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The Indefinite within Descartes' Mathematical Physics

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Abstract

Descartes' philosophy contains an intriguing notion of the infinite, a concept labeled by the philosopher as indefinite. Even though Descartes clearly defined this term on several occasions in the correspondence with his contemporaries, as well as in his Principles of Philosophy, numerous problems about its meaning have arisen over the years. Most commentators reject the view that the indefinite could mean a real thing and, instead, identify it with an Aristotelian potential infinite. In the first part of this article, I show why there is no numerical infinity in Cartesian mathematics, as such a concept would be inconsistent with the main fundamental attribute of numbers: to be comparable with each other. In the second part, I analyze the indefinite in the context of Descartes' mathematical physics. It is my contention that, even with no trace of infinite in his mathematics, Descartes does refer to an actual indefinite because of its application to the material world within the system of his physics. This fact underlines a discrepancy between his mathematics and physics of the infinite, but does not lead a difficulty in his mathematical physics. Thus, in Descartes' physics, the indefinite refers to an actual dimension of the world rather than to an Aristotelian mathematical potential infinity. In fact, Descartes establishes the reality and limitlessness of the extension of the cosmos and, by extension, the actual nature of his indefinite world. This indefinite has a physical dimension, even if it is not measurable.

Keywords

Descartes, Aristotle, Nicholas of Cusa, Galileo, potential and actual infinity, mathematical and physical infinity, mathematical physics, space, matter, extension, subtle matter.

Resumen

La filosofía de Descartes contiene una noción intrigante de lo infinito, un concepto nombrado por el filósofo como indefinido. Aunque en varias ocasiones Descartes definió claramente este término en su correspondencia con sus contemporáneos y en sus Principios de filosofía, han surgido muchos problemas acerca de su significado a lo largo de los años. La mayoría de comentaristas rechaza la idea de que indefinido podría significar una cosa real y, en cambio, la identifica con un infinito potencial aristotélico. En la primera parte de este artículo muestro por qué no hay infinito numérico en las matemáticas cartesianas, en la medida en que tal concepto sería inconsistente con el principal atributo fundamental de los números: ser comparables entre sí. En la segunda parte analizo lo indefinido en el contexto de la física matemática de Descartes. Mi argumento es que, aunque no hay rastro de infinito en sus matemáticas, Descartes se refiere a un indefinido real a causa de sus aplicaciones al mundo material dentro del sistema de su física. Este hecho subraya una discrepancia entre sus matemáticas y su física de lo infinito, pero no implica ninguna dificultad en su física matemática. Así pues, en la física de Descartes, lo indefinido se refiere a una dimensión real del mundo más que a una infinitud potencial matemática aristotélica. De hecho, Descartes establece la realidad e infinitud de la extensión del cosmos y, por extensión, la naturaleza real de su mundo indefinido. Esta indefinición tiene una dimensión física aunque no sea medible.

Palabras clave

Descartes, Aristóteles, Nicolás de Cusa, Galileo, infinito en potencia y en acto, infinito matemático y físico, física matemática, espacio, materia, extensión, materia sútil.

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The indefinite within Descartes' mathematical physics

The infinite appears at several levels, including theological and moral, of the Cartesian system. This article focuses on the *indefinite* in Descartes' mathematical physics as a notion standing for the size of the universe. Descartes, like his predecessor, Galileo, and successor, Isaac Newton, wrote a *mathematical physics*, and, therefore, this study of the infinite takes place within the framework that is generally referred to as *mathematical physics*.

How can we define the nature of a *mathematical physics*? There is some agreement that this type of physics uses mathematics to solve physical problems1. Galileo initiated a type of *mathematical physics* by defining the nature of matter by shape, numbers and movements:

> I think that tastes, odors, colors, and so on are no more than mere names so far as the object in which we place them is concerned, and that they reside only in consciousness...To excite in us tastes, odors and sounds, I believe, that nothing is required in external bodies except shapes, numbers and slow and rapid movements. (Galilei, 1957a, p. 274)

Descartes pushed further the mathematisation of nature, and established as one of the main principles of his physics that matter and space are identical to geometrical extension; Newton emphasized the mathematical aspect of physics by using equations to prove his laws.

