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# Exploring Mathematization Processes through Role-Playing Games*

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## Abstract

*Storytelling in role-playing games offers players the opportunity to create and immerse themselves in fictitious worlds. These worlds might include situations, objects, and environments that resemble reality. This study explores how 13 research works that utilized role-playing games to integrate mathematization processes provide learning environments for mathematical modeling. The analysis focuses on the deductive and inductive categories of mathematical statements, student and teacher interventions, and the research conclusions, contributing to the identification of patterns, regularities, and relations. These categories formed the basis for themes revealed by three insights: first, the mechanics, creation, and evolution of characters, scenarios, and narratives in role-playing games offer various possibilities for integrating mathematization processes; second, these processes can be explained by how students and teachers select the models, strategies, and procedures for problem-solving; third, the validation or interpretation of results by players and teachers is crucial for achieving a logical progression of the narrative.*

## Keywords

*Role-playing; educational games; mathematical models; problem-solving; mathematics education.*

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# Exploración de los procesos de matematización mediante juegos de rol

## Resumen

*La narración en los juegos de rol brinda a los jugadores la oportunidad de crear y sumergirse en mundos ficticios que pueden incluir situaciones, objetos y entornos que se asemejan a la realidad. Este estudio explora 13 trabajos de investigación que utilizaron juegos de rol para integrar procesos de matematización y la forma en que estos generan entornos de aprendizaje para la modelación matemática. El análisis se centra en las categorías deductivas e inductivas de los enunciados matemáticos, en las intervenciones de estudiantes y docentes, y en las conclusiones de las investigaciones, a fin de contribuir a la identificación de patrones, regularidades y relaciones. Dichas categorías fundamentaron los temas que surgieron a partir de tres ideas clave: 1) la mecánica, la creación y la evolución de personajes, escenarios y narrativas en los juegos de rol ofrecen diversas posibilidades para integrar procesos de matematización; 2) estos procesos pueden explicarse por la manera en que estudiantes y docentes seleccionan modelos, estrategias y procedimientos para la resolución de problemas; 3) la validación o interpretación de los resultados por parte de jugadores y docentes es crucial para lograr una progresión lógica de la narrativa.*

## Palabras clave

*Juegos de rol; juegos educativos; modelos matemáticos; resolución de problemas; enseñanza de las matemáticas.*

# Exploração dos processos de matematização mediante jogos de papéis

## Resumo

*A narrativa presente nos jogos de papéis oferece aos jogadores a oportunidade de criar e a imersão em mundos fictícios que podem incluir situações, objetos e ambientes que se assemelham à realidade. Este estudo explora 13 artigos de pesquisa que utilizaram o jogo de papéis para integrar processos de matematização e de que maneira essas práticas geram ambientes de aprendizagem voltados à modelagem matemática. A análise foca nas categorias dedutiva e indutiva relacionadas às afirmações matemáticas, nas interações de estudantes e professores, e nas conclusões da pesquisa, com o objetivo de identificar padrões, regularidades e relacionamentos. Essas categorias foram a base para os temas que emergiram de três ideias-chave: 1) a mecânica, criação e evolução de personagens, cenários e narrativas em jogos de papéis oferecem diversas possibilidades para integrar processos de matematização; 2) esses processos podem ser compreendidos a partir da forma como estudantes e professores selecionam modelos, estratégias e procedimentos para a resolução de problemas; 3) a validação ou interpretação dos resultados por jogadores e professores é crucial para alcançar uma progressão lógica da narrativa.*

## Palavras-chave

*Jogos de papéis; jogos educativos; modelagem matemática; resolução de problemas; ensino de matemática.*

## Introduction

Across cultural contexts and educational systems, many mathematics educators have engaged in diverse efforts to uphold an image of mathematics as a valuable intellectual pursuit and an enjoyable school subject. A set of approaches affiliated with establishing "real-world connections" relies on taking up students' everyday practical experiences, introducing realistic problem-solving settings, and encouraging mathematical modeling and mathematizing (Ishibashi & Uegatani, 2022; Jablonka, 1996; Niss & Blum, 2020; Nunokawa, 2005; Shiakalli & Zacharos, 2014; Stillman, 2019; Verschaffel et al., 2009). In this paper, we are interested in those mathematical modeling activities that Kaiser and Sriraman (2006) have categorized as epistemological or conceptual (educational) modeling. These approaches often include, as an important aim, increasing students' motivation and engagement with mathematics as they recognize its use-value (Kaiser, 2020; Smith & Morgan, 2016).

However, if modeling aims at the development and exploration of mathematical concepts or models that offer a route into mathematical theory (not at drawing conclusions about an extra-mathematical situation of immediate interest), the expectation that "real-world connections" automatically create an experience that leads to students' increased engagement with mathematics is not justified (Gerofsky, 2009; Van den Heuvel-Panhuizen & Drijvers, 2020). Moreover, the use of realistic problems that relate to students' out-of-school practices may result in the (re)production of socio-cultural inequalities (Jablonka & Gellert, 2011). Additionally, there is no need to insist on a form of realism in narratives describing the problem settings, which serve as starting points for processes of mathematization. This alludes to the importance of exploring the potential of fictitious scenarios as a starting point for mathematization activities.

Another avenue for engaging students and allowing them to develop their skills in a "playful" manner is the use of games. This approach has attracted resurgent attention in mathematics education with the development of digital media (Bernier & Zandieh, 2024; Bright et al., 1985; Dan et al., 2024; Devlin, 2011; Ernest, 1986; Gee, 2004; Hui & Mahmud, 2023; McFeetors & Palfy, 2018; Mousoulides & Sriraman, 2020). Prior research on games as a pedagogical strategy for teaching mathematics has identified several challenges. For instance, the use of games may not address issues arising from interference with traditional teaching methods, which can negatively affect the autonomy students seek in playing games (Muniz, 2021a, 2021b). Similarly, incorporating mathematical challenges into games can conflict with the idea of playing as an activity solely driven by pleasure and enjoyment (Karimov et al., 2024; Tamayo & Maltempi, 2023a; Tamayo-Martínez, 2025). Some see a major problem in integrating the exploration of abstract mathematical concepts and relationships without disrupting the immersive gaming experience (Devlin, 2011; Gee, 2004; Gros, 2000).

Based on these observations, this paper investigates the potential for integrating mathematization activities into role-playing games (RPGs). RPGs are not exempt from the problems associated with the use of games mentioned above, but their particular form has the potential to enhance productive interactions within the mathematics classroom, engaging both students and teachers (Bressan, 2014; Feijó, 2014; Sobral, 2018).

Considering the various scholarly definitions of RPGs, following Zagal and Deterding (2018), it can be argued that these depend on the interests of specific social groups, including academics, and the usage they seek to make of these games. This dynamic precludes the existence of a single comprehensive definition. However, these authors argue that the term RPG is used to characterize specific ludic activities centered on the performance of characters typically conceived by players, who control their actions and make decisions to attain objectives within structured, fictitious worlds, often set in science-fiction

or fantasy genres. These worlds are overseen by a player known as the "master." Dungeons & Dragons is an example of such a game.

This paper seeks to identify an analytical avenue grounded in the concept of mathematization, as these fictitious realities provide an environment for exploring mathematical concepts, developing models, and developing problem-solving skills. Central to this exploration is the theoretical lens of Realistic Mathematics Education (RME), which offers a nuanced understanding of how mathematical concepts emerge and evolve from meaningful contexts. Because of their openness, RPGs can offer a space for the transformation of meaning during mathematization within fictitious worlds.

