents the function of unconditional marginal density of λ_i and $f(\lambda_i|\theta)$ is the probability of the data observed.

Put simply, the BM, as shown in [24], groups this estimate into two consecutive processes: in the first instance, it estimates the history of accidents for each of the sites (i) in order to define the distribution of probability of the ratio of accidents in each section studied locally. The second step consists of using this local probability distribution and the accident rate of each site (i) in order to obtain a more precise estimate of the probability distribution that is associated with the ratio of accidents of a particular site (i). In this way, it is possible to assess the probability that one of the sections under study may be dangerous. The function of accumulated distribution associated with the accident ratio (λ_i) is represented in the eq. (2):

$$P(\lambda_0 \leq \lambda) = \int_0^\lambda f_i(\lambda|N_i, V_i)\, d\lambda \tag{2}$$

Where V_i is the number of vehicles that transit along section i during the period of study, N_i is the number of accidents that occur in section i studied within the time frame analyzed (for this case, the number of accidents that occurred between January 2007 and August 2013), and f_i ($\lambda \mid N_i, V_i$) is the probability density function associated with the ratio of accidents in section i. Prior research [13] sets out two basic assumptions on which the BM bases its logic:

Assumption 1: In a given place, the occurrence of crashes follows a Poisson-type counting process, where the probability that n accidents occur per unit of time (n = 0, 1, 2,.) is given by the following eq. (3):

$$P(N_i = \frac{n}{\lambda_i} = \lambda, V_i) = \frac{(\lambda V_i)^n \, e^{-\lambda V_i}}{n!} \tag{3}$$

Assumption 2: The probability distribution $F_r(\lambda)$ is of the population of the gamma-distributed sites, where $g(\lambda)$ is denoted as the gamma probability density function (eq. 4) and is typically modeled as a function of the co-variables of the site.

$$F_r(\lambda) = \frac{\beta^{\alpha} * \lambda^{\alpha-1} * e^{-\beta\alpha}}{\Gamma(\alpha)} \tag{4}$$

In these equations, α is the parameter of form and β is the parameter of scale of the gamma function, which can be estimated from the procedures set out by [26]. Finally, the BM permits two types of approximations from which it is possible to identify the accident-prone sections. The first makes use of the eq. 5 to determine them:

$$Prob = 1 - \int_0^{\lambda_p} \frac{\beta_i^{\alpha+\lambda_{cr}*V_i} * (\lambda)^{\alpha+\lambda_{cr}*V_i-1} * e^{-\beta*\lambda}}{\Gamma(\alpha)} d\lambda \tag{5}$$

Where: λ_p is the mean of the ratios of accidents observed for all of the sections studied and λ_{cr} corresponds to the accident-prone rate in each section studied.

For this first approximation, the probability (Prob) of $\lambda_{cr} \leq \lambda_r$ is estimated; this probability is defined according to a 95% confidence interval. In this way, values of λ_{cr} are estimated such that there is a probability of 95% and $\lambda_{cr} \leq \lambda_r$ is compared; if this verification is met, the null hypothesis (H_0: $\lambda_{cr} \leq \lambda_r$) is accepted, and it is said that a section is accident prone.

The second approximation sets out the estimate of probability based on the following model (eq. 6):

$$Prob = 1 - \int_0^{\lambda_r} \frac{\beta_i^{\alpha_i} * (\lambda)^{\alpha_i-1} * e^{-\beta_i*\lambda}}{\Gamma(\alpha_i)} d\lambda \tag{6}$$

Where: λ_r is the accident ratio observed for all of the sections studied in the time period in which the observations were made. In the estimates based on the second approximation, the probability (Prob) is calculated and compared against that established in the 95% probability threshold. If this probability is greater than or equal to 95%, the null hypothesis (H_0: Prob ≥ 95%) is accepted and it is said that a section is accident prone.

2.2. Criteria used to determine a hazard ranking

Once the accident-prone sites have been identified, it is necessary to establish a hazard ranking and, in this way, prioritize investments and the implementation of preventive and/or corrective policies that will maximize benefits associated with road safety. Investment priorities must be based not only on a hazard ranking, but also cost-benefit analysis; however, this aspect was not considered for this research. There are two criteria that allow a hazard ranking to be determined:

2.2.1. Criterion 1

This procedure relates the accident ratio observed in each section studied (λ_r) to the accident-prone rate in each section studied (λ_{cr}). This ratio must be greater than one ($\lambda_r / \lambda_{cr} \geq 1$).

