



# Short-term generation planning by primal and dual decomposition techniques

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## Abstract

This paper addresses the short-term generation planning (STGP) through thermoelectric units. The mathematical model is presented as a Mixed Integer Non Linear Problem (MINLP). Several works on the state of art of the problem have revealed that the computational effort of this problem grows exponentially with the number of time periods and number of thermoelectric units. Therefore, we present two alternatives to solve a STGP based on Benders' partitioning algorithm and Lagrangian relaxation in order to reduce the computational effort. The proposal is to apply primal and dual decomposition techniques, which exploit the structure of the problem to reduce solution time by decomposing the STGP into a master problem and a subproblem. For Benders' algorithm, the master problem is a Mixed Integer Problem (MIP) and for the subproblem, it is a Non Linear Problem (NLP). For Lagrangian relaxation, the master problem and the subproblem are MINLP. The computational experiments show the performance of both decomposition techniques applied to the STGP. These techniques allow us to save computation time when compared to some high performance commercial solvers.

*Keywords:* Benders' algorithm; Lagrangian relaxation; subgradient; decomposition techniques; power generation.

# Planeación de la generación a corto plazo mediante técnicas de descomposición primal y dual

## Resumen

En este trabajo se aborda la planeación de la generación a corto plazo (STGP) a través de las unidades termoeléctricas. El modelo matemático se presenta como un problema no lineal entero mixto (MINLP). Varios trabajos del estado del arte del problema han revelado que el esfuerzo computacional de este problema crece exponencialmente con el número de periodos de tiempo y número de unidades termoeléctricas. Por lo tanto, presentamos dos alternativas para resolver la planeación de la generación a corto plazo (STGP) basadas en el algoritmo de partición Benders y relajación lagrangiana con la finalidad de reducir el esfuerzo computacional. La propuesta consiste en aplicar técnicas de descomposición primal y dual que explotan la estructura del problema para reducir el tiempo de solución mediante la descomposición de la planeación de la generación a corto plazo (STGP) en un problema maestro y un subproblema. Para el algoritmo de Benders el problema maestro es un problema entero mixto (MIP) y para el subproblema es un problema no lineal (NLP). Para la relajación lagrangeana, el problema maestro y el subproblema son MINLP. Los experimentos computacionales muestran el rendimiento de ambas técnicas de descomposición aplicadas a la planeación de la generación a corto plazo (STGP). Estas técnicas permiten ahorrar tiempo de cálculo en comparación con algunos optimizadores comerciales de alto rendimiento.

*Palabras clave:* Algoritmo de Benders; relajación lagrangiana; subgradiente; técnicas de descomposición; generación eléctrica.

## 1. Short-term generation planning

The efficient short term generation planning of available energy resources for satisfying load demand has become an important task in modern power systems [14,17].

A definition of short-term generation planning (Unit commitment) is the operation of generation facilities to produce energy at the lowest cost to reliably serve consumers, recognizing any operational limits of generation facilities. The problem consists of deciding which units must

be used over a given planning horizon, usually 24 time periods.

In the typical Unit Commitment [5], the transmission network is not considered, so for this case we considered network constraints, and the problem consists of determining the mix of generators and their estimated output level to meet the expected demand of electricity over a given time horizon (a day or a week), while satisfying the load demand, spinning reserve requirement and transmission network constraints. An electric network consists of many generation nodes with various generating capacities and cost functions, lines of transmission and nodes of power demand [7,9,12].

Over the past few years, several studies have been conducted to define appropriate models and algorithms in order to obtain the optimal solution of the STGP. There are many optimization techniques for solving this problem, the major solution approaches are decomposition techniques, branch-and-bound techniques, and metaheuristics [15,16].

Since the Short Term Generation Scheduling (STGS) with network constraints is a NP-hard Mixed Integer Non Linear Problem, for large power systems, exact methods proved to be inefficient [10]. Because of this, we present an alternative to generating quality bounds in short computing time based on primal and dual decomposition.