This terrain is not yet well-explored. A literature review conducted in 2022 (Tamayo & Maltempi, 2023b; Tamayo-Martínez, 2025) revealed an unexplored dimension: the challenges posed by integrating problem-solving and mathematization activities in RPGs. Drawing on reports, this paper explores these processes. The analysis aims to uncover the potential of such integration to overcome some of the problems associated with "real-world connections." In the upcoming sections, this paper will begin with insights into the concepts of mathematization and RME theory. Subsequently, insights concerning RPGs and the construction of fictitious worlds used for playing will be provided. The paper will then turn to the methodological approach, followed by a presentation of the outcomes. Ultimately, some conclusions regarding the benefits and drawbacks of integrating mathematization, modeling, and problem-solving into RPGs will be presented.

## Theoretical Framework and Literature Review

### *Mathematization and "Real-World Connections"*

The concept of mathematization holds considerable significance within the field of mathematics education across a range of curricular conceptions, some of which consider it a constituting didactic principle. Broadly, it delineates a dynamic, active process by which something is represented or organized using mathematics as a descriptive language, which can then be further formalized. This process involves students actively structuring and organizing content from a specific reality (real or imagined) into mathematical models. In this process, the descriptions and the mathematical apparatus co-evolve, eventually leading to the creation of new "realities" through applications of mathematics (Jablonka, 1996; Jablonka & Gellert, 2011). The concept of mathematization underpinning RME, as articulated by Freudenthal (2002) and further elaborated by Van den Heuvel-Panhuizen and Drijvers (2020), aligns with this interpretation and holds particular interest for pedagogical strategies involving RPGs.

RME is a domain-specific instruction theory developed in the Netherlands, characterized by giving a prominent position to rich, "realistic" situations in the learning process. These situations serve as a source for developing mathematical concepts, tools, and procedures, and as a context in which students can apply their mathematical knowledge, which gradually becomes more formal and general. Notably, the meaning of "real" in RME derives from a Dutch word meaning "realization" or "being able to conceive or imagine something" (*zich realiseren*). Consequently, the problem descriptions that serve as starting points for modeling activities and mathematization processes can also relate to non-realistic genres, such as fairy tales (Van den Heuvel-Panhuizen & Drijvers, 2020). This unique interpretation of "real" in RME provides a strong theoretical justification for exploring the potential of fictional worlds, like those found in RPGs, as fertile ground for mathematical learning.

Regarding procedural descriptions of modeling in applied mathematics, as outlined by Jablonka (1996), the conception of validating model outcomes deserves attention when simulated within a fiction-

al world, as the supposedly independent reality cannot be conceived as existing outside the players' or the master's narration.

The use of RPGs promises to overcome some challenges and effects related to "real-world connections." One such effect concerns the (re)production of socio-cultural inequalities stemming from particular types of out-of-school practices embedded in realistic narratives (Jablonka & Gellert, 2011). As Gellert and Jablonka (2009) observe, "one of the reasons for the difficulties teachers and students, who are faced with 'realistic' problems, experience, is that school mathematics, if about optimization when shopping, for example, is neither mathematics nor shopping" (p. 2). Gellert and Jablonka (2009) further explain, with examples from classroom interaction, that the invitation to express practical activities in mathematical terms conceals the fact that the mathematical structures projected onto them are not determined by the structure of these activities. This recurrent reference creates a "myth of participation" (Dowling, 1998), where mathematics is presented (particularly to marginalized student groups) as if it addresses practical problems and as if its acquisition were necessary for navigating everyday life (e.g., shopping). This myth obscures mathematics' aim of developing general solution methods while mythologizing mundane activities like shopping. In other words, the models are not reality from a Kantian perspective; they are representations of that reality (Gallego et al., 2006).

Similarly, the "myth of reference" (Dowling, 1998) conveys that mathematical descriptions correspond to non-mathematical realities in an unproblematic way. This myth arises from problem settings that are evidently conceptualized from a mathematical perspective, retaining only traces of non-mathematical significance. What it conceals are the possibilities of alternative, non-mathematical descriptions of the same "reality." This is even more concerning when learners perceive mathematics as a language or technology completely detached from human choices or values, as there is a risk of manipulation by those who claim to have perfect knowledge of supposedly objective mathematics (Wagner, 2011). RPGs, despite being overseen by a "master," may help to challenge these myths through their open principles for determining what constitutes a legitimate (mathematical) move within the game.

Another issue central to this analysis concerns what in RME is conceptualized as a shift from "horizontal mathematization" to "vertical mathematization." As Freudenthal (2002) explains:

> *Horizontal mathematization leads from the world of life to the world of symbols. In the world of life one lives, acts (and suffers); in the other one symbols are shaped, reshaped, and manipulated, mechanically, comprehendingly, reflectingly; this is vertical mathematization. (pp. 41, 42)*

Jablonka and Gellert (2007) connect this distinction to Bernstein's (1999) concepts of vertical and horizontal knowledge structures and hierarchical and horizontal discourses. Horizontal mathematization in RME involves developing local "mathematical models" within horizontal discourses characterized by initially tacit and potentially contradictory meanings across the various sites of everyday practices where they emerge. In contrast, hierarchical discourses, characterized by explicit grammar and systematic organization (such as in the sciences), may—but do not necessarily—result in vertical knowledge structures. These structures consist of hierarchically organized concepts enabling progressive integration into theoretical systems of increasing generality, referred to as "(vertical) mathematization" in RME. To illustrate these concepts, consider the process of a student calculating the required amount of "gold" for their character to purchase an item in an RPG:

- **Horizontal Mathematization Example:** A student engages in horizontal mathematization when they translate the in-game problem (e.g., "My character has 15 gold, and the item costs

20 gold. How much more do I need?") into a simple subtraction problem (20 – 15). This involves organizing the "real-world" quantities (gold, item cost) into a mathematical statement, making sense of the game's economy in mathematical terms. They are creating a local model of the situation.

- **Vertical Mathematization Example:** This involves reflecting on the initial problem of subtracting an amount, developing a more formal understanding of operations or algebraic relationships, and generalizing strategies beyond the immediate calculation of gold, cost, or other "real-world" quantities. Students are systematizing and formalizing their mathematical tools.

A key challenge regarding "real-world connections," as Jablonka and Gellert (2007) elucidate, lies in the widespread, unjustified assumption that "abstraction from extra-mathematical contexts to mathematical concepts and structures is possible and straightforward, but, actually, this process is a step from the contradictory world to a coherently organized esoteric sphere that has long since cut off its everyday roots" (p. 3).

## *"Intrinsic Integration" of Mathematization in RPGs*

In mathematization activities aimed at generalization, where students are encouraged to create models applicable to a range of situations and refine local models into broader mathematical theories, maximizing realism or truthfulness in problem scenarios is not essential. Indeed, for supporting mathematical generalization, specific referentiality needs to be overcome; it is unnecessary to attend to natural or social regularities for meaning-making, as can be observed in historical examples of the "word problems" genre, with numerous versions of the same riddles or puzzles. Hence, the context of RPGs appears to be a promising setting. In these games, a master typically creates and plans the story and structure of a fantastical world, often based on RPG manuals or their own rules, incorporating pedagogical objectives when used in a teaching context (Tamayo & Maltempi, 2023b).

