Robust sample size for weibull demonstration test plan


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Abstract
The efficiency of a Weibull demonstration test plan is completely determined by the total experimental time \( T_a \), which depends on the unknown sample size \( n \) and on the Weibull shape parameter \( \beta \). Thus, once \( \beta \) was selected, \( T_a \) depends only on \( n \). Unfortunately, because \( n \) was estimated by the parametrical binomial approach, then if the confidence level \( C \) was higher than 0.63, \( n \), and as consequence \( T_a \), was overestimated, (For \( C<0.63 \), they were underestimated). On the other hand, in this paper, because the intersection between \( n \) and \( \beta \), for which \( T_a \) was unique, was found with \( n \) depending only on \( R(t) \), then the estimation of \( T_a \) was optimal. On the other hand, since once \( \beta \) was selected, \( \eta \) was completely determined, then \( \beta \) and \( \eta \) were used to incorporate the expected failure times of the operational level in an accelerated life test analysis (ALT). Numerical applications are given.

Keywords: Weibull demonstration test plan, Success run testing, Lipson equality, Accelerated life testing.

1. Introduction

In reliability engineering, because of its flexibility the Weibull distribution is one of the most commonly used probability density functions to model the behavior of a product or process through the time [6]. Moreover, since the lower reliability index could be seen as an index of stability (quality through the time), the Weibull demonstration test plans are performed without failures in order to determine whether the product fulfills its designed reliability \( R(t) \). On the other hand, to perform the test plan, we must know; \( R(t) \), the designed life time \( t_o \), the operational environmental, the desired confidence level \( C \) and the Weibull shape parameter \( \beta \). Regardless of this knowledge, given that the test plan is completely determined by the total experimental time \( T_a \), and because for a known \( \beta \), \( T_a \) depends only on the sample size \( n \), then the efficiency of the test plan depends on the accuracy with which \( n \) is estimated. In practice, the parametrical binomial approach, considering a constant failure rate \( p \) ([2] chapter 9, [5] and section 2.2), is used to estimate \( n \). Unfortunately, if \( C \) higher than 0.63, \( n \) is overestimated, and as a consequence \( T_a \) is overestimated too. To solve this problem, and based on the fact that for constant \( \beta \), \( T_a \) is directly related to the Weibull scale parameter \( \eta \), then based
on the addressed relations among $\eta$, $n$, $R(t)$ and $\beta$, and on the found intersection between $n$ and $\beta$ for which $T_a$ is unique, in this paper a method to estimate $n$ in closed form, but independent of $\beta$, is given. Moreover, because $n$ was found to be depending only on the known $R(t)$ index, the estimated $T_a$ is completely representative of the designed test plan.

On the other hand, in order to show how to proceed under time or lab restrictions, by using the above estimated $n$, the Lipson equality is applied to perform a tradeoff between $n$ and the experimental time $T_i$, for which $T_a$ is constant. Finally, because $n$ is directly related with $R(t)$, in section 5.1, we show how it could be used in the median rank approach to incorporate the expected lifetimes of the operational level if an accelerated life test analysis (ALT) has to be used. The paper structure is as follows. Section 2 addresses the problem statement. Section 3 presents the proposed method. Section 4 offers the application and comparison between the proposed method and the binomial approach. Section 5 outlines the steps to incorporate the failure times of the operational level into the ALT analysis. Section 6 presents the conclusions. Finally, the paper ends with the references in section 7.

2. Problem statement

Since a Weibull demonstration test plan is performed without failures, the Weibull parameters $\beta$ and $\eta$ could not be estimated. Thus, the efficiency of the test plan depends completely on the accuracy on which $T_a$ is estimated. But, because $T_a$ for constant $\beta$ depends only on $n$, then the efficiency of the test plan now only depends on the accuracy with which $n$ is estimated. Regardless of this, since $n$ is estimated by the parametrical binomial approach considering a constant failure rate $p$, and a confidence interval $C$, then when $C$ higher than 0.63 is selected, $n$ is overestimated and for $C$ lower than 0.63 $n$ is underestimated. The overestimation (or underestimation) of $n$ directly implies that $T_a$ is overestimated (or underestimated) also. Observe that this means that the test plan fails to demonstrate whether the product fulfills its designed $R(t)$ index. On the other hand, although in practice, $\beta$ is selected from a historical data set (or engineering knowledge), because its value depends on the material characteristics [18], and on the variability of the manufacturing process [13], here the analysis is presented in two parts. The first is conducted to address the effect that the uncertainty of $\beta$ has on $T_a$, and the second is conducted to statistically identify the disadvantages that the use of the binomial approach has on the estimation of $n$. To do this, let us first present the $\beta$ analysis.

