Heuristic for production scheduling on *job-shop* plants considering preventive maintenance tasks

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Abstract
The simultaneous analysis of production scheduling and preventive maintenance task attracts special attention of researchers due to its complexity and therefore the necessity to seek efficient methods for solving this kind of combinatorial problems. This paper presents a heuristic approach to solve this issue on job shop plants. The solution method includes a linear programming model, based on the Traveling Salesman Problem, where the setup time is considered as distance measure. The method´s aim is to obtain a sequence of production orders and preventive maintenance tasks that reduce the idle time and the backlogs simultaneously, accomplishing the maintenance program. After finding an optimal solution for each machine a *Correction Factor* (*CF*) is determined as new distance measure. The *CF* considers the structure of the initial solution, the machine utilization and the product priorities. Then, the final solution is reached running the linear programming model using the distance updated values. Finally, the proposed heuristic is applied to real case study of the Cuban industry. The experimental results indicated a significant idle time reduction for the company under examination.

Keywords: simultaneous programming; production scheduling; preventive maintenance; linear programming model.

Heurística para la secuenciación de producción en plantas *job-shop* considerando tareas de mantenimiento preventivo

Resumen
El análisis simultáneo de la programación de la producción y las tareas de mantenimiento preventivo atrae especial atención en los investigadores debido a su gran complejidad y por ende la necesidad de encontrar métodos eficientes para resolver este tipo de problema combinatorio. Este artículo presenta un enfoque heurístico para resolver dicha problemática en plantas tipo *job-shop*. El método de solución incluye un modelo de programación lineal inspirado en el problema del Agente Vendedor, donde el tiempo de iniciación es considerado como métrica de distancia. El método persigue obtener una secuencia de las órdenes de producción y tareas de mantenimiento preventivo que reduzcan el tiempo ocioso y los retrasos simultáneamente, cumpliendo con el programa de mantenimiento. Luego de encontrar la solución óptima para cada máquina, un factor de corrección (*CF*) es determinado como la nueva medida de distancia. El factor *CF* considera la estructura inicial de solución, la utilización de las máquinas y las prioridades en los productos. De esta forma, la solución final es alcanzada solucionando el modelo de programación lineal usando los valores de distancia actualizados. Finalmente, la heurística propuesta es aplicada en un caso de estudio real de la Industria Cubana. Los resultados experimentales indicaron una reducción significativa del tiempo ocioso para la compañía objeto de estudio.

Palabras clave: programación simultánea; secuenciación de producción; mantenimiento preventivo; programación lineal.

1. Introduction

One of the elements for optimization, while establishing the manufacturing sequence, is the setup time reduction, due to its influence on equipment downtime. This topic has received a great deal of attention by several current researchers, since great savings are achieved when the setup times are included in the scheduling decisions [1]. In this context, the maintenance tasks programming are inserted, as well as the search for options which allow to generate production–maintenance integrated plans on this level [l-6].

According to [1] most of the researches considering setup

times are aimed to flow shop scheduling, while job shop scheduling had been less studied. In the latter case, we stand out the researches carried out by [7] who proposed a heuristic method to minimize the makespan. Authors in [8] tackled this type of problem, in job shop scheduling, through a mixed integer linear programming, developing a heuristic method for its solution, having likewise as an objective, to minimize the makespan. In [9] this is approached considering features such as: release dates, due dates and transportation times. Also [10] developed a hybrid algorithm based on Genetic Algorithms (GAs) which prioritizes the orders according to the shorter processing time dispatching rule; the first operations for every machine are defined from GA, whereas the remaining operations are programmed through the referred dispatching rule, having as an objective the makespan minimization. Similar to the former evolutionary strategy, [11] proposes an elitist GA, which provides a Pareto solution set minimizing simultaneously the makespan and maintenance total cost.