Tamayo and Maltempi (2023b) analyzed 42 studies on RPGs in mathematics education. Their characterization of RPGs highlights elements relevant to mathematics education, emphasizing storytelling, creativity, and imagination as the most crucial components recognized by educators and researchers. Additionally, they note RPGs' effectiveness in engaging students and facilitating problem-solving within the game environment, a topic pertinent to this discussion.

Another relevant feature of RPGs is their reliance on negotiation, as the story's development is not solely controlled by the master; other players also contribute (Rudich, 2021). This interaction introduces power dynamics among players, who must collaborate to maintain a playful experience, as noted by Hammer et al. (2018). These authors describe three types of power in RPGs: "Power-to might reflect a player's ability to personally influence the game world, power-over might reflect their ability to control or negate the creative contributions of others, and power-with might describe shared creative elements of gameplay" (p. 450).

A prominent challenge in integrating mathematization into RPGs is ensuring that mathematical situations "naturally" fit within the game's narrative and campaigns (Marins, 2020), thereby preventing a scenario in which students feel forced to interrupt gameplay to perform mathematical tasks (Devlin, 2011). The distinction between "natural" and "non-natural integration" aligns with the concept of "intrinsic integration," which involves merging predefined mathematical content with the game's narrative, as explained by Habgood and Ainsworth (2011). They explain that intrinsic integration is achieved when two conditions are met: mathemat-

ics is incorporated into the most enjoyable parts of the game, without diminishing the overall enjoyment, and learning materials become an integral part of the game's structure, including its scenarios and player interactions, which provide the exploration of the content through its core mechanics. These two criteria guided the selection of works analyzed in this paper.

Another challenge involves ensuring consistency, coherence, and logical display of the game's story (Rudich, 2021; Schrier et al., 2018). Even though an RPG's setting evolves or transforms, these changes must adhere to the rules and logic governing the game world. This means that the actions allowed for characters must not break those rules. In other words, since RPG worlds are based on fantastical displays detached from the ordinary lives of players (Zagal & Deterding, 2018), it is easy to assume that any action might be carried out by the characters. However, the narrative's consistency is determined by some implicit or explicit rules that establish limits for players, such as physical laws, character card descriptions and powers, or mappings of the "physical" ambiance of the created world (Schrier et al., 2018). Such coherence allows the master-creator (or the teacher with pedagogical objectives) to design situations in which mathematization becomes possible.

The study by Çakıroğlu et al. (2023) connects modeling, RME, and virtuality. Particularly relevant here is their concept of "virtual RME." The study defines mathematical literacy as the ability to solve "real-life problems," extending beyond school mathematics and emphasizing non-routine problems. Their research reveals that the challenges of identifying real-life objects and problem scenarios can be overcome through the virtual world, where non-routine problems are presented in ways that immerse students and simulate real-world situations. The outcomes indicate that virtual tools possess the potential to generate problem scenarios, enhancing the efficacy of RME while fostering engagement and participation. Notably, students found these virtual problems realistic, and the virtual tools also facilitated formalization.

Lastly, a study by Dalla Vecchia and Maltempi (2014) reveals that mathematical modeling perspectives, although typically associated with the creation, analysis, and validation of models in the real world, can also be extended to virtual reality. The study investigated the modeling process undertaken by graduate students who designed games using Scratch (an educational platform oriented towards programming). Of particular relevance to this paper is that, despite the virtual world's distinction from physical space and time, it offered a platform for exploring modeling scenarios. As the authors elucidate, virtual reality actualized the constructed models derived from "real life." This prospect demands transcending the Cartesian notion of the physical world. This conclusion may, in turn, underscore the significance of conceiving fictional realities open to the processes of mathematization.

## Methods and Procedures

Only a limited number of works will be analyzed in this paper, as there are only a few that use RPGs as a pedagogical strategy within mathematics, and among those, only a small number stem from doctoral research, with a substantial portion arising from master's research (Tamayo-Martínez, 2025). It is important to note that, while research on using games for teaching and learning mathematics is not new, studies involving RPGs are a recent development in this field. Consequently, specialized mathematics education journal articles are underrepresented among the cited sources, plausibly reflecting the emergent status of this line of inquiry.

Through a comprehensive search using the keywords "Role-playing game" and "Mathematics" in Spanish, Portuguese, and English, only 64 works were found in September 2022. The search was conducted in repositories of theses and dissertations, mathematics education journals, and Google Schol-

ar, Redaly c, SciElo, ERIC, and JSTOR. Of those 64 works, 13 were selected based on the following criteria:

1. Explicitly reference and explain the examples used and, if possible, the interactions among participants.

2. To obtain a closer approximation to "intrinsic integration," the mathematical situations in the studied RPG should better satisfy the two characteristics outlined above.

3. The following characteristics led to the exclusion of works:

    - Includes only puzzles (even if they have some connection to the narrative) with little or no relation to mathematical concepts (e.g., passwords based on alternative alphabets or words and *numbers* used solely as symbols).

    - Uses equations, equation systems, or mathematical procedures only to find a number or set of numbers without any connection to the meaning or applicability of mathematical concepts.

    - Only includes routine problems and exercises to obtain objects or progress to the next stage in the game, often presented as multiple-choice questions but lacking any connection to the narrative.

4. In cases where multiple mathematical problems were present in each work, the one that most coherently integrated each criterion was considered.

The research serves an explorative purpose. Our goal is to inform the future development of RPGs for teaching mathematics and the practices that support their productive utilization. The analysis focused on situation statements framed as mathematics problems, modeling scenarios, or questions that require mathematization. Additionally, the examination included reported comments, explicit assumptions, and interactions of students, teachers, and researchers in response to those statements.

In this context, we categorized information units to identify patterns, regularities, and relations between categories, contributing to our research question. We initially defined some deductive categories based on the conceptual underpinnings and distinctions explained above, which were then evaluated for reliability by contrasting them with a portion of the selected material, as Mayring (2000) explained. Inductive categories emerged from the material, helping refine the deductive categories and develop coding rules. Based on those categories, some themes emerged that are presented in the next section. Table 1 summarizes the deductive starting categories that guided the analysis; the inductive development is reported through the themes presented in the next section.

**Table 1. Categories from which Themes Emerged**

| | |
|---|---|
| Group validation/interpretation | Mathematization of everyday math |
| Individual validation/interpretation | Mathematization with concepts |
| Validation/interpretation with the teacher | Mathematization with models |
| Mathematization from characters | Mathematization with problems |
| Mathematization from mechanics | Mathematization with procedures |
| Mathematization from narrative | Model selection/creation by students |
| Mathematization from scenarios | Model selection/creation by the teacher |
| Mathematization of abstract math | Model selection/creation together |

Source: Own elaboration

## Results and Discussion

To present the emerging themes, a general understanding of the stories and the mathematical situations in the games is necessary. Table 2 presents this information. The names of the RPGs described were taken from each work, or, in cases where they were missing, names were created for direct reference.