2.1. Shape parameter analysis

Since for non-failures Weibull analysis $T_a$ is cumulated as

$$ T_a = n \times T_i^\beta $$

(1)

Where $T_i$ is the unite experimental time, which is selected as the designed time ($T_i = t_{d}$), the value of $\beta$ has a big impact on $T_a$, [see [12] and [15] sec. 2.3] and because in the Weibull demonstration test plan there is no failures information to estimate $\beta$, and due to the fact that its estimation depends on the variability of the manufacturing process [13], in practice $\beta$ is selected from tabulated data sets. Moreover, because of the Weibull closure property ($\beta$ has to be constant), once the $\beta$ value is selected it has to be considered constant in the analysis [16]. On the other hand, regardless which value of $\beta$ we had selected, once it was assigned, the efficiency of the test plan depends on the scale parameter $\eta$. Thus, we are now interested in how to estimate $T_a$ in function of $\eta$. However, because there are no failures, the lower expected limit of $R(t)$ has to be used. That is, that the lower limit of $\eta$, here called ($\eta_L$), has to be estimated. According to [10] and [11], $\eta_L$ is given by

$$ \eta_L = \left[ \frac{2 \sum_{i=1}^{n} T_i^\beta}{X_{\alpha,2r+2}} \right] $$

(2)

Where $\alpha$ is the significant level and $r$ is the number of observed failures. On the other hand, since equation (2) for zero failures, with $C$ representing the desired confidence level and $T_i$ equal to $t_{d}$, is given by

$$ \eta_L = \left[ \frac{\sum_{i=1}^{n} T_i^\beta}{-\ln(1-C)} \right]^{\frac{1}{\beta}} $$

(3)

Then by selecting $C = (1 - e^{-1})$ in (3), the relation between $\eta_L$ and $T_a$ is given by

$$ T_a = \eta_L^\beta $$

(4)

Clearly from (4), $T_a$ is in function of $\beta$, which in practice is selected from a data set (or engineering knowledge). Now, let us focus on the uncertainty of $n$.

2.2. Binomial approach analysis

Given that the Weibull demonstration test plan, the parametrical binomial approach used to determine $n$ (see [2] and [5] chapter 9) is based on the binomial distribution given by

$$ p(x) = \frac{n!}{(n-x)!x!} p^x q^{n-x} \quad x=0,1,2,\ldots,n $$

(5)

which, instead of considering the time and the risk function to determine $n$, considers a constant failure rate $p$ and a confidence level $C$ to model the uncertainty on $R(t)$, then $n$ is not optimal. Given this, first let us analyze how the estimation of $n$ is formulated. In doing so, we can see that because no failures are allowed ($x=0$), the lower confidence of $R(t)$ has to be used. Thus, $C$ based on the fact that if $n$ items are tested and $k=1,\ldots, n$ of them fails, $C$ is given by

$$ C = 1 - \sum_{i=1}^{k} \frac{n!}{(n-x)!x!} R^{n-x} (1-R)^x $$

(6)
In (6), R represents the lower confidence of R(t). In (6), R is used instead of R(t) because the binomial approach does not consider the time variable t. Therefore, equation (6) with zero failures \((x = 0)\) is given by

\[
R^n = (1 - C)
\]  

(7)

Finally, by rearranging terms, \(n\) is given by

\[
n = \frac{\ln(1 - C)}{\ln R(t)}
\]  

(8)

Function (8) is known as success run testing [5]. Second, suppose that our customers are asking us to demonstrate a reliability of \(R(t) = 0.96\) for \(t = 1500\) hrs. In addition suppose, that from historical data, we know that \(\beta = 2.5\). Thus, by using (8), we have to test without failures \(n = 57\) pcs for \(1500\) hrs each. Thus, observe from (1) that \(T_a = 57*1500^{2.5} = 4967101142\) and from (4) that \(\eta = 7558.59718\). With this information, since we now know \(\beta\) and \(\eta\), then by using these parameters in the Weibull reliability function given by

\[
R(t) = \exp \left( - \frac{t}{\eta}^\beta \right)
\]  

(9)

We note that the demonstrated \(R(t)\), for \(t = 1500\) hrs, is \(R(t) = 0.9826\), instead of the planned \(R(t) = 0.96\). Thus, because \(R(t)\) and \(\beta\) are known, we conclude that the \(C\) value used in (8) overestimated \(n\), and that, as a consequence, \(T_a\) defined in (1) was overestimated too.