On the other hand, [1] carried out a study applying four metaheuristics to solve a set of instances from the general scheduling problem (including preventive maintenance tasks), considering the machines setup time. Three preventive maintenance (PM) policies were taken into account: 1) PMat fixed predefined time intervals; 2) PM optimum period model, maximizing the machines' availability; 3) maintaining a minimum reliability threshold for a given period. This policies were included in the hybrid metaheuristics: GA–Shorter Processing Time Dispatching Rule (GA-SPT), Simulated Annealing (SA) –Shorter Processing Time Dispatching Rule (SA-SPT), GA adapted for production scheduling and preventive maintenance task simultaneously, from the one proposed by [10] (GA-Ch), and the Immune Algorithm (IA) introduced by [12] for job shop scheduling considering setup times (IA-YZ). After a performance analysis it was concluded that among the metaheuristics, the hybrid method GA-SPT gave the best results and there were no significant differences between the results obtained through SA-SPT and GA-Ch methods; in addition, the IA-YZ showed less solution quality and with the application of the dispatching rule SPT, separately, the worse result was obtained.

Despite the great amount of techniques developed for solving the production scheduling and preventive maintenance problem, including the exact methods [4], heuristics [5] and metaheuristics [13], it cannot be stated that an optimal option for its treatment has been found due to the disadvantages of all of these approximate proposals; for instance, the fact that being so sophisticated, great efforts for its codification and implementation are needed. In other cases the proposals developed lack generality. Many of these methods have specific characteristics, which are rarely applicable to extensions from the original problem.

The latter arguments demonstrate the complexity of the production scheduling and preventive maintenance task problem, considering one or several machines. Furthermore, there exist other real-life characteristics from corporate practice context that raising this complexity when taken into account. Underestimated the aforementioned elements may lead to design models which can give an inappropriate

representation of the real problem and, therefore, the solutions would be quite ineffective.

Considering the above, in this paper a heuristic algorithm is proposed, which is aimed to obtain the effective production scheduling and preventive maintenance task in job shop process, minimizing the equipment downtime as a result from the setup time reduction, and thus, facilitating the accomplishment of products delivery due date, having as a premise the compliance of the established preventive maintenance program.

This paper is structured as follows: Section 2 is dedicated to the description of the optimization problem, considering specifically, the integration of the production order scheduling and the preventive maintenance tasks. It is also presented in this section, the mathematical formulation of the problem, as well as a detailed explanation of the steps for the proposed algorithm. In Section 3 the main experimental results from the application of the heuristic algorithm are presented. Afterwards, a discussion about the achieved results is addressed in Section 4. Finally, some conclusions of this research are presented in Section 5.

2. Problem formulation and algorithmic proposal

2.1. Description of the problem

The production environment is characterized by a set M machines and N types of products. The manufacturing for each product requires its processing in all or some of the machines, according to its technological sequence. According to the delivery due date, the company defines the priority of each product. The processing time and the setup times for the machines are known. The time for the preventive maintenance task for each machine is known according to the maintenance program.

Correspondingly to the established manufacturing strategy in the company, the planning objectives in this level are aimed to achieve an optimal exploitation of the production capacity, as well as complying with the delivery due dates. These two decisions can be formulated according to the criteria of minimizing the machines setup time and, simultaneously, minimizing the delivery backlogs.

Thus, the proposed problem consists on establishing the production order sequence (batches of each type of product) and preventive maintenance task on every machine, which minimizes their setup time, accomplishing the product delivery dates and the preventive maintenance program established for each machine.

As a model premise, should be considered that: the manufacturing for a product batch must be not started if its execution time overcomes the remaining time to start the preventive maintenance task. Production interruptions are not acceptable once a batch processing has started, except for failures.

