1. Introduction

In numerous countries the main goal of monetary policy is to maintain price stability. To do so, the central bank (CB) follows a policy rule enjoying a substantial degree of independence. Suitably designed, monetary policy rules may deliver price stability as well as maintain output close to its potential. The ongoing worldwide financial crisis has made clear that, beyond price stability, financial stability (comprising the provision of CB liquidity and the use of prudential rules) is and remains an essential objective. In recent years, there has been a sizeable increase in the provision of lender of last resort (LOLR) services to individual commercial banks, whereby CBs stand ready to inject high-powered money into the banking system.
whenever a bank is solvent but suffers from temporary liquidity problems.\footnote{Even prior to the latest crisis, there is empirical evidence of LOLR transactions from a wide variety of experiences (Bordo, 1986; Dowd, 1999; Eichengreen and Portes, 1987; Humphrey and Keleher, 1984).} LOLR services to individual commercial banks have been a common practice, although in theory failures of banks could be prevented by implementing appropriate systems of bank regulation and supervision or private safety nets. These instruments are thus deemed insufficient to prevent CBs from intervening in the banking sector.

Despite the relevance of financial stability considerations, the economics profession does not offer a workhorse model for how macroprudential actions interact with the more traditional inflation-fighting role of monetary policy. It has been emphasised that, in the present context, multiple objectives require multiple instruments (Blanchard et al., 2012). But a better understanding is needed of issues such as what instruments should monetary and other authorities use to achieve these macroprudential goals, how large are the relevant trade-offs between macroeconomic performance and financial stability, and how economic uncertainty affects the conduct of CB policies.\footnote{There have been changes in the institutional arrangements for handling financial crises. While CBs tackled such crises in most countries largely on their own, crises from a wide variety of experiences (Bordo, 1986; Dowd, 1999; Eichengreen and Portes, 1987; Humphrey and Keleher, 1984).} It has been argued that the CB should provide liquidity to the market and should not lend to individual banks, which would be able to borrow in the interbank market if they are considered to be solvent (Goodfriend and King, 1988). This view, however, assumes that interbank markets work perfectly and that the market is as well or better informed than the CB about the relative solvency of a bank short of liquidity. Moreover, LOLR transactions could obey to a macro rather than a micro motivation. Four valuable formal approaches have deviated from such view and contributed to understanding why CBs provide LOLR services:

- First, some studies shed light on the question of why commercial banks might be reluctant to make use of LOLR services in connection with a coordination failure (Rochet and Vives, 2004). A coordination problem may arise when there is any large-scale need to redirect reserves, but there is no incentive for any individual commercial counterparty to sort out the problem by assuming the credit risk and undertaking the transaction costs involved. This occurred, for example, when the operation of many markets were severely disrupted as the Bank of New York computer malfunctioned in 1985 and in the events of September 11 (2001), when the Federal Reserve System hugely expanded its discount window lending to many individual banks (McAndrews and Potter, 2002).

- Second, some authors focus on the micro-aspects of CB intervention in dealing with market failure. Using the framework of Diamond and Dybvig (1983), Freixas et al. (2004) analyse the moral hazard problem caused by bank managers' incentive to choose an inefficient technology that gives them some private benefit.\footnote{There have been changes in the institutional arrangements for handling financial crises. While CBs tackled such crises in most countries largely on their own, crises arise are becoming increasingly managed by a committee of public sector agencies. Thus far, the literature has not addressed the challenge of formally modelling such a development.} In this asymmetric information context, liquidity shocks affecting banks might be undistinguishable from solvency shocks.\footnote{Moral hazard is a standard feature in models of bank behaviour. Moral hazard arises in the presence of informational frictions and ‘agency problems’ between bank managers and owners (Gorton and Rosen, 1995). Better capitalised banks have less moral hazard incentives (Jeitschko and Jeung, 2005) and are more prone to adopt careful practices to reduce costs. Regulators can force banks to increase the amount of capital commensurably with the amount of risk taken (Gropp and Heider, 2010).} Bianchi et al. (2012) model the interaction between credit frictions, financial innovation, and learning.\footnote{For an alternative approach, see Cordella and Levy Yeyati (2003).} In the decentralised equilibrium each household fails to internalise the effect of its borrowing decisions on asset prices, leading to excessive debt accumulation and too frequent crises. When the CB has better information than banks about the economic outcome, macroprudential policy would be justified since it can help offset the pecuniary externality generated by the collateral constraint.