In contrast, observe that for \(C = 0.50\), \(n = 17\) pcs, \(T_a = 1481416130\) and \(\eta = 4658.7652\), with \(R(t) = 0.9428\). That is to say that for \(C = 0.50\), \(n\) was underestimated. Thus, a \(C\) value between 0.5 and 0.9 for which \(n\) is optimal exists. In the next section, this value is statistically addressed and generalized to any desired \(R(t)\) and \(\beta\) value.

3. Proposed method

Given that the goal of the proposed method is to remain practical, first let us show how the binomial approach, the Lipson equality, and the Weibull reliability function are related. In doing so, we can first see that because \(T_a\) completely determines \(R(t)\), the analysis is based on \(T_a\). Second, we can see that for constant \(\beta\), \(T_a\) depends only on \(n\). Finally, note that regardless how \(\beta\) and \(n\) were estimated, once their values were selected, a tradeoff between \(n\) and \(T_a\), for which \(T_a\) remains constant could be made by applying the Lipson equality (see [5]) as follows. The Lipson equality is formulated by replacing \(R\) given in (7) with the Weibull reliability function defined in (9). After the replacement, the equality is given by

\[
R(t) = (1 - C)^n = \exp \left( - \left( \frac{t}{\eta} \right)^\beta \right)
\]  

(10)

From (10), by taking logarithms and by rearranging terms, the sample size \(n\) in the Lipson equality is given by

\[
n = \frac{\ln(1 - C)}{L^\beta \ln R(t)}
\]  

(11)

In (11), \(L\) represents one life of the product \(L = t_d\). Function (11) is known as Lipson equality (or extended life approach) and it relates the binomial approach and the Weibull reliability function. Finally, from (11), it is clear that it works regardless of how \(n\) and \(\beta\) were selected. Thus, in the same way as \(T_a\), its efficiency depends on how \(n\) is estimated.

On the other hand, in order to estimate \(T_a\) accurately, by substituting (8) in (2), \(\eta_L\) is found to be depending only on \(R(t)\) and \(\beta\) as in (12).

\[
\eta_L = \left[ \frac{T_a^\beta}{-\ln(R(t))} \right]^{1/\beta}
\]  

(12)

Then based on (12), and by selecting \(n\) as in (13),

\[
n = -\frac{1}{\ln(R(t))}
\]  

(13)

the relation between \(T_a\) and \(\eta_L\) as in (4), is given by

\[
T_a = \eta_L^\beta \left[ \frac{n * T_i^\beta}{-\ln(R(t))} \right]
\]  

(14)

From (14), \(T_i\) is given by

\[
T_i = \left[ \frac{-t_d^\beta}{n * \ln(R(t))} \right]^{1/\beta}
\]  

(15)

And clearly, since there no failures are allowed, then we can select \(T_i = t_d\), thus \(\eta\) is given by

\[
\eta = n^{1/\beta} T_i = n^{1/\beta} t_d
\]  

(16)

From (16), since \(T_i = t_d\) is known, and \(n\) is directly related to \(R(t)\) as in (13), \(\eta\) now depends only on the selected \(R(t)\) value. On the other hand, observe that \(n\) in (13) is estimated regardless of the value of \(\beta\). Seeing this numerically, suppose we are determining \(n\) defined in (12) by a designed time \(t_d = 1500\) hrs, and suppose we desire to demonstrate a reliability of \(R(t) = 0.90\). Furthermore, suppose that from historical data (or engineering knowledge), we know that \(\beta\) ranges from 1.5\(\leq\beta\leq3\). Then by testing different \(n\) values for \(\beta = 1.5\) and \(\beta = 3\), as in Table 1, we found that \(T_i\) shifts its behavior from higher to lower, implying that an intersection for which \(T_i\) is equal to both \(\beta\) values exists. And because, this intersection corresponds exactly to the \(n\) value defined in (13), and it does not depend on \(\beta\), we conclude that by estimating \(n\) using (13), the proposed method is robust under the uncertainty that \(\beta\) has over \(T_a\). In particular, we can see from Table 1 and Fig. 1 that for \(T_i = t_d\) \(n\) is as in (13) and \(R(t)\) is as expected for both values of \(\beta\), and that \(\eta_L\) is as in (12).
In order to demonstrate whether a product fulfills its designed reliability, the following steps were taken:

1. Determine the reliability level $R(t)$ index to be demonstrated, the operational environmental to be tested and the design time $t_d$.
2. Determine the value of $\eta$ to be used. A base line product, where engineering knowledge or historical data could be used to select the most suitable $\eta$ value.
3. By using (13) with the $R(t)$ level of step 1, determine the sample size $n$ to be tested without failures during the $t_d$ lifetime each.
4. If there are experimental or time restrictions, perform the desired tradeoff between $n$ and $t_d$, using (11) with $C=0.63212$.
5. Test each specimen by $t_d$ lifetime and, if neither of them fails, go to step 6. If one of them fails, go to step 7.
6. By using (12) or (16), estimate the expected $\eta_L$ value. And by using $\eta_L$, $t_d$ and the selected $\beta$ value in (9), determines the demonstrated $R(t)$ value, and draw your conclusions.
7. Correct and reinforce the design (or process) and go to step 1.
8. If you are performing an accelerated life testing, and the normal operational conditions could not be applied to the experiment, follows the steps given in section 5.1.

### 3.1. Steps of the proposed method

In order to demonstrate whether a product fulfils its designed reliability, the following steps were taken.

1. Determine the reliability level $R(t)$ index to be demonstrated, the operational environmental to be tested and the design time $t_d$.
2. Determine the value of $\beta$ to be used. A base line product, where engineering knowledge or historical data could be used to select the most suitable $\beta$ value.
3. By using (13) with the $R(t)$ level of step 1, determine the sample size $n$ to be tested without failures during the $t_d$ lifetime each.
4. If there are experimental or time restrictions, perform the desired tradeoff between $n$ and $t_d$, using (11) with $C=0.63212$.
5. Test each specimen by $t_d$ lifetime and, if neither of them fails, go to step 6. If one of them fails, go to step 7.
6. By using (12) or (16), estimate the expected $\eta_L$ value. And by using $\eta_L$, $t_d$ and the selected $\beta$ value in (9), determines the demonstrated $R(t)$ value, and draw your conclusions.
7. Correct and reinforce the design (or process) and go to step 1.
8. If you are performing an accelerated life testing, and the normal operational conditions could not be applied to the experiment, follows the steps given in section 5.1.

### 4. An application

As an application, first consider the data in section 2.2. $(R(t)=0.96, C=0.90$ and $C=0.50$, $t_f=1500$hrs and $\beta=2.5$). With this information obtained by using (13) in step 3, we have to test $n=24.49\approx25$pcs for 1500hrs each. And by using (1) or (14) with $\beta=2.5$, $T_a=2134685635$hrs and by using (12) or (16), $\eta_L=5391.797$. Thus, by applying (9) the demonstrated reliability, is $R(t)=0.96$ as was planned.

On the other hand, suppose that because of the lab capacity, we can run each test for no more than 3000hrs; $(L=2$lifes$)$, then by using (11) with $C=0.63212$, we have to run $n=4.33\approx5$pcs without failures for 3000hrs each, and from (1) or from (14) $T_a=5*3000=5^5=2464751.509hrs$ and from (12) or (16) $\eta_L=5710.96hrs$, and by using $\eta_L$ in (9), the demonstrated reliability, as planned, is $R(t)=0.96$.

Finally, since $n$ in (13) depends only on $R(t)$, and because $R(t)$ is used in the response variable of the median rank approach, in the next section, $n$ is used to incorporate the expected normal operational lifetimes into an accelerated life time analysis.

### 5. Weibull accelerated life test planning

In Weibull accelerated life test analysis (ALT) for constant and interval valued variables, the shape parameter $\beta$ is considered constant in the analysis due to the Weibull closure property [16]. Thus, the reliability index $R(t)$ depends only on $\eta$ ([4], [7] and [9]), which in ALT is estimated as a linear function of the covariates by using a life/stress models $r\{X_L(t)\}$ [1], [3], [7] and [10], as follows

$$\eta_{Li} = r\{X_{Li}(t)\}$$

(17)

Here, it is important to note that in (17) for constant over time stress variables $r\{X_{Li}(t)\}$ is parametrized as

$$r\{X_{Li}(t)\} = e^{-\beta Z}$$

(18)