2.2. Heuristic algorithm proposed

The heuristic algorithm proposed in this paper consists of four general steps, which are described below:

Step 1.Initial solution search

It refers to the search of an initial sequence for jobs processing (product batches), independently, on each machines. This initial sequence will minimize the setup time and backlogs. Then, the algorithm is as follows:

For each machine $m_i \in M$ ***do***:

P1.1Initialize $N_{Si} = 0$ (Initial number of products)

P1.2 Implement the model as follows:

Parameters:

n: Number of products (production orders) to be scheduled.

i, j, k: Index referring to products.

m: Index referring to preventive maintenance task.

l: Total orders to be scheduled in the machine, including production and preventive maintenance task.

t_{ij}: Machine setup time between ending order i and beginning order j.

t_{km}: Machine setup time between ending order k and beginning preventive maintenance task.

$t0_j$: Processing time of order$j$.

Pr_i: Priority of order i.

Pr_j: Priority of order j.

Pr_k: Priority of order k.

TMP: Time between preventive maintenance tasks.

Sn: n-th component of the subcircuit or analyzed circuit.

TS: Length of a subcircuit or circuit.

Decision Variables:

x_{ij}: Binary variable, which indicates if the order j is processed {1} or not {0} after the order i.

x_{km}: Binary variable, which indicates if the preventive maintenance task$m$will be scheduled {1} or not {0} in the sequence after the order k.

x_{0k}: Binary variable, which indicates if the order k will be the first {1} or not {0} in the production sequence.

H: Slack variable.

Objective function:

To reduce the setup time (z) in the equipment,

$$\min z = \sum_{i=0}^{n} \sum_{j=1}^{l} (t_{ij} * x_{ij}) \\ + \sum_{k=1}^{n} (100 * t_{km} * x_{km}) \quad (1) \\ i \neq j; \quad l = 1, \dots, n, m$$

It was used the constant value of 100 in order to ensure that the model sequences the major quantity of orders until the moment for the preventive maintenance task occurs. This value was defined after some *a priori* values previously tested.

Constraints:

1) To ensure the accomplishment for the preventive maintenance program,

$$\sum_{i=0}^{n} \sum_{j=1}^{l} (t_{0j} * x_{ij}) + H \leq TMP \quad i \neq j \quad (2)$$

2) To ensure that one order can be preceded by one order only,

$$\sum_{j=0}^{n} x_{ji} = 1 \quad \forall i, i = 1, \dots, n, m \quad i \\ \neq j \quad (3)$$

3) To ensure that from a node only comes out one arc (i.e. an order can be preceded by one order only),

$$\sum_{j=1}^{M} x_{ij} = 1 \quad \forall i, i = 0, \dots, n \quad i \neq j \quad (4)$$

4) To avoid subcircuits and circuit. Specifically for all of the long subcircuits TS, TS $= \{2, \dots, n\}$ and circuit,

$$x_{ij} + x_{jk} +, \dots + x_{ts,i} \leq |TS - 1| \\ \forall i = \{1, \dots, (Sn - 2)\}, \\ j = \{i + 1, \dots, (Sn - 1)\}, \quad (5) \\ k = \{j + 1, \dots, Sn\}$$

5) To ensure a logical sequence,

$$x_{ik} - \sum_{j=1}^{M} x_{kj} \leq 0 \\ \forall i = \{0, \dots, n\}, \quad (6) \\ k = \{1, \dots, n\} \\ i \neq j; \quad k \neq j; \quad i \neq j$$

$$\sum_{i=1}^{n} x0_i = 1 \quad (7)$$

If N_{Si} is lower than the number of products N_{ei} to be manufactured in the machine m_i, repeat step P1.2 with the remaining products for sequencing; otherwise go to Step 2.

Step 2. Obtaining the relative importance for every machine

The relative importance (weight) for every machine can be computed based on various criteria. One of them, which is the one adopted in this paper, is the capacity utilization rate, which aims to favor the most loaded machines in the obtained final sequence. For the weight determination, any of the qualitative or quantitative current methods can be applied.