- Third, Goodhart and Huang (2005) assess the role of both contagious risks and moral hazard at the macro-level. If an illiquid, but solvent, bank is forced into closure, it is more likely that this will have significant adverse implications for the financial system as a whole the bigger that bank is. Thus, Goodhart and Huang's (2005) model in a static setting suggests that the CB would only rescue banks that are “too big to fail”.\footnote{In the presence of learning distinguishes Bianchi et al. (2012) from studies assuming that agents form rational expectations with full information, such as Benigno et al. (2013), Bianchi (2011), Bianchi and Mendoza (2010), Jeanne and Korinek (2010), Korinek (2013), Lorenzoni (2008), and Stein (2012). Concerning the role of credit frictions and imperfect information, Bianchi et al. (2012) relate to the financial accelerator models of Aiyagari and Gertler (1999), Bernanke et al. (1999), and Kiyotaki and Moore (1997).} Authors find that this result is broadly robust to a dynamic extension, in which setting contagious considerations dominate the role of moral hazard.

- Fourth, Berlemann and Zeidler (2009) propose a model where the primary motive for providing LOLR transactions is macro rather than micro. The authors argue that, following the closure of commercial banks, fractional reserve banking systems are prone to liquidity crises whenever the public changes its preferences towards holding more high-powered money. In such a setting, LOLR transactions can contribute to lowering uncertainty about the money multiplier, and thus dampening variability in both inflation and output. Given the costs incurred in unsuccessful LOLR transactions, CBs have an incentive to save only the large banks, while the small banks are closed. A benevolent CB will thus accept greater macroeconomic variability when facing such LOLR costs.

The present paper examines the role of LOLR provision for macroeconomic stabilisation. LOLR provision, by adjusting banks' access to liquidity, can be combined with standard monetary policy to rebalance macroeconomic and financial stability. The main object is to investigate how the trade-off between financial instability and macroeconomic variability (as created —say— by financial shocks) can be improved by dampening fluctuations in inflation and output via adjustments in monetary institutions and the economic structure. Institutions (as given by the monetary policy setup, wage bargaining and the non-cooperative game involved between the CB and wage setters) affect the ability of policymakers to successfully undertake LOLR actions,\footnote{As large banks have a higher chance of being saved, this may trigger bank mergers, implying lower funding cost (Hunter and Wall, 1989; Boyd and Graham, 1991; Benson et al., 1995; Penas and Unal, 2004; DeYoung et al., 2009, and Rose and Wieladek, 2012). Another rationale for large bank size is inadequate corporate governance enabling bank managers to pursue high-growth strategies at the expense of shareholders. In the latest crisis, the too-big-to-fail argument may have been mitigated by the severe deterioration in the public finances, which reduces countries' ability to guarantee bank liabilities and makes large banks subject to greater market discipline (Demirgüç-Kunt and Huizinga, 2013a and 2013b; see also Acharya et al., 2013).} as does the economic structure (as captured by the link between trade openness and the responsiveness of aggregate supply).

We start from a setup where the CB provides LOLR services to banks on top of its standard stabilisation policy, in the context of an endogenous determination of output distortions (operating via the labour market). Concerning the representation of the banking sector, we follow Goodhart and Huang (2005), who—in the presence of bank closures— assume that the public may move out of failing
bonds’ deposits into cash, thereby reducing the money multiplier. Heightened financial strain raises both financial instability (directly) and macroeconomic variability (via bank closures and their implications for fluctuations in the money multiplier). LQR transactions are aimed at mitigating part of the rise in overall instability. Moreover, we introduce imperfect monetary policy transparency and an explicit consideration of wage setters’ preferences. This draws on Spyromitros and Zimmer (2009) and Sánchez (2011 and 2013), who study games between the CB and trade unions where the CB may be ambiguous about the policy preferences. This draws on Spyromitros and Zimmer (2009) and policy transparency and an explicit consideration of wage setters’ overall instability. Moreover, we introduce imperfect monetary transactions are aimed at mitigating part of the rise in financial crises of Mexico (1995), Asia (1997-1998), Russia (1998), and Argentina (2000-2001), we are able to capture the partial aspect of financial disintermediation triggered by the banking panic.

postulate a relationship between the responsiveness of aggregate supply and trade openness.