**Table 2. Description of RPGs' Narratives and Mathematical Statements**

| RPG | Description of the story and mathematical situations analyzed |
|---|---|
| Gauss' Fortress (Albino, 2020) | Players awake in a room that is part of a fortress from which they need to escape by passing through various doors that must open by solving puzzles. In one room, they find four prisms. One prism has a circular base, with the diameter displayed. The other three prisms have polygonal bases, and the lengths of their sides are also displayed. To open the door, they must place the prism with the largest volume on a device, which unlocks the door. |
| Magic Book (Azevedo, 2017) | The players need to find some pages of a magic book and defeat three monsters that escaped. Players represent heroes with different skills. In the battles, the monsters roll dice with six faces, each with a number between 1 and 11, except one with two failures (number 0). Also, players roll the dice; they roll a number between 1 and 5, and one failure. The original energy of the monsters is 20, 30, and 30. Each player has an initial energy of 13 and forms groups of three. In the battles, students must reduce the energy of the monsters to 0 before the monsters completely dismiss the energy of the three heroes. Students have two opportunities to use the power of a fairy to restore their energy by rolling a die with numbers 10-15 on it. |
| Ceiba Mission (Cano, 2014) | An ecologist writes a letter to the students because he wants to save a Ceiba tree before a bridge is built in the school neighborhood. He asks the students to find any construction irregularities to stop the bridge project and save the tree. Students need to find the number of vehicles (N) transiting some streets during an interval of time (T). Those streets might affect the bridge's construction. This data is used to define the traffic volume (Q) at the intersection of those streets using the equation: $Q = N/T$. |
| Medieval Mystery (Bressan, 2014) | In a medieval context, players are adventurers contracted by the mayor of a city to investigate the disappearance of citizens. The mayor gives the adventurers 250 golden coins. Each group of students (3 to 5 members) needs to divide the coins among their characters and decide what to do with the remainder if the division is not exact. |
| Criminal Case (Feijó, 2014) | Players need to find a serial killer by following a series of clues. Players find a bomb with a hexagonal prismatic shape, with a base side measuring 5cm and a height of 50 cm. They must calculate the effect of the bomb if a previous explosion produced by a cylinder of 8 $cm^3$ affected 30 $m^2$. |
| Medieval Fantasy (Honorio, 2015) | A feudal lord contracts some heroes (the players' characters) to test a series of challenges that suitors, who want to marry his daughter, need to pass. Those challenges consist of several rooms with magic and technological devices that open the following door. A light ray needs to be redirected to a target on the door using mirrors that reduce its original temperature (200 °C) by 5 % each time it is reflected. Each mirror has a device that shows the angle at which it is graduated. The door opens when the temperature of the ray on the target is between 130 °C and 150 °C. |
| Medieval World (Leitão, 2020) | In a medieval world, students need to solve mysteries and defeat some enemies. Players must roll dice to determine the value of their characters' skills and test the success of their actions in the game. They use basic arithmetic operations to do so. |

(Continued)

(Continued)

| RPG | Description of the story and mathematical situations analyzed |
|---|---|
| Ancient Civilization (Machado et al., 2013) | Players start the story in an ancient civilization. At some point, they need to pass through portals called "palindrome portals." To pass through, they must understand why those portals are named that. They roll three dice, one at a time, obtaining a three-digit number. Then they must invert the digits and add the two numbers. By analyzing the result (a palindrome), they need to find the meaning of the portal's name. |
| RPG Makers (Rosa, 2004) | Groups of students design their own RPGs. Each game must help students of the same grade learn about integer numbers. One student discusses the concept of multiplying negative integers by considering debts to people who have just died. What he owes is negative (-4 reales), and the number of people is also negative because they are dead (-3 people). So, the amount the student still has is positive (12), because he does not have to pay now. |
| War of Canudos (Santos, 2011) | Players take part in some of the main campaigns in a historical war against the abuse of power in a region of Brazil (*Guerra de Canudos*). They analyze the characteristics of role dice (with the shape of platonic solids) used to test actions and solve events in the game. |
| Laboratory Attack (da Silva, 2014) | Players are part of a special group of heroes with superpowers. They are contracted to help defend a laboratory against a supervillain attack. Later, they discover that the villains have a secret plan to destroy the city. Players find some documents during the campaign. One document shows the graph of the function $f(x)= -0.05x^2 + x$, which describes the trajectory of a missile. They need to find the zeros of f(x) and locate the villains. |
| Pirate Adventure (Sobral, 2018) | A descendant of Barba Negra is looking for treasures with special powers. She contracts a group of pirates (the players) to help her find those relics. Players must go to the highest hill on an island. All the slopes of the hills have an inclination of 30°. However, there is a certain number of steps (e.g., 84, 72, 60, etc.) to reach the top of each hill. Some information in the game also provides the distance to the midpoint of each hill's base (e.g., 72, 63, 51 steps). Each step represents 3 meters. |
| 3D Tournaments (Nishizawa & Yoshioka, 2008) | The characters that players design participate in several tournaments to improve their skills and power. The tournaments are divided into three classes: sports, music, and puzzles. Students create a character based on strength, thinking ability, and musical skills. Each characteristic is a vectorial component of a 3D vector (CV) of a given length that represents students' characters. In some battles, a sum vector is to be defined from various CVs of players. In the battles, each CV, or the sum vector, is compared with the unit vector of the battlefield (which the teacher changes each match) using a dot product. Winners grow their CVs by 20%, and losers shrink them by 20%. |

Source: Own elaboration

Based on these works, three main themes could explain differences and similarities in the integration of mathematization processes to provide learning environments in RPGs.

## *Mathematization from Mechanics, Characters, Scenarios, or Narratives*

The first possibility relates to where the mathematization processes emerge and are contextualized in the games.

Mechanics in an RPG are the rules and conditions that govern what can be done in the game, including players' decisions about their actions, tests, combat results, etc. The process of mathematization in RPG mechanics might be interpreted as a natural necessity to keep the games going. For example, in the case of *Magic Book*, primary school students had to add and subtract the results of dice rolls to determine whether their characters could defeat the monsters. The same processes were more auto-

matic and precise for older students from secondary and middle school in the case of *Medieval World*. In this instance, "mathematization" resembles the application of a skill, as those operations were well understood by students of this age (which does not exclude the fact that students who demonstrated difficulties improved).

In *3D Tournaments*, students needed to understand the composition of the vectors that described their characters, interpret visualization programs that helped them represent those vectors, and use the dot product to determine match results. Here, as in the other two examples, the game could not continue without the application of these concepts or skills.

Most of the other games also used dice and basic arithmetic operations in their mechanics. However, beyond merely playing and resolving the tests or fights in the games, teachers (masters), researchers (who were sometimes also masters), and students were occasionally interested in understanding the principles behind the concepts and procedures based on their experience of playing. The games initially presented a horizontal discourse that assumed the ability to read results from throwing dice or the components of characters' strength. However, understanding the character's strength as a vector with three components involves making explicit the mathematical principles behind this construction of the "character" to provide the means for actions and fights within the game. Hence, this process involves some vertical mathematization.

Mathematics was integrated into the mechanics when needed to make decisions in the game, such as fighting monsters or other opponents. Something different happened in the *War of Canudos*. While a (vertical) mathematization process was indeed needed when students had to define the characteristics of the dice they used to assess the success of their actions, its integration appeared to be lacking. The analysis of the dice was irrelevant for determining what happened with the results after rolling them, since those geometrical characteristics were not used to continue the game. In contrast, if students had used that analysis to select a die to improve the probability of a better outcome after rolling, the integration with mechanics would have been clearer. The authors do mention that a student made that kind of decision; however, it was not clear how it happened.

Mechanics in RPGs include character creation, which defines their skills, other attributes, and evolution. However, a distinction needs to be made regarding the relevance of the mathematization processes for the performance of characters in this type of game. In most RPGs, and as described in *Medieval World*, basic mathematics is used to distribute, add, or subtract the results of the dice among the characters' different skills. A special case is also presented in *3D Tournaments*, where students need to create their characters using a vector with three skill components. The length of that vector increases or decreases by 20 %, reflecting the characters' evolution throughout the game. The mathematization allows players to model their characters with the vector, a process in which the mathematical recontextualization of the horizontal discourse that explains their strength mutates into a vertical discourse in linear algebra that defines it in terms of the mathematical object (components and length). In general terms, here the mathematization of characters is not only used to describe them but also to evolve them, enabling this progression.