Step 3. Obtaining a corrected distance measure

With this new measure, it is attempted to correct the initial distance measure adopted in Step 1. To obtain it, a coefficient FO_{ij} is proposed (order factor for product j with respect to machine i. This coefficient integrates three factors:

- FPE_j: Priority factor for product j based on its delivery date. This factor is calculated only once for each product, regardless the machines.

- FPT_{ij}: Technological priority factor for product j with respect to the machine i. It constitutes a proximity measure between the position of product j, according to the initial sequence, with respect to the machine i, and the weighted position of the product considering all the machines, also based on the initial sequence. This factor is calculated only once, for each product depending on

every machine, using the information supplied by the initial sequence of every machine.

- FI_{ij}: Starting factor for product j with respect to the machine i. It is a measure for the availability of the product j, in a specific moment, to start its processing in a machine i after being processed in the machine $i-1$ according to its operation sequence. The starting factor must be calculated for every machine as the machine is going to be sequenced. This factor ranges from $1/n$ to $(2n-1)/n$.

The proposed mathematical expressions to calculate these factors and the corrected distance are the following:

$$DC_{ij} = FO_{ij} * tp_{ij}$$
$$\forall i = 1, \dots, m \quad and \quad \forall j = 1, \dots, n \qquad (8)$$

$$FO_{ij} = FPE_j * FPT_{ij} * FI_{ij}$$
$$\forall i = 1, \dots, m \quad and \quad \forall j = 1, \dots, n \qquad (9)$$

If it is assigned $Pr_j = n$ to the product with the highest priority,

$$FPE_j = 1 + \frac{(n - Pr_j)}{n}$$
$$\forall i = 1, \dots, m \quad and \quad \forall j = 1, \dots, n \qquad (10)$$

If it is assigned $Pr_j = 1$ to the product with the highest priority,

$$FPE_j = 1 + \frac{(Pr_j - 1)}{n}$$
$$\forall i = 1, \dots, m \quad and \quad \forall j = 1, \dots, n \qquad (11)$$

$$FPT_{ij} = 1 + \frac{[OSIMSM - \sum_{i=1}^{m}(PPSI_{ij} * PM_i)]}{n + 1}$$
$$i = 1, \dots, m \quad and \quad \forall j = 1, \dots, n \qquad (12)$$

$$FI_{ij} = 1 + \frac{OPMAS - OSIMSM}{n + 1}$$
$$\forall i = 2, \dots, m \quad and \quad \forall j = 1, \dots, n$$
$$if \quad i = 1; \ FI_j = 1 \ \forall j = 1, \dots, n \qquad (13)$$

where:

DC_{ij}: Corrected distance for the product j used for obtaining the final sequencing with respect to the machine i.

FO_{ij}: Order factor for product j with respect to machine i.

tp_{ij}: Setup time in the machine when product i is changed for product j according to the initial sequence.

$PPSI_{ij}$: Position for product j according to the initial sequence of the machine i.

PM_i: Weight for the machine i.

n: Number of products to be manufactured.

m: Number of machines

Pr_j: Priority order given to the product j according to its delivery date. $Pr_j = \{1, \dots, n\}$.

$OPMAS$: Order in which the product would be processed in the previously sequenced machine to the one that is being sequenced at the present moment, according to the initial sequence.

$OSIMSM$: Order of the product in the initial sequence for the machine that is being sequenced at the present moment.

Step 4. Obtaining the final solution

For every machine $m_i \in M do$:

P4.1 Initialize $N_{Si} = 0$

P4.2 Implement the proposed mathematical model in P1.2, but taking into account as a distance measure the corrected distances (DC_{ij}).

If N_{Si} is lower than the number of products N_{ei} (products to be manufactured in the machine m_i) repeat P4.2 with the remaining products for sequencing; otherwise take the solution obtained for every machine as a final integrated sequence for production orders and preventive maintenance.