2.1. Macroeconomic Block

Aggregate demand and aggregate supply are assumed to be given, respectively, by

\[
\pi = m - y 
\]

\[
y = -\alpha(w - \pi) 
\]

where \( \pi \) is inflation, \( y \) is the aggregate output (in logs), \( m \) is the money supply (in logs) and \( w \) is the wage level (in logs). In (2), parameter \( \alpha \) is a positive constant that is likely to reflect structural characteristics of the economy. In particular, we assume that \( \alpha \) is inversely related to trade openness. That is, trade openness makes \( \alpha \) smaller and thus the supply schedule steeper. The reason is that a monetary expansion increases output at home relative to output abroad and thus, with imperfect substitutability between domestic and foreign goods, reduces the relative price of domestic goods (i.e., produced a real exchange rate depreciation). Furthermore, for a given real depreciation associated with output expansion, the inflationary effect is larger the more open the economy is (see, e.g., Romer, 1993).

In equilibrium inflation and output can then be computed as

\[
\pi = (1 - \beta) m + \beta w
\]

\[
y = \beta(m - w)
\]

where \( \beta = \alpha / (1 + \alpha) \in (0,1) \). The larger \( \beta \), the flatter the aggregate supply curve; \( \beta \) is thus an inverse measure of the aggregate supply slope. Parameter \( \beta \) is positively related to \( \alpha \), and thus —given the hypothesised link between \( \alpha \) and the degree of trade openness— also decreasing in the latter.

We assume that the level of the money supply is determined as the product of base money and a money multiplier, so that in log terms we get

\[
m = b + \varpi
\]

where \( b \) denotes base money (in logs) and \( \varpi \) the money multiplier (in logs).

2.2. Commercial Banks

The banking sector is supposed to consist of \( N \) banks which own no capital of their own. They make investing decisions funding themselves with deposits \( D_i \) gathered from the non-banking private sector. Banks are assumed to maximise expected profits under risk neutrality. Every single bank \( i \) chooses an optimal portfolio of two investment projects: a safe one with a return of 1 with certainty and a risky one described by a production function with decreasing expected marginal returns:

\[
E[Q(C_i)] = [b_i + E(\xi_i)] C_i - b_i C_i^2
\]

8. In a context of “liquidity trap”, the role of base money (which underlies the concept of money multiplier) can be rationalised by the standard argument that money is a better indicator of monetary policy stance than the interest rate. In more normal times, there should be an inverse relation between interest rates and money supply.


10. Sánchez (2008) uses this feature of the Phillips curve within a simple model. This feature conveys an open economy flavour, even if ours is a closed economy model. To some extent a similar interpretation can be applied to the assumption that, following a bank closure, the public suddenly changes its preferences towards holding domestic currency. Although we abstract from the possibility that depositors switch out of the deposits of banks perceived as risky into foreign exchange, as has occurred in the financial crises of Mexico (1995), Asia (1997-1998), Russia (1998), and Argentina (2000-2001), we are able to capture the partial aspect of financial disintermediation triggered by the banking panic.

11. Since this is a static model we use inflation \( \pi \) interchangeably with the price level \( m \) (in logs), resorting to the normalisation that the previous period’s price level is zero.

12. To derive (3) and (4), first plug (2) into (1) to get \( \pi = (m + \alpha w)/(1 + \alpha) \), which —using the definition of \( \beta \) — yields (3). Eq. (4) can be derived by inserting (3) into (2), which gives \( y = \alpha (1 - \beta) m - w \); given that the definition of \( \beta \) implies that \( \alpha (1 - \beta) \beta = \beta / 2 \), then \( y \) can be expressed as in (4).

13. This simplified formulation could be derived by assuming decreasing marginal returns for each loan to non-banking firms, in conjunction with an exogenous allocation of loans across banks. A more general model would allow for credit demand considerations, endogenising the allocation of loans across banks (see Schnabl, 2012).
with \( b_i \) and \( b_i \) are positive constants, \( C_i \) the capital invested in the risky project, \( Q_i \) the marginal returns on \( C_i \), and \( \lambda_i \) is a bank-specific random variable. As in Berlemann and Zeidler (2009), we assume for simplicity that these investments are made outside the country under consideration, which allows us to neglect their influence on domestic output. In order to ensure that at least some investment in the risky project is profitable, we also assume \( b_i + E(\lambda_i) > r_i \). Provided \( D_i \) is sufficiently large to make the optimal investment in the risky project possible, the optimal investment in the risky project \( C_i^* \) is given by

\[
C_i^* = \frac{b_i + E(\lambda_i) - r_i}{2b_i} \tag{7}
\]

Banks are allowed to differ in the number of depositors they attract, and thus in their sizes.\(^{14}\) Otherwise, the banking sector is homogeneous, i.e., we assume \( E(\lambda_i) \) and \( \text{Var}(\lambda_i) \) to be identical for all banks \( i \). Therefore, from (7), \( C_i^* \) is uniform, so in what follows we set \( C_i^* = C^* \). We shall later briefly refer to the case of moral hazard, which opens the possibility that some banks’ behaviour is influenced by the LOLR option and may deviate from a uniform volume of risky investment.