## 2 The Model

Parameters:

- $A_j$  Start up cost of power plant  $j$ .
- $B_{nm}$  Susceptance of line  $n - m$ .
- $C_{nm}$  Transmission capacity limit of line  $n - m$ .
- $D_{nk}$  Load demand at node  $n$  during period  $k$ .
- $E_j(t_{jk})$  Nonlinear function representing the operating cost of power plant  $j$  as a function of its power output in period  $k$ .
- $E_{j1}$  Linear coefficient of operating cost for plant  $j$ .
- $E_{j2}$  Quadratic coefficient of operating cost for plant  $j$ .
- $F_j$  Fixed cost of power plant  $j$ .
- $V_{jk}^j$  Parameter that is equal to 1 when plant  $j$  is committed in period  $k$  after dual subproblem is solved.
- $Y_{jk}$  Parameter which is equal to 1 when plant  $j$  is started up at the beginning of period  $k$  after dual subproblem is solved.
- $K_{nm}$  Conductance of line  $n - m$ .
- $R_k$  Spinning reserve requirement during period  $k$ .
- $T_j$  Maximum power output of plant  $j$ .
- $\bar{T}_j$  Minimum power output of plant  $j$ .
- $\bar{n}\bar{r}$  Reference node with angle zero.

Variables:

- $t_{jk}$  Power output of plant  $j$  in period  $k$ .
- $v_{jk}^j$  Binary variable equal to 1 when plant  $j$  is committed in period  $k$ .
- $y_{jk}$  Binary variable equal to 1 when plant  $j$  is started up at the beginning of period  $k$ .
- $\delta_{nk}$  Angle of node  $n$  in period  $k$ .
- $\lambda_{nk}$  Lagrangian multiplier associated to a power balance constraint.

- $\mu_k$  Lagrangian multiplier associated to a spinning reserve requirement.
- $\gamma_{nk}, \beta_{nk}$  Lagrangian multipliers associated to transmission capacity limits.

Sets:

- $J$  Set of indices of all plants.
- $K$  Set of period indices.
- $N$  Set of indices of all nodes.
- $\Lambda_n$  Set of indices of the power plants  $j$  at node  $n$ .
- $\Omega_n$  Set of indices of nodes connected and adjacent to node  $n$ .
- $\Phi$  Set of Benders' iterations.

The problem STGP is defined as follows:

$$\text{Min } Z = \sum_{k \in K} \sum_{j \in J} [F_j v_{jk} + A_j y_{jk} + E_j(t_{jk})] \quad (1)$$

$$\sum_{j \in \Lambda_n} t_{jk} + \sum_{m \in \Omega_n} B_{nm} [\delta_{nk} - \delta_{mk}] - \sum_{m \in \Omega_n} K_{nm} [1 - \cos(\delta_{nk} - \delta_{mk})] = D_{nk} \quad (2)$$

$$\forall n \in N, \forall k \in K$$

$$\sum_{j \in J} \bar{T}_j v_{jk} \geq \sum_{n \in N} D_{nk} + R_k \quad \forall k \in K \quad (3)$$

$$\bar{T}_j v_{jk} \leq t_{jk} \leq \bar{T}_j v_{jk} \quad \forall j \in J, \forall k \in K \quad (4)$$

$$y_{jk} \geq v_{jk} - v_{jk-1} \quad \forall j \in J, \forall k \quad (5)$$

$$-C_{nm} \leq B_{nm} [\delta_{nk} - \delta_{mk}] \leq C_{nm} \quad \forall n \in N, \forall k \in K, \forall m \in \Omega_n \quad (6)$$

$$-\pi \leq \delta_{nr} \leq \pi \quad \forall n \in N \setminus \{nr\}, \forall k \in K \quad (7)$$

$$\delta_{nrk} = 0 \quad nr \in N, \forall k \in K \quad (8)$$

$$v_{jk}, y_{jk} \in \{0,1\} \quad \forall j \in J, \forall k \in K \quad (9)$$

The objective eq. (1) minimizes the start up cost  $A_j y_{jk}$  and operating cost of each plant. The operating cost of each plant  $j$  is included a fixed cost  $F_j v_{jk}$  and a variable cost  $E_j(t_{jk})$ .

There is a power balance constraint eq. (2) per node and time period. In each period, the production has to satisfy the demand and losses in each node. Power line losses are modeled through cosine approximation and it is assumed that the demand for electric energy is known and is discretized into  $t$  periods. There are many approximations to model power line losses, some of them are linear and non-linear approximations. Further details of the cosine approximation

can be found in [1].

Spinning reserve requirements are modeled in eq. (3). In each period the running units have to be able to satisfy the demand and the prespecified spinning reserve.