3. Experimental results

Step 1. Implementation: Searching for an initial solution

The production environment is characterized by a set of five machines ($M = 5$): mechanical saw ($m1$), conventional lathe ($m2$), milling machine ($m3$), grinding machine ($m4$), sharpening machine ($m5$); and a set N of six (6) types of products: punches ($n1$), matrix ($n2$), dies ($n3$), pintles ($n4$), blades ($n5$) and nozzles ($n6$). The processing sequence and priority for every product is shown in Table 1.

Table 2 contains the processing time of each product on every machine; whereas Tables 3, 4, 5, 6 y 7 show the setup time of every machine to change from one product to another, as well as the time between orders and preventive maintenance task.

Table 1.
Technological sequence and priority for every product.

Products	m1	m2	m3	m4	m5	Priority
n1	1	2	3	4	5	1
n2	1		2	3	4	4
n3	1	2	3			3
n4	1		2	3		2
n5	1		2	3		6
n6	1	2	3	4		5

Source: The autors

Table 2.
Product processing time (h/batch).

Products	m1	m2	m3	m4	m5
n1	10	33.3	25	43.2	1
n2	3.2		33.3	60	20
n3	10	26.6	5.8		
n4	33		125	75	
n5	6.6		225	40	
n6	16.6	30	50	86.7	

Source: The autors

Table 3.
Setup time among the products and among themselves and the preventive maintenance task (IP) in m1 (h/batch).

	n1	n2	n3	n4	n5	n6	IP
n1	-	1	1.5	1.5	1.5	1.3	0.5
n2	1	-	2	2.5	1	1.25	1
n3	3	2.5	-	1.5	1.5	1.5	0.5
n4	2	1.5	2	-	2	1.5	1.5
n5	1.5	1.5	2.5	1	-	1.8	1
n6	2	1.3	1	1.5	1	-	0.5
Start	1	0.5	1.5	1	1.5	0.8	-

Source: The autors

Table 4.
Setup time among the products and among themselves and the preventive maintenance task (IP) in m2 (h/batch).

	n1	n3	n6	IP
n1	-	1.6	1.4	0.8
n3	1	-	2	0.8
n6	1.5	1.25	-	1
Start	0.5	0.2	0.8	-

Source: The autors

Table 5.
Setup time among the products and among themselves and the preventive maintenance task (IP) in m3 (h/batch).

	n1	n2	n3	n4	n5	n6	IP
n1	-	2.5	2	1.5	1	0.8	0.6
n2	1	-	2	1.8	1	1	0.8
n3	1.5	2	-	1	1.5	1	0.8
n4	0.8	1.3	1.5	-	2.5	1.5	1
n5	1.25	1.3	1.5	1	-	1.5	0.7
n6	2	2.3	1	1.25	0.8	-	0.5
Start	0.3	0.2	0.5	0.5	0.6	0.25	-

Source: The autors

Table 6.
Setup time among the products and among themselves and the preventive maintenance task (IP) in m4 (h/batch).

	n1	n2	n4	n5	n6	IP
n1	-	1.2	1.3	1.5	1	0.7
n2	0.5	-	1	1.5	1.25	1
n4	0.8	1.7	-	0.75	2	1.25
n5	2	1.6	1	-	1.3	0.5
n6	0.7	1	1.25	2	-	0.5
Inicio	0.3	0.25	0.5	0.4	0.3	-

Source: The autors

Table 7.
Setup time among the products and among themselves and the preventive maintenance task (IP) in m5.

	n1	n2	IP
n1	-	2.5	0.8
n2	2	-	1.3
Inicio	0.25	0.2	-

Source: The autors

The preventive maintenance task, in every one of the five machines taken into account, must be done every 72, 72, 240, 192 y 24 operating hours, respectively. The initial sequence of production orders and preventive maintenance task (IP) for each one of the 5 machines is showed in Table 8.

Table 8.
Initial product sequence and preventive maintenance tasks.