Where $\beta$ is a vector of regression coefficients to be estimated and $Z$ is a vector of the effect of the related stress variable (e.g. in Arrhenius $Z=1/T$ where $T$ is the temperature in Kelvin degrees). Thus, by using the Weibull density function given by

$$f(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} \exp \left\{ -\left( \frac{t}{\eta} \right)^\beta \right\}$$

(19)

the Weibull/life/stress parameters are estimated by substituting $\eta$ in (17) with the corresponding $r\{X_{Li}(t)\}$ model defined in (17) or (18). For example suppose that the stress variable is the temperature, then the Arrhenius model is used. The Arrhenius model is given by
The Weibull/Arrhenius model is given by

\[ f(t) = \frac{\beta}{A} \left( \frac{t}{A} \right)^{\beta-1} \exp \left\{ -\left( \frac{t}{A} \right)^{\beta} \right\} \]

in (21) the Weibull/Arrhenius parameters are estimated in joint form by using the maximum likelihood method.

Thus, the Weibull/Arrhenius model is given by

\[ \eta = r[X_L(t)] = C e^{\left( \frac{B}{A} \right)} \]

(20)

Thus, the Weibull/Arrhenius model is given by

\[ \ln(-\ln(R(t))) = -\beta \ln(\eta) + \beta \ln(t) \]

(23)

The expected times of the operational level can be estimated and incorporated to the ALT analysis as in the next section.

5.1. Application to ALT analysis.

As an application, let us use data given in Table 3. Data was published by [17]. Suppose the normal operational temperature level is 323K and the design time is \( t_d = 15000 \) hrs. The analysis to incorporate the expected lifetimes of the 323K level into the analysis is as follows.

1. For each ALT level, by applying (13) with the desired \( R(t) \) level, determine the number of replicates to be tested. Here \( R(t) = 0.90 \) was used, thus \( n = 10 \) pcs.
2. Perform the experiment, and for each replicated level, estimate the Weibull parameters \( \beta \) and \( \eta \) (here \( \beta \) should be constant).
3. Using the estimated \( n \) and \( \beta \) value in (16), estimate the corresponding \( \eta L \) value. (here it is \( \eta L = 101/4.2206 \) (5600)=25883.58312hrs).
4. Using (22) with \( n \) given by (13), estimate the expected \( R(t) \) value for each of the order statistics. Then by using these \( R(t) \) values, the \( \beta \) and the \( \eta L \) values in (23), solve (23) to \( \ln(t) \) and then determine the expected failure times of the operational level to each order statistic. (In our case, the expected failure times appear in the last row of Table 3).
5. By using the incorporated expected times, and the experimented accelerated lifetimes, perform the estimation of the Weibull/Life–stress parameters. Here, the analysis, by using the ALTA routine and (21), yield to \( \beta = 4.3779, B = 2308.4884 \) and \( A = 19.8667 \).
6. Finally, by using the parameters of step 5, determine the desired reliability indexes.

To conclude, observe that the data in Table 3, for \( t_d = 15000, R(t) \) is of \( R(t) = 0.90 \) as was expected. In particular, note that without incorporating the operational times, the estimated \( R(t) \) for the operational level is of 323K, instead of the designed \( R(t) = 0.90 \), would be \( R(t) = 68.33\% \).

Table 2.

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Source: The authors

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Source: The authors
6. Conclusions

In Weibull demonstration test plans, the binomial approach for confidence levels higher (lower) than 0.63212, overestimate (sub-estimate) $R(t)$; thus, $C$ should not be selected lower than 0.63212, and if it is selected higher, it should be selected close to 0.63212, say 0.7. In the proposed method, given that $n$ depends only on $R(t)$ which is always known, the designed and the demonstrated $R(t)$ value always holds. On the other hand, observe that because data is gathered by using an experiment design where each row represents a different form to run the process, then the $\beta$ parameter does not remain constant and as a consequence, the multivariate approach using the Taguchi method as in [14], should be used. Given that in the estimation process, the value of $\beta$ depends on the variance of the logarithm of the failure times, which depends on the control of environmental factors (see [13] and [14]), its value must be selected from a data set that covers the variability of the manufacturing process in its interval.

Although, the proposed method allows practitioners to incorporate the expected operational times to the ALT analysis, it is important to note that their efficiency corresponds to the efficiency of the median rank approach and on the assumption of a constant $\beta$. Finally, it is important to note that although it may seem that $n$ could be used to incorporate the expected times in the ALT analyses where the Weibull closure property does not hold ($\beta$ is not constant), as is the case of ALT analysis with several variables, more research is required.

References


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