2.2.2. Criterion 2

This criterion is defined by the eq. 7:

$$(\lambda_r - \lambda_{cr}) * a \tag{7}$$

Where the Average Daily Transit (ADT) and the time (years) for which there are accident records are related, estimated as follows (eq. 8):

$$a = \frac{ADT * T * 365}{1.000.000} \tag{8}$$

2.3. Data and sections studied

A database was prepared of the records of accidents that occurred in the urban perimeter of the municipality of Ocaña between January 2007 and August 2013. Based on other studies, time periods of between 3 and 6 years are suitable for this type of study [13]. In parallel, and based on prior studies [27] the BM was applied in 15 roads corridors of Ocaña. The length of the sections (L) is a variable identifying each road section. Each section is classified as homogeneous in terms of geometric and operational characteristics. However, at present the effects of the length of the identification section on the hotspot are still not as clear [28]. Some other evidence from the literature shows the variation in the length of sections of road [28-30].

Since the BM allows relating the number of accidents allocated to each section with vehicle volumes, it was necessary to estimate the ADT for each of the 15 corridors to be studied. In summary, a total of 1,062 accidents were reported, spread out among the 15 sections studied. It must be clarified that in countries such as Colombia (particularly in Ocaña), accident records are obtained from the National Police and other entities, such as the Volunteer Firefighters' Corps and/or Ocaña Civil Defense, which are entities that deal with accidents. This involves some disadvantages, because there is no linkage and/or agreement between those reported by medical sources and those from police records, resulting in underestimates of the records; in addition, records of accidents with minor injuries, single-vehicle accidents and cyclist accidents are sometimes not reported [31,32].

3. Study area

Ocaña is a city located in the northwestern region of Colombia in the Norte de Santander department. It is the

Figure 1. Geographic Location of Ocaña (Colombia).
Source: The Authors

second largest town of the department after Cúcuta, with approximately a population of 100.000 including rural areas. It has elevation relative to sea level of 1202 m and a land area of 460 km^2, representing 2,2% of the surface of department. The geographic location of Ocaña is presented in Fig. 1:

4. Results and discussion

As was already mentioned in the previous section, two types of approximations were used to determine whether or not a section is accident prone in terms of road safety. Parameters were estimated that allow for estimating the critical status of the section from the approximation where it is verified that $\lambda_{cr} \leq \lambda_r$. The evaluation and identification of the accident-prone section and the estimate of the other parameters can be seen in Table 2.

Table 2.
Identification of Sections λcr ≤ λr

Section (i)	N	ADT	L	λr	λcr	Prob	State
1	45	13.380	751,24	1,3163	2,1712	95,00%	Non-Critical
2	38	20.047	1.007,66	0,7419	2,0941	95,00%	Non-Critical
3	28	8.607	1.141,10	1,2733	2,2784	95,00%	Non-Critical
4	226	14.158	1.801,56	6,2476	2,1594	95,00%	Critical
5	170	49.506	1.407,54	1,3440	2,0157	95,00%	Non-Critical
6	179	10.791	4.084,26	6,4923	2,2200	95,00%	Critical
7	41	12.342	1.233,64	1,3002	2,1888	95,00%	Non-Critical
8	13	18.568	685,18	0,2740	2,1074	95,00%	Non-Critical
9	22	12.193	355,85	0,7062	2,1915	95,00%	Non-Critical
10	4	15.692	374,85	0,0998	2,1386	95,00%	Non-Critical
11	57	9.632	2.524,63	2,3162	2,2483	95,00%	Critical
12	25	15.745	732,16	0,6215	2,1380	95,00%	Non-Critical
13	47	18.381	684,50	1,0008	2,1092	95,00%	Non-Critical
14	17	10.958	667,54	0,6072	2,2163	95,00%	Non-Critical
15	150	26.278	1.283,53	2,2341	2,0513	95,00%	Critical