In order to appreciate the meaning of the *indefinite* in Cartesian physics, we analyze the nature of the infinite first in his mathema-

[1] The Journal of Mathematical Physics, published by the American Institute of Physics, defines mathematical physics as: "[…] the application of mathematics to problems in physics and the development of mathematical methods suitable for such applications and for the formulation of physical theories [...]".

tics, and second in his *mathematical physics*. More specifically, we try to understand the neologism "indefinite" that Descartes created to describe the size of the universe. Unlike most Descartes' scholars and historians of the Infinite who take the position that Descartes only conceived the infinity of the world as being *potential*, I show that the Cartesian "*indefinite"* just as it is within his *mathematical physics* is not *potential* but *actual* and effective. My purpose is to develop Descartes' concept of the *indefinite* as an *actual dimension*, and show why Descartes, in contrast to Nicholas of Cusa and Galileo, could accommodate it in his physics.

The Cartesian mathematical conception of the Infinite

The first occurrence of the infinity in Descartes' works refers to mathematical infinity. In mathematics infinity is traditionally treated as a quantity and according to Descartes' mathematics quantities are analyzed as proportions. For Descartes the pure science of mathematics covers geometrical analysis and algebra, and consists solely of taking into account the various relations or proportions between quantities. It is in this context that Descartes approaches the problem of infinity in his mathematics; he writes the following:

> For I notice first that it was no more difficult to discover what twice 6 is than twice 3, and that whenever we find a ratio between any two magnitudes we can always find, just as easily, innumerable others which have the same ratio between them. (Descartes, 1999, p. 23)

We may consider that there is an infinite number of magnitudes whose ratio is a given number, but not for Descartes; that's why he used the term *innumerable* instead of infinite. Obviously, Descartes' lack of ability to understand and formulate a mathematical infinity derives from his mathematics of proportion and relation. If there is an infinite number, he reasons, we should be able to

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discover it within these relations. But, as Descartes explains, it is not possible to combine different infinities:

> With regard to infinity, you asked me a question in your letter of 14 March, which is the only thing I find in it which is not in the last letter. You said that if there were an infinite line it would have an infinite number of feet and of fathoms, and consequently that the infinite number of feet would be six times as great as the number of fathoms. I agree entirely. Then this latter number is not infinite. I deny the consequence. But one infinity cannot be greater than another. Why not? Where is the absurdity? Especially, if it is only greater by a finite ratio, as in this case, where multiplication by six is a finite ratio, which does not in any way affect the infinity. In any case, what basis we have for judging whether one infinity can be greater than another or not? It would no longer be infinity if we could grasp it. (Descartes, 2000b, pp. 28-30)

If an infinite line exists, it implies that it can consist of an infinite number of feet or fathoms. If we were to compare these different infinities, the infinity of the foot and the infinity of the fathom, we would conclude that the first is greater than the second, which is, by definition, not the case, because one infinity cannot be larger than another.

For Descartes, the nature of the infinite is to be incomparable, and therefore cannot be a number or a magnitude. In contrast to Descartes, Cantor would later assert it as a basis to demonstrate the existence of the infinite as a number comparable to other infinite numbers2. Furthermore, unlike Aristotle, Descartes does not recognize the existence of a *potential infinity* in mathematics. For Aristotle, an infinite number does not exist as such, only

[2] In the 1891 paper "Über eine Elementare Frage der Mannigfaltigkeitslehre", Cantor proved that the set of real numbers has larger cardinality (is "more numerous") than the set of natural (and rational) numbers; this showed, for the first time, that there exist infinite sets of different sizes (Cantor, 1915, pp. 103-110).

the process of counting exists, and infinity is just a potentiality. Aristotle writes:

> In a way the infinite by addition is the same as the infinite by division. In a finite magnitude, the infinite by addition comes about in a way inverse to that of the other. For in proportion as we see division going on, in the same proportion we see addition being made to what is already marked off. For if we take a determinate part of a finite magnitude and add another part determined by the same ratio (not taking in the same amount of the original whole), and so on, we shall not traverse the given magnitude. But if we increase the ratio of the part, so as always to take in the same amount, we shall traverse the magnitude, or every finite magnitude is exhausted by means of any determinate quantity however small. The infinite, then exists in no other way, but in this way it does exist, *potentially* and by reduction. (Aristotle, 2006, p. 38)

For Aristotle, mathematical infinity does exist as a process, as an act of adding or dividing which never ends, and therefore, it exists only as a *potential* and not as an *actual* or real, because it is never completed.

Descartes denies any potential or actual existence of an infinite number and does not consider the infinite as such in his mathematics. We can even define Cartesian mathematics as mathematics *of the Finitude*. In this regard, Descartes offers the infinite an even smaller role in his mathematics than does Aristotle (Monnoyeur, 1992).