Mathematization can also emerge from scenarios, which include not only the features of the fictitious environments where the game takes place but also the objects placed within them, often characterized by elements of realism. Among the works studied, this was the most prominent means by which mathematics emerged. In the case of scenarios, players needed to define the height of mountains using geometric or trigonometric theorems (*Pirate Adventure*) or determine the number of vehicles passing along a street to calculate traf-

fic density (*Ceiba Mission*). Regarding the objects placed in the game environments, students had to calculate the volume of a specific cup among others (*Calculus Geniuses*), find the volume of a prism to determine the effect of a bomb explosion (*Criminal Case*), to determine the intensity of a light ray after being reflected in several mirrors to open a door using percentages (*Medieval Fantasy*), to divide golden coins received as a reward for undertaking a mission among a group of heroes (*Medieval Mistery*), or analyze the trajectory of a missile represented by a function (*Laboratory Attack*).

In general terms, mathematization from scenarios uses fictive environments, with their associated fictive characteristics and objects, to integrate mathematics into RPGs. In the analyzed works, this kind of mathematization often does not differ much from those statements used in textbooks, and students might face the same difficulties, as shown by Gellert and Jablonka (2009). Additionally, we may still see these games as conveying a myth of reference, though the overall narrative of the game provides a different sense and context: a virtual world that simulates the advantage of a mathematical perspective without alternatives. In contrast to those vignettes from school classrooms, the fantasy in these environments does not always aim to simulate students' daily life. Further, it is conceptualized in a way that either maintains some features of the real world or does not. These allow for maintaining some of the characteristics as a base for establishing a vertical discourse. Again, *3D Tournaments* serves as a good example. Here, the characteristics of the battlefields are represented strictly by a unit vector, which the teacher can modify according to the type of tournament, the conditions of which are defined solely by the game's narrative. Through such constructions, RPGs overcome the mythologizing of students' everyday practical lives.

In this context, RPG narratives also allow for mathematization processes in which mechanics, characters, or scenarios are not explicitly involved, and only the overall narrative indicates a mathematical problem. For example, in *Ancient Civilization*, there is no device that alludes to palindrome numbers or even a sign with the names of the portals; it is the narration that demands finding out what "palindrome" means to cross the portals and the process through which students can identify the characteristics of the concept. Similarly, in *RPG Makers,* the idea of dead people and debts is conveyed only through the narrative.

In general, mathematization from narratives uses the narrative's fantasy to make a mathematical situation plausible or verisimilar. The narrative appears to support the mathematical recontextualization of the statements through this fantasy, which is open to any narrative structure that justifies the emergence of the problem. In the case of *Medieval Mystery*, for instance, dividing money among characters is perfectly plausible in a medieval mission. It not only does not require much explanation about why this needs to be done, but the problem is also accepted as natural in the story. In the case of *Gauss' Fortress*, it might not be an important issue for the players that the glasses have perfect geometric shapes, since that is how it must be in the story. The same happens with the energy of the monsters in *Magic Book*, since there is no need for an explanation of why the monsters or the heroes lose their power. In this sense, through these games, the myths of participation or reference are abolished.

### *Selection of Models, Strategies, and Procedures by Students, Teachers, or Both*

The mathematization process can also be explained by selecting models, strategies, and procedures to solve the problems featured in the games. These problems are generally designed by teachers or researchers (except in the special case of *RPG Makers*). However, teachers do not always use mathematical methods to solve problems, as the development of the story often involves negotia-

tion and complex power dynamics between teachers and players, as noted by Hammer et al. (2018) and Rudich (2021).

In some instances, students actively interacted with the problem, often with validation from teachers or the rest of the group. In *Ancient Civilization*, when students needed to figure out what the palindrome portal meant, they asked for help after demonstrating that everything was done correctly after adding the two numbers and that they could read the result of the addition forward and backward. Eventually, after the researchers insisted that the answer be found independently, students deduced that if the numbers could be read that way, then the palindrome portals were round-trip. Notably, the research done by students in their groups was based on different numbers, as each group rolled different results.

A similar situation occurred with students in *Medieval Mystery*. Even though the groups had the same number of golden coins to divide, the number of members ranged from three to five, and in cases of remainders, they could do whatever they wanted with the rest of the money. Multiple solutions were also possible in *Medieval Fantasy*, where students could place different numbers of mirrors to reflect the ray and achieve a correct temperature, as long as it was between 130 °C and 150 °C.

In those examples, the teachers' intervention needed to be minimal, only mediating or asking questions to help students find the answers themselves. An example of gradual teacher intervention is presented in *Calculus Geniuses*. The teacher used the narrative itself to offer some help by introducing a character (a fictitious tablet that shows some texts) that provided information regarding the problems that needed to be solved. Students could also roll dice, and depending on the results, the teacher, performing as the tablet character, offered additional help. This ranged from a geometrical description of the solids whose volumes had to be found, to the formula for calculating the area of their bases, to characteristics of their height, and even concluding that, as the solids had the same height, it was only necessary to find the area of their bases to determine which container had the larger volume.

This example suggests a slightly different situation in which the RPG itself serves as the mediator and offers possible models. This is perfectly possible within the context of the narrative, for example, using a character represented by the teacher or showing clues that students can find around. However, any help the game offers diminishes students' possibilities to discover procedures or models. In some cases, the teacher's intervention could be too much. In *Ceiba Mission*, for example, the narrative presented students with a list of instructions to determine traffic volume, including the formula for relating volume and time.

A similar case is *Magic Book*, in which the researcher and teacher did not allow students to use the appropriate procedures (addition or subtraction) to obtain results in most situations. The correct answer after rolling the dice was not open to discussion, as it was only to determine whether the monsters were eliminated. In those instances, the students could only perform the operations mechanically. Clearly, the master missed the opportunity to teach players when to add or subtract after rolling the dice. A situation with a different teacher intervention occurred in *Medieval World*. Students also needed to perform basic operations to find results after rolling dice, but they had to determine the appropriate procedure using their own strategies, such as mental calculation.

In most other games, it made sense for the students to discover the results or mathematical meanings because the narrative demanded it of them, not of the master. They could earn a reward for their characters by choosing to continue the game or increase their power. If the narrative conveys that players must understand why something

happens (e.g., the round trip of palindrome portals), it loses its meaning if students do not find their own answers. Otherwise, there is a risk of inconsistency, incoherence, and an illogical display of the story, elements highlighted by Rudich (2021) and Schrier et al. (2018) as characteristic of RPGs.

This lack of coherence is not only perceived by the teacher, who has pedagogical aims, but also by students, who understand that their role in the game requires them to find solutions, and receiving answers is akin to cheating. In these cases, students' mathematical solutions were mandatory within the context of the characters, scenarios, or narrative. In a few cases, the students' ideas were combined with the teachers' explanations when selecting the procedures. For instance, in *War of Canudos*, the teacher asked about the characteristics of the dice that the students were using. The students presented some characteristics when they remembered them; when they did not, the researcher described these characteristics or encouraged the students to look them up on the internet. After identifying them, the teacher guided the students to compare them and draw conclusions. In *Criminal Case*, after realizing that the students did not remember the formula for finding the volume of a prism, the teacher wrote down a general version ($v = b \times h$) and drew a representation of the shape on the board. He also drew a cylinder, allowed the students to find the volume, and encouraged them to compare the effects of bombs with different volumes when using the same explosive, offering occasional help and validating the procedures.