Source: The Authors

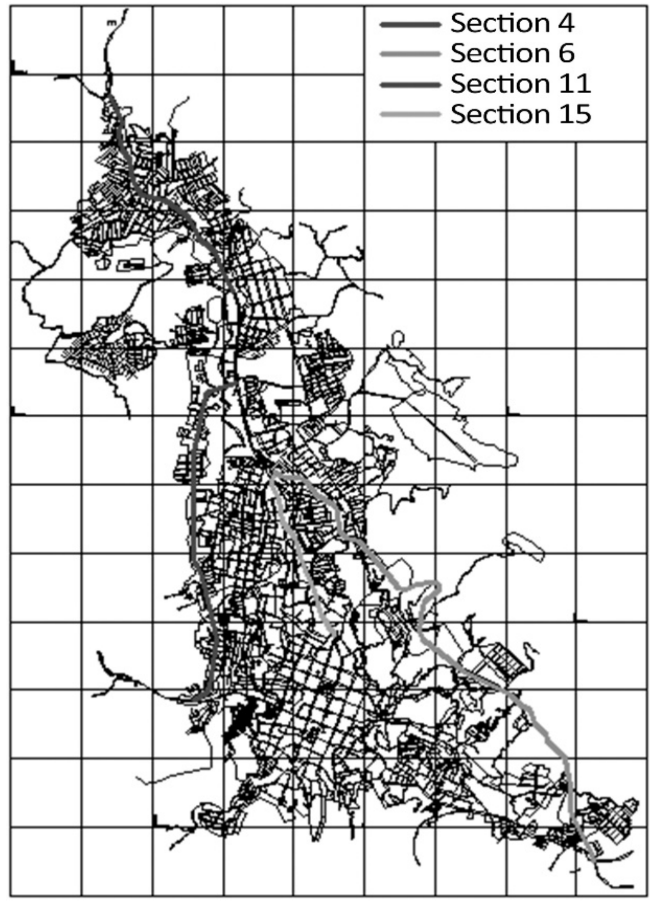

Figure 2. Location of the Accident-prone Sections. Source: The Authors

Table 3.
Identification of Sections Prob ≥ 95%

Section (i)	N	ADT	L	λr	Prob	State
1	45	13.380	751,24	1,3163	1,66%	Non-Critical
2	38	20.047	1.007,66	0,7419	0,00%	Non-Critical
3	28	8.607	1.141,10	1,2733	2,94%	Non-Critical
4	226	14.158	1.801,56	6,2476	100,00%	Critical
5	170	49.506	1.407,54	1,3440	0,38%	Non-Critical
6	179	10.791	4.084,26	6,4923	100,00%	Critical
7	41	12.342	1.233,64	1,3002	1,69%	Non-Critical
8	13	18.568	685,18	0,2740	0,03%	Non-Critical
9	22	12.193	355,85	0,7062	0,00%	Non-Critical
10	4	15.692	374,85	0,0998	0,13%	Non-Critical
11	57	9.632	2.524,63	2,3162	96,96%	Critical
12	25	15.745	732,16	0,6215	0,01%	Non-Critical
13	47	18.381	684,50	1,0008	0,00%	Non-Critical
14	17	10.958	667,54	0,6072	0,00%	Non-Critical
15	150	26.278	1.283,53	2,2341	99,65%	Critical

Source: The Authors

From the analysis and the estimates made with the first approximation, it can be observed that four sections were identified as accident-prone or dangerous sections. The accident-prone sections correspond to those identified as 4, 6, 11 and 15, which are shown in Fig. 2.

Table 4.
Hazard Ranking Criterion 1

Section (i)	λr	λcr	(λr/λcr)>1	Ranking
4	6,25	2,16	2,89	2
6	6,49	2,22	2,92	1
11	2,32	2,25	1,03	4
15	2,23	2,05	1,09	3

Source: The Authors

Table 5.
Hazard Ranking Criterion 2

Section (i)	λr	a	λcr	(λr-λcr)*a	Ranking
4	6,25	36,17	2,16	147,89	1
6	6,49	27,57	2,22	117,79	2
11	2,32	24,61	2,25	1,67	4
15	2,23	67,14	2,05	12,28	3

Source: The Authors

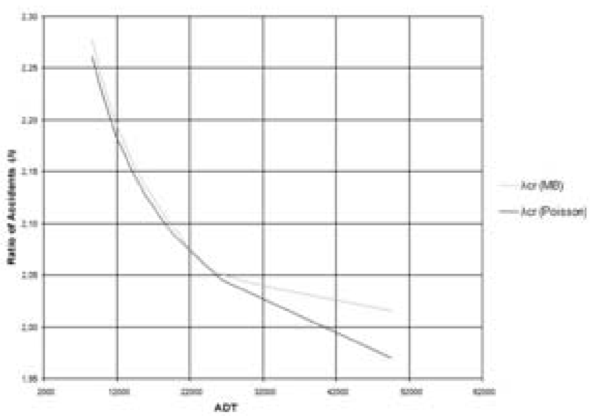

Figure 3. λcr (BM) and λcr (Poisson) Trends.
Source: The Authors

The estimates, presented in Table 3, show the accident-prone sections to be those given by the second approximation; these results show that the BM is an accurate and reliable methodology. The BM offers reductions of 50% in false positives and false negatives as identified by other methods [13].