How does Descartes reconcile his *mathematics of finitude* with the "*indefinite"* of his mathematical physics? Jacob Klein claims that Descartes' *mathesis universalis* comes to identify itself with the physical world through the imagination. He writes:

> How can the *mathesis universalis* turn, with its aid, into the true physics as well? In respect to the intellect, the imagination is defined by its "service" function, which insures the possibility of symbolic knowledge in general and, in particular, of the *mathe-*

> *sis universalis* as a general theory of proportions and equations. (Klein, 1992, p. 208)

Through the imagination, the *mathesis universalis* defined as a general theory of proportions and equations relates to the physical world in terms of proportions and equations. It is because for Descartes, the nature of the *indefinite* cannot be of a magnitude or a number; it does not have a place in his mathematics. Jacob Klein takes for granted that Descartes' Mathesis matches his physics. Following our previous demonstration, the Cartesian mathematics of proportions and equations does not acknowledge any type of infinite. Therefore, if, as Jacob Klein postulates, there is a real match between mathematics and physics, there cannot be a kind of infinite in physics. How then can we understand the status of the "indefinite" in Descartes mathematical physics?

Descartes' physical conception of infinity

Does the absence of an expression of the infinite in Descartes' mathematics mean that an *actual indefinite* in his *mathematical physics* is impossible? The answer is no.

a) Meaning of the Cartesian *indefinite*

In an article of the *Principles of Philosophy* Descartes includes the statement that the extension of the world is *indefinite*:

> We further discover that this world or the whole (*universitas)* of corporeal substance, is extended without limit, for wherever we fix a limit, we still not only imagine beyond it spaces *indefinitely extended*, but perceive these to be truly imaginable, in other words, to be in reality such as we imagine them; so that they contain in them *corporeal substance indefinitely extended*, for, as has been already shown at length, the idea of extension which we conceive in any space whatever is plainly identical with the idea of corporeal substance. (Descartes, 2000c)

The meaning of this *indefinite extension* of space and matter has generally been interpreted as referring to a *potential infinity*. In fact, most of the commentators consider that Descartes, when creating the expression *indefinite* for the world, refuses to recognize effective existence of an infinite extension of the world (McGuire, 1983, pp. 69-112). For instance, in his famous book, *From the Closed World to the Infinite Universe* Alexander Koyré states:

> The Cartesian distinction between the infinite and the indefinite thus seems to correspond to the traditional one between actual and potential infinity, and Descartes' world, therefore, seems to be only potentially infinite. (Koyre, 1957, p. 79)

I argue that Descartes has an actual and positive concept of *indefinite* in his physics, and that he develops this conception in response to a view that the world is a *potential* and *negative infinite,* the one namely developed by Aristotle and Nicholas of Cusa. At this stage, it is necessary to refute first that in the Cartesian Physics the *indefinite* of the world means *potential*, and second that it is a kind of *negative infinity* in comparison with God Infinity.

What do the traditional *potential/actual infinity* and *positive/negative infinity* of the world mean? We explained previously how the distinction *potential/actual* was developed by Aristotle trying to count to the infinite, which was *potential* or unreal. The Aristotelian notion of *potential infinity* was reinterpreted by Nicholas of Cusa and renamed *negative infinity* in the case of the world. Nicholas of Cusa compared the real world with the divine infinity, and, consequently, called it *negative infinity*:

> From the point of view of God's infinite or limitless power, the universe could be greater; but from the point of view of possibility or matter, which is incapable of actual, infinite extension, it cannot be greater. Since, then, nothing that would be a limit to the universe, by being greater than it, is able actually to exist, we may call the universe limitless and so privately infinite. (Cusa, 1954, p. 71)

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In this context, the *negative infinity* of the world solely exists because it is considered being in connection to the Infinity of God. As God is infinite, His creation incorporates something of this infinity, but is by definition less perfect than God's infinity. In a letter to Chanut, Descartes defines the *indefinite* as follow:

> In the first place, I remember that Cardinal Cusa and several other scholars have assumed the world to be infinite, without ever having been censured by the Church […].Thus, having no argument to prove, and not even being able to conceive, that the world has limits, I call it *indefinite*. But for all that I cannot deny that it may perhaps have some limits known by God, although they are incomprehensible to me: this is why I do not say absolutely that it is infinite. (Descartes, 2000d, p. 277-288)

We understand that any limitation to the infinity of the world can come only from us as we are unable to assign limits to the physical world. This is the reason why Jean-Baptiste Jeangène Vilmer emphasizes that the *indefinite* needs to be understood literally, which means as neither finite nor infinite (Vilmer, 2008).

b) *Indefinite* within the Cartesian physics

Does the Cartesian expression of the *indefinite* in the *Principles of Philosophy* have some connection with the Aristotelian and Cusian potential/actual and positive/negative infinity?