In eq. (4), each unit has a technical lower and upper bound for the power production. Transmission capacity limits of lines eq. (5) avoid dynamic stability system problems. The constraint eq. (6) holds the logic of running, start-up and shut-down of the units.

A running unit cannot be started-up. Finally, angle in all buses has a lower and upper bound eq. (7).

### 3. Benders' Decomposition

Benders Decomposition has been successfully applied to take advantage of underlying problem structures for various optimization problems, such as planning of power systems. The basic idea of this method is the generation, at each iteration, of an upper bound and lower bound on the sought solution of the problem. The upper bound results from the primal subproblem, while the lower bound results from the master problem [1].

Benders' Master Problem:

$$\text{Min}_{\theta, v_j(k), y_j(k)} f_{Mast}^v = \theta + \sum_{k \in K} \sum_{j=J} [F_j v_j(k) + A_j y_j(k)]$$

subject to :

$$\theta \geq f_{sub}^{(v-1)} + \sum_{k \in K} \sum_{j=J} \lambda_j^{(v-1)}(k) [v_j(k) - V_j^{(v-1)}(k)] \quad (10)$$

$$\sum_{j \in J} \bar{T}_j v_j(k) \geq \sum_{n \in N} D_n(k) + R(k)$$

$$y_j(k) \geq v_j(k) - v_j(k-1)$$

Benders Subproblem:

$$\text{Min}_{t_j(k), \delta_n(k)} f_{sub}^v = \sum_{k \in K} \sum_{j=J} [E_j(t_j(k))]$$

subjecto a :

$$\sum_{j \in \Lambda_n} t_j(k) + \sum_{m \in \Omega_n} B_{nm} [\delta_m(k) - \delta_n(k)] - \sum_{m \in \Omega_n} K_{nm} [1 - \cos(\delta_m(k) - \delta_n(k))] = D_n(k)$$

$$\bar{T}_j v_j(k) \leq t_j(k) \leq \bar{T}_j v_j(k)$$

$$-C_{nm} \leq B_{nm} [\delta_m(k) - \delta_n(k)] \leq C_{nm}$$

$$-\pi \leq \delta_n(k) \leq \pi$$

$$v_j(k) = V_j^v(k) : \lambda_j^{(v)}(k)$$

$$y_j(k) = Y_j^v(k)$$

(11)

### 4. Lagrangian Relaxation

Lagrangian relaxation [2,8,11] decomposes the STGP into a master problem and makes it easier to solve subproblems separately. The subproblems are linked by Lagrange Multipliers that are added to the master problem to yield a dual problem. The dual problem has lower dimensions

than the primal problem and is easier to solve. The multipliers are updated through different methods, usually a subgradient method. The major difficulty of this method is associated with obtaining solution feasibility because of the dual nature of the algorithm.

Dual Master Problem:

$$\text{Max}_{\lambda_j(k), \mu_i(k), \gamma(k), \beta(k)} g^v = \sum_{k \in K} \sum_{j=J} [L_n^v] \quad (12)$$

subject to :

$$\lambda_j(k), \mu_i(k), \gamma(k), \beta(k) \geq 0$$

Dual Subproblem:

$$L_n(t_{jk}, v_{jk}, y_{jk}, \delta_{nk}, \lambda_k, \mu_k, \gamma_k, \beta_k) =$$

$$\min \left\{ \sum_{k \in K} \sum_{j \in \Lambda_n} [F_j v_{jk} + A_j y_{jk} + E_j(t_{jk})] \right.$$

$$+ \sum_{k \in K} \lambda_k [D_{nk} - \sum_{j \in \Lambda_n} t_{jk} - \sum_{m \in \Omega_n} B_{nm} (\delta_{mk} - \delta_{nk}) + \sum_{m \in \Omega_n} K_{nm} (1 - \cos(\delta_{mk} - \delta_{nk}))]$$

$$+ \sum_{k \in K} \mu_k [D_{nk} + R_k - \sum_{j \in J} \bar{T}_j v_{jk}] + \sum_{k \in K} \gamma_k [C_{nm} - \sum_{m \in \Omega_n} B_{nm} (\delta_{mk} - \delta_{nk})]$$

$$\left. + \sum_{k \in K} \beta_k [-\sum_{m \in \Omega_n} B_{nm} (\delta_{mk} - \delta_{nk}) - C_{nm}] \right\} \quad (13)$$

Subject to eq. (4, 5, 7, 8, 9)

### 5. Computational experimentation

Three test systems are presented to evaluate the performance of the proposed AGS algorithm.