	Machines				
Products and IP	m1	m2	m3	m4	m5
n1	1	2	4	2	2
n2	6	-	3	1	1
n3	4	4	1	-	
n4	2	-	2	5	
n5	7	-	7	6	
n6	3	1	5	3	
IP	5	3	6	4	3

Source: The autors

In Table 9 are presented possible performance measures for each one of the machines, based on the initial sequence obtained: the total setup time (TTP), the idle time while waiting for the moment to start the maintenance task, given by the slack variable (H) used in the mathematical model and,

Table 9.
Performance measures of the initial solution.

Machines	TTP [h]	H [h]	A [%]
m1	66.7	2.4	96.5
m2	91	8.7	91.3
m3	55.5	0.9	98.4
m4	53.85	2.1	96.2
m5	82.2	24.8	76.8

Source: The autors

Table 10.
Relative importance of each machine.

Machines	Q [h]	Weight
m1	79.4	0.08
m2	89.9	0.09
m3	464.1	0.48
m4	304.9	0.32
m5	21	0.02

Source: The autors

Table 11.
Priority factor according to the delivery date.

Products	FPE_j
n1	1
n2	1.50
n3	1.33
n4	1.16
n5	1.83
n6	1.66

Source: The autors

Table 12.
Technological priority factor of each product in every machine.

		Machines				
		m1	m2	m3	m4	m5
Products	n1	0.79	0.79	1.22	0.79	0.94
	n2	1.35		0.92	1.06	0.64
	n3	1.35	1.07	0.92		
	n4	0.85		0.85	1.28	
	n5	1.06		1.06	0.92	
	n6	0.89	1.04	1.18	0.75	

Source: The autors

the corresponding capacity utilization rate (A), resulting from the latter measure (idle time).

Once all products in every machine have been sequenced, the **Step 2** is applied.

Step 2. Implementation: Obtaining the relative importance of every machine

The relative importance (weight) of each machine is established according to its workload (Q) expressed in hours, so the most loaded machines obtain more weight. The results are shown in Table 10.

Step 3. Implementation: Obtaining the corrected distance for each product on every machine

In Table 11 is presented priority factor for product j based on its delivery date (FPE_j). Then, in Table 12, is shown technological priority factor for product j with respect to the machine i (FPT_{ij}).

Table 13.
Starting factor of each product in every machine.

		Machines			
	m1	m2	m3	m4	m5
n1	1	1	0.57	1.43	0.86
n2	1		1.43	0.86	1.43
n3	1	1.29	1.14		
n4	1		1	0.57	
n5	1		1	1.14	
n6	1	0.86	0.86	1.43	

Source: The autors

Table 14.
Order factor of each product in every machine.

		Machines			
	m1	m2	m3	m4	m5
n1	0.79	0.79	0.69	1.13	0.8
n2	2		1.97	1.37	1.37
n3	1.8	1.84	1.39		
n4	1		1	0.85	
n5	1.94		1.94	1.92	
n6	1.48	1.48	1.68	1.78	

Source: The autors

Table 15.
Setup and starting times corrected in m1.

	n1	n2	n3	n4	n5	n6	IP
n1	-	2	2.7	1.5	2.91	1.92	0.5
n2	0.79	-	3.6	2.5	1.94	1.85	1
n3	2.37	5	-	1.5	2.91	2.22	0.5
n4	1.58	3	3.6	-	3.88	2.22	1.5
n5	1.19	3	4.5	1	-	2.66	1
n6	1.58	2.6	1.8	1.5	1.94	-	0.5
Starting	0.79	1	2.7	1	2.91	1.18	-

Source: The autors

Table 16.
Setup and starting times corrected in m2.

	n1	n3	n6	IP
n1	-	2.94	2.07	0.8
n3	0.79	-	2.96	0.8
n6	1.19	2.3	-	1
Starting	0.4	0.37	1.18	-

Source: The autors

Table 17.
Setup and starting times corrected in m3.