Based on the previous discussion and the power dynamics described by Hammer et al. (2018), a question arises: How do players' power dynamics influence the formulation of problems and the selection of solution methods in the games? In most cases analyzed, the teacher-master exerts influence over the game world by formulating the problems and introducing solutions, aligning with a "power-to" dynamic that shapes the recreated narrative. Students also affect the game world through their interactions with the problems, their decisions, and the methods they select to solve them. When teachers assist students in finding solutions or when students reach agreement on responses, the "power-with" dynamic becomes more apparent in the game interactions. This type of power aligns with the collaborative nature of RPGs and the teacher-master's goal of encouraging students to find the answers independently. Lastly, "power-over" was not clearly observed in the analyzed examples, except in instances where the teacher-master influenced students by approving and validating their interactions and responses, which leads to the final theme.

### *Validation or Interpretation by Students with Teachers or Players in Their Groups*

The discussion above shows that, after solving the mathematical problems, validating or interpreting the solutions players find becomes especially important in RPGs. Players need to confirm that their results or conclusions are correct in order to continue the game, make decisions, or determine the outcomes of battles or character actions. These processes of validation or interpretation were carried out through interactions between players and the teacher or within the player groups themselves. Although individual validation was expected during game analysis, it was not clearly identified in any of the studies.

Due to the teachers' pedagogical interests, validation by them occurred more frequently during gameplay. This was evident in all the analyzed works, except in *Ancient Civilization* and *3D Tournaments*. In both cases, the interactions between students and teachers were not extensively presented. However, it is possible to infer that, if the students successfully passed through the portals after understanding the meaning of a palindrome, the answer was likely satisfactory to the teacher. In the case of *3D Tournaments*, it seems the teacher played a significant role in approving the creation and modification of the character vectors. Generally, this type

of validation occurs because the teacher's role is crucial to the game's development; their judgment (also serving as the master) determines whether the game's results align with the expected, implicit, or explicit rules.

To illustrate this type of validation, in *Pirate Adventure*, the researcher presented two possible outcomes based on players' decisions after analyzing the problem. If the students chose to ascend any hill without a specific criterion, the game would continue, but failing to select the highest one would lead to a difficult battle, risking the loss of an important letter containing clues for the next adventure. However, if students correctly identified the highest hill, they could avoid the mortal combat and easily obtain the letter.

On the other hand, validation occurred only within groups, without the teacher's clear intervention, when students corrected a peer's answer or, through interaction, collectively found a solution to a problem. As might be expected, this type of validation occurred in gameplay situations in which the teacher did not select the model, procedure, or strategy to solve the problem.

Additionally, in only three of the analyzed works was group validation supplemented by teacher validation. In these cases, the teacher complemented the group's findings with explanations or encouraged them to explain their reasoning to others. In contrast, in *Magic Book*, after students discussed the results of the dice among themselves, the teacher, instead of encouraging them to delve deeper into their reasoning, directly presented the necessary procedure (adding or subtracting) to determine the results after rolling the dice.

## Conclusions

From the 64 works identified in the literature review supporting this paper, seven were research studies without empirical evidence, meaning they did not provide examples of integrating mathematization processes within the context of RPGs used in direct interaction with students. This left 57 works that contained mathematical statements to analyze. However, only 13 of these works included statements that more closely approximated the criteria for intrinsic integration, meaning they did not include only puzzles with limited connection to mathematical concepts or routine problems that were weakly or entirely unrelated to the RPGs' narratives. This finding highlights the challenges of effectively implementing RPGs that integrate mathematization processes while meeting these criteria. This difficulty may stem from the fact that, like many educational video games designed for mathematics learning, RPGs often focus primarily on developing algorithmic skills rather than fostering mathematical thinking, which is indeed more challenging to teach (Devlin, 2011). Employing RPGs for such limited purposes risks wasting their potential to create rich learning environments.

A deeper understanding of the potential for RPGs in teaching mathematics can be gained by examining their characteristics. RPGs underscore a robust relationship among gaming, modeling, and problem-solving:

- The mechanics (including character creation and evolution), scenarios, and narratives provide numerous opportunities to integrate mathematization processes.
- The models, strategies, and procedures used to solve problems present various challenges for teachers and students during gameplay.
- Learning can be enhanced depending on how validation or interpretation of the results is carried out by players and the master, typically represented by teachers or researchers, though students can also direct their own games.

From analyzing these aspects, it is evident that mathematical situations—particularly those involving intrinsic integration—unfold organically within

RPGs. Teachers do not need to artificially create scenarios to formulate isolated problems, which could compromise the coherence of mathematical statements. Instead, they can use the narratives, mechanics, and scenarios of RPGs, whether or not there is an explicit attempt to connect with students' everyday experiences. Within their recreated realities, RPGs align with the notion of "real" in RME, which emphasizes the capacity to imagine or conceive something within a context.

These fictional worlds, while flexible and able to tolerate certain contradictions akin to real life, also present challenges. A didactic RPG still requires translating the game's horizontal discourse into the vertical discourse of mathematics. Nevertheless, the creative and imaginative nature of RPGs can align with the rules of vertical discourse, partially mitigating these contradictions.

Despite the juxtaposition of horizontal and vertical discourses, the recontextualization involved in RPGs presents a unique condition: the problem exists authentically within the fictional world. It is not artificially created solely to contextualize vertical discourse within a horizontal context; rather, it emerges as part of the game's development. While students must still transition from horizontal to vertical discourse, the mathematical statements in RPGs are supported by fictional worlds offering endless possibilities. In these narratives, problems arise from pure invention, providing a different kind of recontextualization.

Therefore, the "myth of participation," that might lead students to believe that their mathematical knowledge is insufficient to deal with everyday situations (Dowling, 1998), may be more easily overcome, as the link with those domestic situations is openly broken by fantasy and imagination. RPGs also offer a space where the "myth of reference"—the idea that the meaning of mathematical concepts is inherent in their applications (Dowling, 1998)—can be challenged. In RPGs, the rules and mechanics are explicitly negotiated and agreed upon by players, demonstrating that the meaning of mathematical concepts is not fixed but emerges from their use in specific contexts. Furthermore, the narrative frameworks of RPGs allow for alternative, non-mathematical descriptions of recreated realities, broadening the scope for mathematical exploration and creativity.

This perspective challenges the traditional education system, which often prioritizes factual information at the expense of exploring the potential offered by fictional worlds in the classroom. Despite being rooted in imaginative realities, RPGs can effectively engage students in modeling, problem-solving, and learning. This is possible because the game's outcomes—whether as rewards or as mechanisms that shape subsequent scenarios—are vital to gameplay progression.

However, incorporating RPG environments into mathematics education is not without its challenges. The creation and design of these games demand significant effort from teachers and require a paradigm shift in educational practices. This shift involves placing students at the center of the interaction as active participants, with mediation taking precedence over traditional knowledge transmission. Additionally, embedding abstract mathematical concepts into the gaming context requires a creative and rigorous approach. Teachers may also face time and curricular constraints that limit the feasibility of using RPGs in their classrooms.