- The IEEE 24 bus test system with 24 nodes, 24 thermal units and 38 transmission lines [3,6].
- A portion of bus electric energy system of Mainland Spain with 104 nodes, 62 thermal units and 160 transmission lines [1].
- The IEEE 118 bus test system with 118 nodes, 54 thermal units and 186 transmission lines [13].

For Lagrangian relaxation, the mathematical model of STGP were implemented in GAMS [4] using the DICOPT solver for solving the MINLP problems (dual subproblems). CONOPT for solving the NLP problems (primal subproblems) and CPLEX for MIP problems (master primal problem).

All the models have been solved on an AMD Phenom™ II N970 Quad-Core with a 2.2 GHz processor and 8 GB RAM.

NLP and MINLP solvers were tuned to obtain solutions with a tolerance of 10% optimality. Therefore, the master and subproblems solutions obtained from the solvers are almost feasible and good solutions.

The iterative procedure continues until some stopping criterion is reached. For this case, the stopping criterion was 0.1 % of optimality of the MINLP solver.

Table 1.  
Lagrangian Relaxation

System	Gap %	vio rel %	CPU Time
IEEE-24	4.47	0.66 (2)	8'30''
IEEE-118	13.8	0.88 (2)	17'25''
SIS-104	0.4	1.34 (2)	30'23''

Source: The authors

Table 2.  
Benders' Decomposition

System	Gap %	vio rel %	CPU Time
IEEE-24	9.3	0.03 (2)	36'57''
IEEE-118	4.2	0.02 (2)	38' 03''
SIS-104	1.9	0.07 (2)	1hr 27'