In the *Principles of Philosophy Part II* the *indefinite* appears independently of the divine infinite and refers only to the *indefinite* extension of material substance. Nevertheless, the *indefinite* in the physics of *Part II* is developed within a metaphysical framework introducing the principles of the Cartesian mechanism such as: matter space, movement, extension, subtle matter, inexistence of vacuum, and indefinite. The expression of *research programme* tailored by Karl Popper matches well Descartes' physics in the Principles of Philosophy. Popper writes:

> In using this term I wish to draw attention to the fact that in almost every phase of the development of science we are under the sway of metaphysical, that is, untestable-ideas. (Popper, 2000, p. 161)

Several commentators have emphasized that Descartes does not have a real *mathematical physics,* but instead, a metaphysical physics. Their position is that Cartesian physics is fundamentally metaphysical because the *Principles of Descartes' physics* are not proven by equations (see, for instance Garber [1992] and Gaukroger [1980, p. 97]). They point out that the laws and rules governing the motion of natural bodies are merely metaphysical principles and are not principles derived from geometry or algebra, as Descartes himself aspired to. I would like to add that the metaphysical character of Cartesian physics is largely derived from the content and presentation of its principles. The Cartesian principles, such as extension of matter and space, inexistence of vacuum, indefinite and movement from cause to effect, are interconnected and organized into a logical framework. For example, the inexistence of vacuum and indefinite extension follows from the reduction of matter and space to extension. In this context, the *indefinite* appears to be part of the identification of matter with space and becomes a principle of Descartes' physics as applied to the world. Descartes writes in his Principles, Part II 21:

> We further discover that this world or the whole (*universitas)* of corporeal substance, is extended without limit, for wherever we fix a limit, we still not only imagine beyond it spaces *indefinitely extended*, but perceive these to be truly imaginable, in other words, to be in reality such as we imagine them. (Descartes, 2000c)

How does the *indefinite* become part of the Cartesian physics of the *Principles*, Part II? The function of the *Principles of philosophy Part II*, as Descartes recalls above is to identify corporeal extension

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with the real world. Notions such as material and spatial extension, subtle matter and indefinite are not only models, but they are also shown to correspond to reality. Descartes had already prepared in his Meditations the expression of reality in physical terms by reducing the secondary qualities to geometrical extension. As a follower of Copernicus and Galileo, he promotes a new physics based upon the reduction of space and matter to a geometrical extension. Galileo paved the way for Descartes by discovering that qualities of corporeal substance or matter are to be bounded and shaped, and he refused to identify bodies with their colors, taste, sound, or fragrance. In his *Second Meditation*, Descartes demonstrates that in order to define the qualities of corporeal substance or matter, we should only consider their extension in space:

> The truth of the matter perhaps, as I now suspect, is that this wax was neither that sweetness of honey, nor that pleasant odor of flowers, nor that whiteness, nor that shape, nor that sound, but only a body which a little while ago appeared to my senses under these forms and which now makes itself felt under others…Let us considerate attentively and, rejecting everything that does not belong to the wax, see what remains. Certainly nothing is left but something extended, flexible, and movable. (Descartes, 1901b, p. 230)

If bodies are extended, they occupy a certain place, which means a certain portion of space. How then should we define the space between these different bodies? Descartes believes in the existence of an extended body between the bodies:

> We have almost all fallen into this error from the earliest age, for, observing that there is no necessary connection between a vessel and the body it contains, we thought that God at least could take from a vessel the body which occupied it, without it being necessary that any other should be put in the place of the one removed. But that we may be able now to correct this false opinion, it is necessary to remark that there is in truth no

> connection between the vessel and the particular body which it contains, but that there is an absolutely necessary connection between the concave figure of the vessel and the extension considered generally which must be comprised in this cavity; so that it is not more contradictory to conceive a mountain without a valley than such a cavity without the extension it contains, or this extension apart from an extended substance, for, as we have often said, of nothing there can be no extension. And accordingly, if it be asked what would happen were God to remove from a vessel all the body contained in it, without permitting another body to occupy its place, the answer must be that the sides of the vessel would thus come into proximity with each other. For two bodies must touch each other when there is nothing between them, and it is manifestly contradictory for two bodies to be apart, in other words, that there should be a distance between them, and this distance yet be nothing; for all distance is a mode of extension, and cannot therefore exist without an extended substance. (Descartes, 1901a, p. 342)