	n1	n2	n3	n4	n5	n6	IP
n1	-	4.93	2.78	1.5	1.94	1.34	0.6
n2	0.69	-	2.78	1.8	1.94	1.68	0.8
n3	1.04	3.94	-	1	2.91	1.68	0.8
n4	0.55	2.56	2.09	-	4.85	2.52	1
n5	0.86	2.56	2.09	1	-	2.52	0.7
n6	1.38	4.53	1.39	1.25	1.55	-	0.5
Starting	0.21	0.39	0.7	0.5	1.16	0.42	-

Source: The autors

Table 18.
Setup and starting times corrected in m4.

	n1	n2	n4	n5	n6	IP
n1	-	1.64	1.11	2.88	1.78	0.7
n2	0.57	-	0.85	2.88	2.23	1
n4	0.9	2.33	-	1.44	3.56	1.25
n5	2.26	2.19	0.85	-	2.31	0.5
n6	0.79	1.37	1.06	3.84	-	0.5
Starting	0.34	0.34	0.43	0.77	0.53	-

Source: The autors

Table 19.
Setup and start times corrected in m5.

	n1	n2	IP
n1	-	3.43	0.8
n2	1.6	-	1.3
Starting	0.2	0.27	-

Source: The autors

Table 20.
Final processing sequence and preventive maintenance task

		Machines			
Products and IP	m1	m2	m3	m4	m5
n1	4	2	4	2	2
n2	6	-	1	1	1
n3	2	4	2	-	
n4	3	-	3	6	
n5	7	-	7	5	
n6	1	1	5	3	
IP	5	3	6	4	3

Source: The autors

Table 21.
Performance measures of the final solution obtained.

Machines	TTP [h]	H [h]	A [%]
m1	58.46	2.4	96
m2	91.44	8.7	91.3
m3	56.96	0.9	98.4
m4	183.11	2.1	98.9
m5	81.87	25.1	76.5

Source: The autors

In Table 13 is shown the starting factor for product j with respect to the machine i (FI_{ij}). Thereafter, in Table 14, is shown the order factor for product j with respect to machine i (FO_{ij}).

In Tables 15, 16, 17, 18 and 19 are shown the corrected representative distance measures of the setup times in each machine (in hours).

Table 22.
Start and end time of each product on every machine and machines downtime (t_i) according to the initial sequence.

Products and IP	Machines				
	m1	m2	m3	m4	m5
n1	1 / 11	95.9 / 129.2	496.8 / 521.8	556.55 / 599.75	578.25 / 579.25
n2	79.6 / 82.8	-	462.5 / 495.8	496.05 / 556.05	556.25 / 576.25
n3	64.6 / 74.6	134.2 / 160.8	161.3 / 167.1	-	-
n4	12.5 / 45.5	-	168.1 / 461.2	692.7 / 767.7	-
n5	83.8 / 90.4	-	577.7 / 802.7	768.45 / 808.45	-
n6	47 / 63.6	64.4 / 94.4	522.6 / 572.6	601 / 687.7	-
IP	75.1 / 79.1	130 / 134	573.1 / 577.1	688.2 / 692.2	580.05 / 582.05
t_i	7	66.2	166.5	499.55	559.05

Source: The autors

Table 23.
Start and end time of each product on every machine and machines downtime (t_i) according to the final sequence.

Products and IP	Machines				
	m1	m2	m3	m4	m5
n1	64.9 / 74.9	75.4 / 108.7	273.4 / 298.4	298.7 / 341.9	342.15 / 343.15
n2	79.9 / 83.1	-	83.3 / 116.6	116.85 / 176.85	177.05 / 197.05
n3	18.4 / 28.4	113.7 / 140.3	140.8 / 146.6	-	-
n4	29.9 / 62.9	-	147.6 / 272.6	620.7 / 695.7	-
n5	84.1 / 90.7	-	354.3 / 579.3	579.7 / 619.7	-
n6	0.8 / 17.4	18.2 / 48.2	299.2 / 349.2	349.5 / 436.2	-
IP	75.4 / 79.4	109.5 / 113.5	349.7 / 353.7	436.7 / 440.7	343.95 / 347.95
t_i	7.3	46.4	111.2	386.8	322.9

Source: The autors

In Table 20 the final processing sequence and preventive maintenance tasks for every machine is shown. In Table 21 are shown the performance measures previously described.