Having identified the four accident-prone sections of the 15 originally defined sections, it is necessary to determine a hazard ranking of accident-prone sections, given that investments in road safety are very scarce and the efficiency and optimization of these resources is necessary in order to maximize their benefits in terms of road safety. The estimated results per criterion 1 and criterion 2 of the hazard ranking are shown in Table 4 and Table 5. Note that the two analyzed criteria differ. According to criterion 1, the most dangerous section is 6, followed by 4, while criterion 2 puts section 4 in first place in the ranking and then section 6. This situation may be due to criterion 2 giving more weight to the parameter associated with the ADT and its relation to the accidents, thus producing the discrepancy in the ranking of the described situation; however, to be clear, other sections analyzed have the same place in the ranking under both criteria. The sections identified as 4 and 6 correspond to the road section between La Ondina until Defensa Civil and Avenida Circunvalar, respectively.

It was possible to estimate the parameters λ_{cr} following a Poisson-type counting process, which is suitable for this type of analysis. These values are graphed with the values of λ_{cr} obtained by the BM. The comparison of both trends is observed in Fig. 3, in which the adjustment of both curves to a logarithmic model is easily predictable, where the curve corresponding to λ_{cr} (Poisson) is below the curve λ_{cr} (BM). This observation has a direct relationship to the effect of regression towards the mean, thereby producing a more conservative curve, as is also shown by [24].

5. Conclusions

It was possible to identify four accident-prone sections from the application of the approaches used ($\lambda_{cr} \leq \lambda_r$ and Prob $\geq$ 95%). The two types of Bayesian approximations were used in this research in order to identify four accident-prone sites in which similar results were found, minimizing in this way the identification of false positives or false negatives that would influence the results of the research and divert investment of resources to road sections where it is not necessary. These approximations also allow controlling for the effect of regression towards the mean, which is very common in this type of modeling. The estimation results with the MB allow be certain of which are the true accident-prone sections in the municipality of Ocaña.

It was possible to apply the hazard ranking to the four sections identified as accident prone using two criteria. Although identical results were not obtained using both criteria. They are similar, however, and their use is recommended for the prioritization of investments, the explanation of their importance having already been provided.

The methodological approach of the MB applied to the urban area of the municipality of Ocaña gives coherent and accurate results, corroborating that this method contributes to and is suitable for studies of accident rates, and, more specifically, for the identification of accident-prone sites.

It must be clarified that accident data were used in this research, i.e. those that had occurred in the field (not simulated). This is an advantage, given that when one works with real data it is possible to identify accident-prone sections, whereas in the use of other approaches uncontrolled observational environments are evident [23].

The parameter estimation λ_{cr} (Poisson) and λ_{cr} (BM) shows consistency of results and relevance in the use of the methodology applied; the behavior of the curve for both approaches was as expected and supports the results obtained.

Future research may measure the effectiveness of the BM against other methods such as Quantile Regression, Confidence Intervals or the Classification Method.

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References

[1] Instituto Nacional de Medicina Legal y Ciencias Forenses, Forensis 2012 Datos para la vida, Bogotá D.C., 2012.