According to Descartes there is nothing that exists as an empty space. The interstices between bodies are filled with invisible corpuscles called *subtle matter*:

> As for me, since I never suppose a vacuum anywhere, but on the contrary have explicitly said that all the spaces from the sun to our bodies are full of a body that is indeed very fluid, but for that very reason more continuous (and which I have called subtle matter) […]. (Descartes, 2000a, p. 85)

When Descartes tells us about the universe or the world, he talks about an extension of matter made from the extension of the bodies added to the extension of subtle matter. This whole extension is infinitely divisible. Galileo had proved by means of various astronomical observations that the matter of the heavens is the same as the matter of the earth:

> For if it is true, as ancient philosophers believed, that there is a single kind of matter in all bodies, and those bodies are heavier which enclose more particles of that matter in a narrower space. (Galilei, 1960, pp. 14-15)

Descartes also considers that subtle matter is of the same matter as the matter of bodies. Consequently, he posits that the whole extension of the world, the division of the parts of the matter and the number of the stars, is *indefinite*. In fact, Descartes establishes the reality and limitlessness of the extension of the cosmos and, by extension, the actual nature of his indefinite world. This indefinite has a physical dimension, even if it is not measurable.

Insisting on the *potential* nature of the *indefinite*, Koyré did not understand that it applies to a real thing, the physical world. Nicholas of Cusa emphasized the negative nature of the infinity of the world because he compared it to the infinity of God. We have shown how in *Part II* the physical *indefinite* is not only a mode but a principle and has no relation to the Infinite God. In fact, calling the world *indefinite*, Descartes, in contrast to Galileo, makes a decision about the size of the world. Galileo, who also had been developing a *mathematical physics* a few decades before Descartes, refused to decide whether the world was finite or infinite:

> Neither you nor any one else has ever proved that the world is finite and figurate or else infinite and indeterminate. Many and subtle reasons are given for each of these views but none of them, to my mind, leads to a necessary conclusion, so that I remain in doubt about which of the two answers is the true one. There is only one particular argument of mine that inclines me more to the infinite and interminate than to the terminate […]: I feel that my incapacity to comprehend might more properly be referred to incomprehensible infinity, rather than to finiteness, in which no principle of incomprehensibility is required. But this is one of those questions happily inexplicable to human reason, and similar perchance to predestination, free-will and such others in which only Holy Writ and divine revelation can give an answer to our reverent remarks. (Galilei, 1957b, p. 73)

Condemned in 1633 by the Inquisition for his defense and teaching of the Copernicus' system, Galileo was willing to be prudent. In any case, his indecision concerning the infinity of the universe had no influence on his previous scientific conclusions. Galileo seems to be able to treat the problem of the infinite as a matter of opinion or religion that should not be a topic for scientists. In opposition to Cusa and Galileo, Descartes developed a physical conception of the infinity as *indefinite*.

Descartes does not encounter any difficulty making a decision about the extension of the world. For Descartes, the universe or the world is understood as something composed of homogeneous matter: The earth, planets, constellations and subtle matter, but it is not something mathematically measurable. In fact, Descartes conceives space and matter as extensions of different kinds: Spatial extension is mathematical and material extension is physical. In this case we see that the equation of material to spatial extension does not work completely, and that material extension is primarily anchored in the physical world. This discrepancy between material and spatial extension explains the non measurability of the world from a mathematical point of view, but its *indefinity* as a reality in the physical world. The *indefinite* world is a conquest of the metaphysical Cartesian physics of matter and subtle matter. This incursion of physics into a realm traditionally occupied by theology symbolizes the Cartesian revolution of the indefinite in the realm of physics.

In conclusion, when Descartes uses the term *indefinite* to qualify the extension of the world, he is not refusing, like Galileo, to decide about the extension of the world. When he chooses to qualify the world as *indefinite*, he recognizes its physical homogeneity but, at the same time, he acknowledges that it is not measurable. Cartesian physics needs either to involve the infinity of God or to define a mathematical conception of the infinite. The impossibility for the physical world to be entirely measurable is solid reason for Descartes to consider it to be *actually indefinite,* because it is

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applied to the infinite matter of the cosmos. We hope to have shed some light about why Descartes could afford to assert the *indefinite* of the world within the context of his physics and why Galileo could not.

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Sun Tzu dijo:

"El empuje del agua que zarandea las rocas.
Eso es shih.
El golpe del halcón que mata a la serpiente.
Eso es nudo.
El shih es como tensar el arco.
El nudo es como disparar la flecha".