To address these challenges, innovative perspectives and further research are necessary to expand our understanding of the role RPGs can play in mathematics education. The findings of this paper were based on specific criteria for selecting gaming experiences, resulting in a limited number of examples. Future research could build upon these insights by comparing them with studies that examine broader uses of RPGs in integrating mathematization processes. Such research might include

instances that do not strictly adhere to the criteria used here, such as routine problems or those with weaker connections to the game's narrative, thereby broadening the scope of inquiry into the potential of RPGs as learning tools.

## Authors' Contributions

Edwin Tamayo: conceptualization, methodology, formal analysis, and investigation; writing—original draft preparation—reviewing and editing, funding acquisition, project administration; Marcus Maltempi: supervision, writing—review & editing; Eva Jablonka: supervision, writing—review & editing.

## Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## AI Usage

The authors declare that they have not used any AI tools to prepare this paper

## References

Albino, H. H. (2020). *Gênios do Cálculo RPG: Uma ferramenta para o ensino de matemática* [Master's thesis, Universidade Federal de Juiz de Fora]. https://repositorio.ufscar.br/handle/ufscar/13620?show=full

Azevedo, K. de L. (2017). *Jogo de tabuleiro com elementos de RPG "aventura de um livro mágico": contribuições para a educação matemática* [Master's thesis, Universidade Federal de Pernambuco]. https://repositorio.ufpe.br/handle/123456789/25198

Bernier, J., & Zandieh, M. (2024). Comparing student strategies in a game-based and pen-and-paper task for linear algebra. *The Journal of Mathematical Behavior, 73*, 1-16. https://doi.org/10.1016/j.jmathb.2023.101105

Bernstein, B. (1999). Vertical and horizontal discourse: An essay. *British Journal of Sociology of Education*, *20*(2), 157-173. https://doi.org/10.1080/01425699995380

Bressan, R. (2014). *RPG como estratégia no ensino das operações elementares em matemática* [Master's thesis, Universidade Cruzeiro do Sul]. https://ptdocz.com/doc/1339335/artigo-completo---universidad

Bright, G. W., Harvey, J. G., & Wheeler, M. M. (1985). Learning and mathematics games. *Journal for Research in Mathematics Education. Monograph*, *1*, i. https://doi.org/10.2307/749987

Çakıroğlu, Ü., Güler, M., Dündar, M., & Coşkun, F. (2023). Virtual Reality in Realistic Mathematics Education to Develop Mathematical Literacy Skills. *International Journal of Human-Computer Interaction*, 1-13. https://doi.org/10.1080/10447318.2023.2219960

Cano, N. A. (2014). *Juegos de rol y análisis de modelos: El contexto del puente de la madre Laura Montoya Upegui* [Master's thesis, Universidad Nacional de Colombia]. https://repositorio.unal.edu.co/handle/unal/51912

Da Silva, F. Q. (2014). *Usando RPG no ensino da matemática* [Master's thesis, Universidade Federal de Juiz de Fora]. https://repositorio.ufjf.br/jspui/handle/ufjf/746

Dalla Vecchia, R., & Maltempi, M. V. (2014). O Modelo na Modelagem Matemática na Realidade do Mundo Cibernético. *Acta Scientiae*, *16*(4), 199-213. http://www.periodicos.ulbra.br/index.php/acta/article/view/1271

Dan, N. N., Trung, L. T., Nga, N. T., & Dung, T. M. (2024). Digital game-based learning in mathematics education at primary school level: A systematic literature review. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(4), em2423. https://doi.org/10.29333/ejmste/14377

Devlin, K. (2011). *Mathematics Education for a New Era: Video Games as a Medium for Learning*. A. K. Peters. https://doi.org/10.1201/b10816

Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/pedagogical texts*. Falmer Press.

Ernest, P. (1986). Games. A Rationale for Their Use in the Teaching of Mathematics in School. *Mathematics in School*, *15*(1), 2-5. https://doi.org/10.1093/teamat/5.3.97

Feijó, R. O. (2014). *O uso de role playing games como recurso pedagógico nas aulas de matemática* [Master's thesis, Universidade Federal de Rio Grande do Sul]. https://lume.ufrgs.br/handle/10183/108424

Freudenthal, H. (2002). *Revisiting Mathematics Education* (Vol. 9). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47202-3

Gallego, A., Gallego, R., & Pérez, R. (2006). ¿Qué versión de ciencia se enseña en el aula? Sobre los modelos científicos y la didáctica de la modelación. *Educación y Educadores*, *9*(1), 105-116. https://educacionyeducadores.unisabana.edu.co/index.php/eye/article/view/650

Gee, J. P. (2004). *What video games have to teach us about learning and literacy* (1st ed.). Palgrave Macmillan. https://doi.org/10.1145/950566.950595

Gellert, U., & Jablonka, E. (2009). "I am not talking about reality": Word problems and the intricacies of producing legitimate text. In L. Verschaffel, B. Greer, W. van Dooren, & S. Mukhopadhyay (Orgs.), *Words and worlds: Modelling verbal descriptions of situations* (pp. 39-53). Brill. https://doi.org/10.1163/9789087909383_004

Gerofsky, S. (2009). Genre, simulacra, impossible exchange, and the real: How postmodern theory problematises word problems. In L. Verschaffel, B. Greer, W. van Dooren, & S. Mukhopadhyay (Orgs.), *Words and worlds: Modelling verbal descriptions of situations* (pp. 21-38). Brill. https://doi.org/10.1163/9789087909383_003

Gros, B. (2000). La dimensión socioeducativa de los videojuegos. *Edutec. Revista Electrónica de Tecnología Educativa*, *12*. https://doi.org/10.21556/edutec.2000.12.557

Habgood, M. P. J., & Ainsworth, S. E. (2011). Motivating Children to Learn Effectively: Exploring the Value of Intrinsic Integration in Educational Games. *Journal of the Learning Sciences*, *20*(2), 169-206. https://doi.org/10.1080/10508406.2010.508029

Hammer, J., Beltrán, W., Walton, J., & Turkington, M. (2018). Power and control in role-playing games. In J. P. Zagal & S. Deterding (Orgs.), *Role-playing game studies: transmedia foundations* (pp. 448-467). Routledge. https://doi.org/10.4324/9781315637532-27

Honorio, B. G. (2015). *Observar com sentido: um experimento com estudantes de licenciatura em matemática envolvendo a utilização do RPG* [Master's thesis, Universidade Luterana do Brasil]. http://www.ppgecim.ulbra.br/teses/index.php/ppgecim/article/view/220/212

Hui, H. B., & Mahmud, M. S. (2023). Influence of game-based learning in mathematics education on the students' cognitive and affective domain: A systematic review. *Frontiers of Psychology*, *14*,1105806. https://doi.org/10.3389/fpsyg.2023.1105806

Ishibashi, I., & Uegatani, Y. (2022). Cultural relevance of validation during mathematical modeling and word problem-solving: Reconceptualizing validation as an integration of possible fictional worlds. *The Journal of Mathematical Behavior*, *66*, 1-19. https://doi.org/10.1016/j.jmathb.2022.100934

Jablonka, E. (1996). *Meta-analyse von zugänger zur mathematischen modellbildung und konsequenzen für den unterricht* [Doctoral dissertation, Technischen Universität Berlin]. https://doi.org/10.1007/BF03338842

Jablonka, E., & Gellert, U. (2007). Mathematisation - Demathematisation. In E. Jablonka & U. Gellert (Orgs.), *Mathematisation and Demathematisation: Social, Philosophical and Educational Ramifications* (pp. 1-18). Sense Publishers. https://doi.org/10.1163/9789460911439_002