In Tables 22 and 23 it is shown the moment where each product begins its processing in each one of the machines and the corresponding downtime of each one of these machines, obtained from the initial and final sequence, respectively. Considering that the processing starts at zero time and there are no other products in process, the initial solution reported a total downtime of 1298.3 hours.

According to Table 23 the total downtime is 874.6 hours. Therefore, the proposed heuristics achieves a reduction of 423.7 hours respecting to the initial solution, which corresponds to the generation of local optimum solutions in each one of the machines. The expression (14) allows quantifying the total weighted disruption time, based on the fact that one of the defined objectives is to achieve the maximum equipment utilization, giving a major priority to the most loaded machines.

Replacing the results of the initial solution in the expression (14), a total weighted disruption time of 257.5 h. is obtained. This performance measure, according to the final solution provided by the proposed method is reduced to 188.4 h.

$$T_{ip} = \sum_{i=1}^{m} t_i \cdot p_i \qquad (14)$$

where:
T_{ip}: Total weighted downtime.
t_i: Downtime of machine i.
p_i: Weight of the machine i according to its utilization level (Table 10).

4. Discussion

Despite that the proposed method does not consider many of the situations that can occur when the production scheduling and preventive maintenance tasks are undertaken in an integrated manner, such as random failure events and their impact on manufacturing technology and even on the maintenance and production program obtained, a scheduling horizon definition which covers more than one preventive maintenance task, among others, is considered an efficient tool for scheduling within the treated study, being its extension possible to productive contexts with similar characteristics, where the machine setup time takes the significant values within the shift work. It is important to establish the objectives to the scheduling process, and the priority to achieve them, as one of the basic heuristic inputs, since the structure of the final solution obtained will depend greatly on it.

In the examined case study, since it was more important for the decision maker to minimize the downtime than complying with the delivery dates, it can be observed how the major priority products ($n1$ y $n4$) are not the first to be completed, which could be interpreted as an inconsistency in the proposed method; however, this result responds to the hierarchical structure which takes the branch and bound method used to solve the linear integer programming model. For subsequent papers it will be interesting to consider the variations that could be obtained in the solution structure if only one objective function was applied in the mathematical model, which could integrate the objectives that are expected to be optimized, assigning them different weights in the equation in order to model the hierarchical importance that the decision maker adopts for them, besides considering the

possibility of assigning different weights to the components which conform the order factors used to obtain the corrected distance measure.

5. Conclusions

In the literature, several proposals can be found focused on obtaining production order sequences and preventive maintenance tasks in an integrated manner, which include mathematical models and proper heuristic methods from the field of Artificial Intelligence, due to the complexity that this type of problem entails; however, it cannot be stated that an optimal option for its treatment has been found due to the disadvantages of all of these proposals, for instance, the fact that being so sophisticated, great efforts for its codification and implementation are needed and, in other cases, many of these methods have specific characteristics which are rarely applicable to branches from the origin problem.

The heuristic method proposed in this paper allows obtaining the optimal processing sequence and preventive maintenance tasks, so the equipment setup time is minimized and the downtime as well, contributing to achieve the delivery dates compliance for the products and the maintenance plan previously established is kept. It includes the design of a mathematical model based on the general scheme of the model used in the Traveling Salesman Problem, setting up certain constraints to consider the time between preventive maintenance tasks on the machines, besides avoiding circuits, which would conform an incoherent solution for the tackled problem.

The validation of the proposed method was carried out from its application in a productive context characterized by five machines and six types of products, considering a preventive maintenance task for every machine. This solution reported a reduction of 423.7 hours in the total downtime.

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