[2] Karlaftis, M.G. and Golias, I., Effects of road geometry and traffic volumes on rural roadway accident rates, Accident Analysis and Prevention, 34, pp. 357-365, 2002.
[3] Chang, L.Y., Analysis of freeway accident frequencies: Negative binomial regression versus artificial neural network, Safety Science, 43, pp. 541-557, 2005.
[4] García, R.A., Delgado, D.E., Díaz, E.E. y García, R.E., Caracterización de la accidentalidad vehicular y análisis de las causas en la provincia de Villa Clara, Cuba, DYNA, 79 (175), pp. 191-200, 2012.
[5] Winslott, L., Estimating the relationship between accident frequency and homogeneous and inhomogeneous traffic flows, Accident Analysis and Prevention, 36, pp. 985–992, 2004.
[6] Ayati, E. and Abbasi, E., Investigation on the role of traffic volume in accidents on urban highways, Journal of Safety Research, 42, 209-214, 2011.
[7] Venkataram, S., Mannering, F. and Barfield, W., Effect of roadway geometrics and environmental factors on rural freeway accident frequencies, Accident Analysis and Prevention, 27 (3), pp. 371-389, 1995.
[8] Ossiander, E.M. and Cummings, P., Freeway speed limits and traffic fatalities in washington state. Accident Analysis and Prevention, 34 (1), pp. 13-18, 2002.
[9] Rangel, T., Vassalloa, J.M. and Herraiz, I., The influence of economic incentives linked to road safety indicators on accidents: The case of toll concessions in spain, Accident Analysis and Prevention, 59, pp. 529-536, 2013.
[10] Hauer, E., Harwood, D., Council, F. and Griffith, M., The empirical bayes method for estimating safety: A tutorial. Transportation Research Record 1784, pp. 126-131, 2002.
[11] Persaud, B. and Lyon, C., Empirical Bayes before–after safety studies: Lessons learned from two decades of experience and future directions, Accident Analysis and Prevention 39, pp. 546-555, 2007.
[12] Lan, B., Persaud, B., Lyon, C. and Bhim, R., Validation of a full Bayes methodology for observational before–after road safety studies and application to evaluation of rural signal conversions. Accident Analysis and Prevention 41, pp. 574-580, 2009.
[13] Cheng, W. and Washington, S.P., Experimental evaluation of hotspot identification methods, Accident Analysis and Prevention, 37, pp. 870-881. 2005.
[14] Organización Panamericana de la Salud – OPS, Informe sobre el estado de la seguridad vial en la Región de las Américas, Washington, D.C., 2009.
[15] Persaud, B.N. and Hauer, E., Comparison of two methods for debiasing before and after accident studies, Transportation Research Record, 975, pp. 43-49. 1984.
[16] Persaud, B.N., Safety migration, the influence of traffic volumes and other issues in evaluating safety effectiveness. Transportation Research Record, 1086, pp. 33-41, 1986.
[17] Laughlin, J. C., Hauer, L. E., Hall, J. W., & Clough, D. NCHPR Report 162: Methods for Evaluating Highway Safety Improvements. Washington, D.C.: National Research Council, 1975.
[18] Qin, X., Ng, M. and Reyes, P.E., Identifying crash-prone locations with quantile regression. Accident Analysis and Prevention, 42, pp. 1531-1537, 2010.
[19] Geurts, K. and Wets, G., Black spot analysis methods: Literature review. Diepenbeek (Belgium): Flemish Research Center for Traffic Safety, 2003.
[20] Miaoua, S.P. and Song, J.J., Bayesian ranking of sites for engineering safety improvements: Decision parameter, treatability concept, statistical criterion and spatial dependence, Accident Analysis and Prevention, 37, pp. 699-720, 2005.
[21] Miranda-Moreno, L.F., Statistical Models and methods for the identification of hazardous locations for safety improvements, PhD. Thesis, Department of Civil Engineering, University of Waterloo, Waterloo, Canada, 2006.
[22] Geurts, K., Wets, G., Brijs, T., Vanhoof, K. and Karlis, D., Ranking and selecting dangerous crash locations: Correcting for the Number of passengers and Bayesian ranking plots. Journal of Safety Research, 37, pp. 83-91, 2006.
[23] Miranda-Moreno, L.F., Labbe, A. and Fu, L., Bayesian multiple testing procedures for hotspot identification. Accident Analysis and Prevention, 39, pp. 1192-1201, 2007.
[24] Feria-Torres, R.F. y Timaná-Rojas, J.A., El método de Bayes aplicado en la identificación de las localizaciones más propensas a sufrir accidentes de tránsito, Revista del Encuentro Científico Internacional, 5 (2), pp. 21-26. 2008.
[25] Carlin, B. and Louis, T., Bayes and empirical Bayes methods for data analysis, London, Chapman and Hall/CRC Press, 2000.
[26] Higle, J. and Witkowski, J., Bayesian identification of hazardous locations, Transportation Research Records, 1185, pp. 24-36, 1988.
[27] Guerrero, T.E., Gallardo, R.J. and Córdoba, J.A., Análisis estadístico y espacial de corredores viales críticos de accidentalidad en Ocaña, Colombia, Revista Institucional Universidad Tecnológica del Chocó, 2, pp. 103-115, 2012.
[28] Yu, H., Liu, P., Chen, J. and Wang, H., Comparative analysis of the spatial analysis methods for hotspot identification, Accident Analysis and Prevention, 66, pp. 80-88, 2014.
[29] Washington, S., Haque, M., Oh, J. and Lee, D., Applying quantile regression for modeling equivalent property damage only crashes to identify accident blackspots, Accident Analysis and Prevention, 66, pp. 136-146, 2014.
[30] Park, B.J., Lord, D. and Lee, C., Finite mixture modeling for vehicle crash data with application to hotspot identification, Accident Analysis and Prevention, 71, pp. 319-326, 2014.
[31] Also, J. and Langley, J., Under reporting of motor vehicle traffic crash victims in new zealand. Accident Analysis and Prevention, 33, pp. 353-359, 2001.
[32] Brenac, T. and Clabaux, N., The indirect involvement of buses in traffic accident processes, Safety Science, 43, pp. 835-843, 2005.

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