Jablonka, E., & Gellert, U. (2011). Equity Concerns About Mathematical Modelling. In B. Atweh, M. Graven, W. Secada, & P. Valero (Orgs.), *Mapping Equity and Quality in Mathematics Education* (pp. 223-236). Springer Netherlands. https://doi.org/10.1007/978-90-481-9803-0_16

Kaiser, G. (2020). Mathematical Modelling and Applications in Education. In S. Lerman, B. Sriraman, E. Jablonka, Y. Shimizu, M. Artigue, R. Even, & M. Graven (Orgs.), *Encyclopedia of Mathematics Education* (pp. 553-561). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_101

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM*, *38*(3), 302-310. https://doi.org/10.1007/BF02652813

Karimov, A., Saarela, M., & Kärkkäinen, T. (2024). Serious games in science and mathematics education: a scoping umbrella review. *International Journal of Serious Games*, *11*(4). https://doi.org/10.17083/ijsg.v11i3.765

Leitão, L. N. P. (2020). *Role-playing game (RPG) na aprendizagem das quatro operações aritméticas: uma interlocução winnicottiana* [Bachelor's thesis, Universidade Federal da Paraíba]. https://repositorio.ufpb.br/jspui/handle/123456789/17951?locale=pt_BR

Machado, S. R. A., Coqueiro, V. dos S., & Hermann, W. (2013). Os desafios de se ensinar matemática por meio de jogos de interpretação de personagem em sextos anos do ensino fundamental de uma escola da rede pública. *Revista NUPEM*, *5*(8). https://doi.org/10.33871/nupem.v5i8.193

Marins, P. N. de. (2020). O RPG de mesa como estratégia para o ensino de matemática: Alguns exemplos de uso. *Revista Mais Dados*, *10*(1), 18-30.

Mayring, P. (2000). Qualitative content analysis. *Forum Qualitative Social Research Forum: Qualitative Social Research*, *1*(2). https://www.researchgate.net/publication/215666096_Qualitative_Content_Analysis

McFeetors, P. J., & Palfy, K. (2018). Educative experiences in a games context: Supporting emerging reasoning in elementary school mathematics. *The Journal of Mathematical Behavior, 50*, 103-125. https://doi.org/10.1016/j.jmathb.2018.02.003

Mousoulides, N., & Sriraman, B. (2020). Mathematical Games in Learning and Teaching. In *Encyclopedia of Mathematics Education* (pp. 538-540). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_97

Muniz, C. A. (2021a). Bases conceituais: conceito de jogo como fonte de aprendizagem matemática. In C. A. Muniz (Org.), *Aprendizagem matemática em jogo* (pp. 14-24). Rede pedagógica.

Muniz, C. A. (2021b). Educação lúdica da matemática, educação matemática lúdica. In C. A. Muniz (Org.), *Aprendizagem matemática em jogo* (pp. 316-337). Rede Pedagógica.

Nishizawa, H, & Yoshioka, T. (2008). A proposal to teach 3d vector operations in a role-playing game. In *Proceedings of the 13th Asian Technology Conference in Mathematics (ATCM 2008)* (pp. 364-369). Bangkok, Thailand. https://atcm.mathandtech.org/ep2008/papers_full/2412008_15294.pdf

Niss, M., & Blum, W. (2020). *The learning and teaching of mathematical modelling*. Routledge. https://doi.org/10.4324/9781315189314

Nunokawa, K. (2005). Mathematical problem solving and learning mathematics: What we expect students to obtain. *The Journal of Mathematical Behavior, 24*(3-4), 325-340. https://doi.org/10.1016/j.jmathb.2005.09.002

Rosa, M. (2004). *Role Playing Game Eletrônico: uma tecnologia lúdica para aprender e ensinar matemática* [Master's thesis, Universidade Estadual Paulista]. http://hdl.handle.net/11449/91089

Rudich, C. M. (2021). Rol, ¿literatura?: la estructura de las narraciones en los juegos de rol. *Lanzando los dados: aproximaciones* (pp. 254-262). Universidad Iberoamericana Ciudad de México.

Santos, C. (2011). *RPG Geo Moodle: Modelagem dos dados de múltiplas fases* [Master's thesis, SENAI/CIMATEC]. http://repositoriosenaiba.fieb.org.br/handle/fieb/725

Schrier, K., Torner, E., & Hammer, J. (2018). Worldbuilding in role-playing games. In J. P. Zagal & S. Deterding (Orgs.), *Role-playing game studies: transmedia foundations* (pp. 349-363). Routledge. https://doi.org/10.4324/9781315637532-20

Shiakalli, M. A., & Zacharos, K. (2014). Building meaning through problem solving practices: the case of four-year olds. *The Journal of Mathematical Behavior, 35*, 58-73. https://doi.org/10.1016/j.jmathb.2014.05.001

Smith, C., & Morgan, C. (2016). Curricular orientations to real-world contexts in mathematics. *The Curriculum Journal, 27*(1), 24-45. https://doi.org/10.1080/09585176.2016.1139498

Sobral, P. M. (2018). *Role playing game (RPG) como método avaliativo no processo de ensino e aprendizagem de saberes matemáticos de forma lúdica* [Master's thesis, Universidade Federal de Pernambuco]. https://repositorio.ufpe.br/handle/123456789/38893

Stillman, G. A. (2019). State of the Art on Modelling in Mathematics Education—Lines of Inquiry. In G. A. Stillman & J. P. Brown (Orgs.), *Lines of Inquiry in Mathematical Modelling Research in Education* (pp. 1-20). Springer International Publishing. https://doi.org/10.1007/978-3-030-14931-4_1

Tamayo, E., & Maltempi, M. (2023a). Entre juego como actividad libre y juego serio: articulación de contenidos matemáticos a un juego de rol. *Paradigma*, *44*(3), 334-353. https://doi.org/10.37618/PARADIGMA.1011-2251.2023.p334-353.id1454

Tamayo, E., & Maltempi, M. (2023b). RPG (role-playing games) na educação matemática. *Anais do Encontro Paulista de Educação Matemática*. www.even3.com.br/ebook/xv-epem-encontro-paulista-de-educacao-matematica-294768/616382-RPGS-(ROLE-PLAYING-GAMES)-NA-EDUCACAO-MATEMATICA

Tamayo-Martínez, E. (2025). *El libro de Morfeo: experiências de aprendizagem mediada em um role-playing game matemático* [Doctoral dissertation, Universidade Estadual Paulista (Unesp)]. https://hdl.handle.net/11449/259872

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic Mathematics Education. In S. Lerman, B. Sriraman, E. Jablonka, Y. Shimizu, M. Artigue, R. Even, & M. Graven (Orgs.), *Encyclopedia of Mathematics Education* (pp. 713-717). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_170

Verschaffel, L., Greer, B., van Dooren, W., & Mukhopadhyay, S. (Orgs.). (2009). *Words and worlds: Modelling verbal descriptions of situations*. Brill. https://doi.org/10.1163/9789087909383

Wagner, D. (2011). Warm Bodies Using Cold Mathematics. *Antistasis, 1*(2), 7-9. https://journals.lib.unb.ca/index.php/antistasis/article/view/18521

Zagal, J., & Deterding, S. (2018). Definitions of "Role-Playing Games." In J. Zagal & S. Deterding (Orgs.), *Role-playing game studies: transmedia foundations* (pp. 19-51). Routledge. https://doi.org/10.4324/9781315637532