Source: The authors

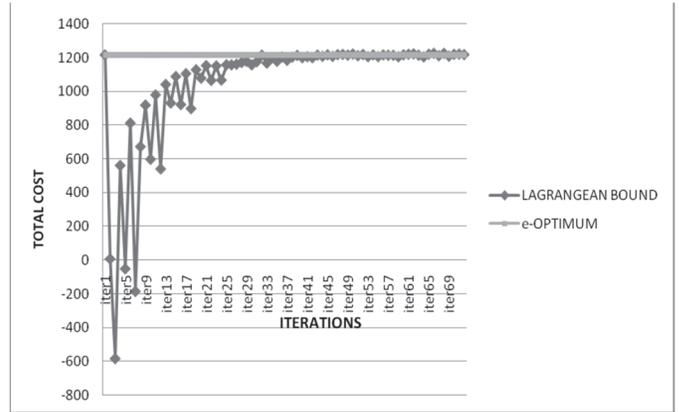


Figure 3. Lagrangian bound of IEEE-118  
Source: The authors

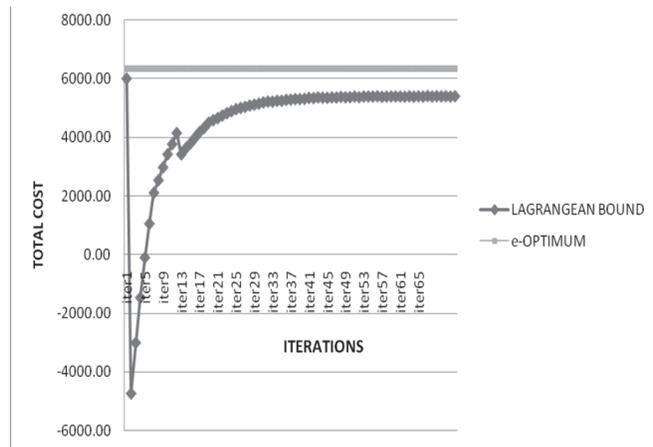


Figure 1. Lagrangian bound of IEEE-24.  
Source: The authors

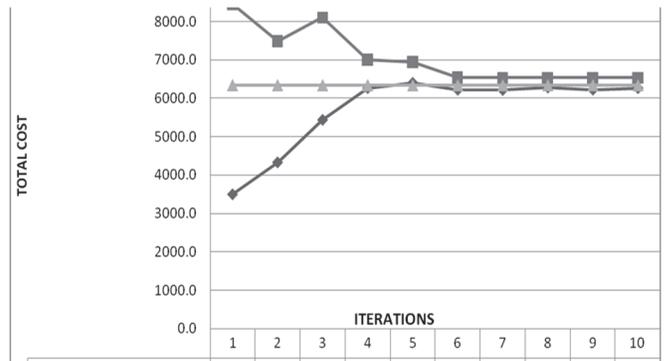


Figure 4. Benders bounds of IEEE-24  
Source: The authors

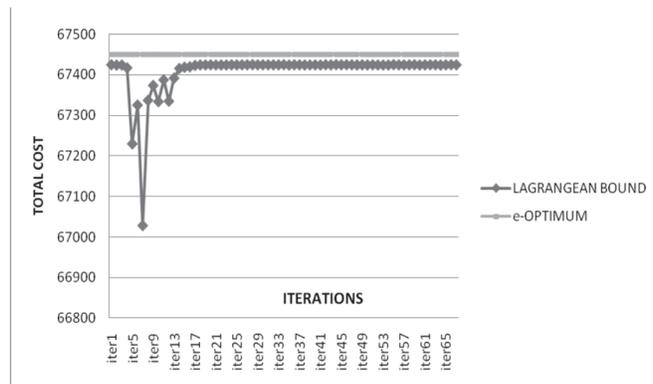


Figure 2. Lagrangian bound of SIS-104.  
Source: The authors

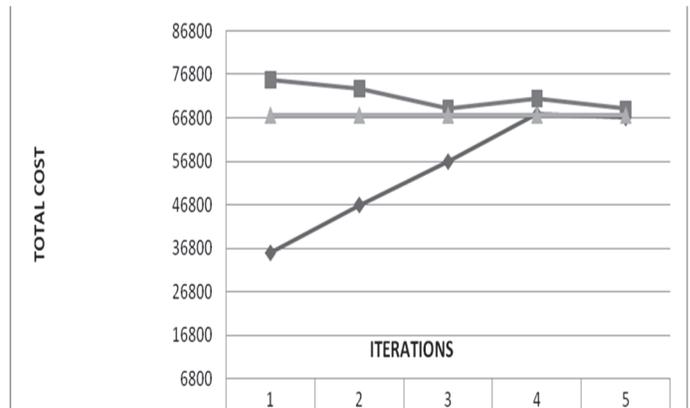


Figure 5. Benders bounds of Sis-104.  
Source: The authors

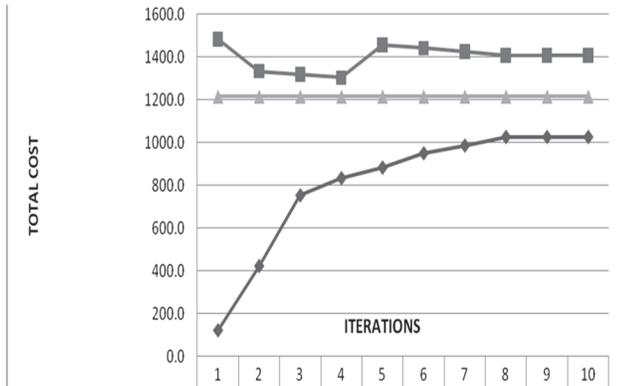


Figure 6. Benders bounds of IEEE-118

Source: The authors

## Conclusions

We evaluated the performance of primal and dual decomposition techniques in order to compare the quality of the solutions they provide. Three test systems are presented to evaluate the performance of the proposed solution methods. Although for the MINLPs' problems global optimality is not guaranteed, the proposed strategies show good convergence properties and provide better results than those obtained solving the problem using other methods. Thus, this application can be extended to large scale problems.

Through the results, we see a decrease in computational time by Lagrangian relaxation of the problem, as well as lower Gap. See Figs. 1, 2 and 3.

However, the percentage of *vio rel* increases due to relaxation of system constraints. The solution obtained by Lagrangian relaxation is sometimes infeasible to the original problem, so to achieve the feasibility of the problem we propose using lower thermal plants cost as part of the vector of solutions that would provide electricity generation to meet demand requirements in all periods.

By contrast, in Benders' decomposition, there is no feasibility loss because the problem retains the same set of constraints. However, through experimentation, we observed a small percentage of relative deviation attributable to the optimizer. See Figs. 4, 5, 6.

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