The initial investment decision is supposed to be irreversible. During the period, project banks are assumed to receive a signal \( X_i \) on the status of the risky project.\(^{15}\) The signal can be of two types: a high-profitability signal \( X_i = X \), or a low-profitability signal \( X_i = \bar{X} \). When the former signal is received, we assume that the project is always finished with high success and thus earns enough profits to pay back all deposits at the end of the period. A healthy bank like this survives without ever encountering liquidity problems.\(^{16}\) When a bank receives a signal \( X_i = \bar{X} \) of low success, there are two possibilities. If this downside risk materialises, the bank is insolvent and thus has to be closed. We assume that in this case (and when the bank refrains from investing additional capital in the project) the remaining funds of the bank are evenly distributed among its depositors base. Alternatively, the project may end with high success, which requires that the bank invests some additional capital in the project. We shall assume that the necessary additional capital (to be provided by LOLR services) is fraction of the capital already invested in the project.

### 2.3. Depositors

The non-banking private sector consists of a number of \( H \) depositors for a given bank \( i \), who divide their liquid assets between cash and bank deposits. We do not differentiate between depositors, who are assumed to each own the same volume of bank deposits.\(^{17}\) The desired cash-to-deposits ratio is supposed to remain constant if no bank goes bankrupt. When there are bank closures, we assume that the public may move out of failing banks’ deposits into cash, no bank goes bankrupt. When there are bank closures, we assume that the public may move out of failing banks’ deposits into cash, \( n = 0 \) if bank \( i \) is closed and with \( l_i \) is a dummy variable with the value \( l_i = 0 \) if bank \( i \) is closed and with \( l_i = 1 \) if bank \( i \) survives.

Among the set of \( n \) illiquid banks, we shall distinguish between the \( n_{\text{LOLR}} \) \( n_{\text{LOLR}} \) banks that are provided assistance and the remaining \( n - n_{\text{LOLR}} \) \( n - n_{\text{LOLR}} \) ones that are denied access to LOLR resources.\(^{18}\)

Eq. (8) follows Berlemann and Zeidler (2009) in abstracting, for simplicity, from contagion effects that would induce even depositors of initially healthy banks to raise their cash-to-deposit ratio. The \( e_i \)'s are assumed to be uncorrelated across banks and depositors. Finally, \( e_i \) is connected to bank size in two ways, namely, with both its expected value and its variance being increasing in the number of depositors, i.e., \( \text{d}E[e_i]/\text{d}H_i > 0 \) and \( \text{d}\text{Var}(e_i)/\text{d}H_i > 0 \).

### 2.4. Central Bank

The CB is assumed to dislike deviations in both inflation and output from their targeted levels. The inflation target is set to \( \pi \) and the output target equals the socially optimal level of zero. In order to achieve its goals, the CB has at its disposal two instruments, namely, by controlling base money \( B \) and by having enough LOLR resources that may be provided to illiquid commercial banks. The CB loss function can be expressed as the sum of macroeconomic performance terms (capturing fluctuations in inflation and output) and a component measuring the cost of saving banks that are ultimately unsuccessful. More concretely, the CB loss function can be expressed as

\[
L = \left( \frac{\pi - \pi^*}{2} \right)^2 + Z \pi^2 + n_{\text{LOLR}} F_h C^* \tag{9}
\]

where \( Z \) is the policy weight on output variability, and \( F \) is a scaling parameter for LOLR-related costs.\(^{20}\) A CB with a high value of \( Z \) is “activist” as it places a relatively high weight on reducing output volatility. Eq. (9) corresponds to the benchmark double-quadratic case. In the last term of (9), the LOLR-related component is proportional to the amount of liquidity provided and the odds—captured by \( h \in (0,1) \)—that a bank will end up unsuccessful in its risky investment. As the funds used for LOLR transactions cannot be used for alternative purposes, unsuccessful LOLR transactions trigger opportunity costs. We abstract from the opportunity costs of successful LOLR transactions, which are normally of a short term nature, as well as from reputation costs facing the CB in the event of ultimately closing rescued banks (because there is no systematic error involved).

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14. Since it is not the aim of this paper to explain the structure of the banking sector we stick to the assumption that bank sizes are given exogenously.

15. The signal \( X_i \) is directly related to the actual realisation of the bank-specific variable \( \lambda_i \), which is the only random element weighing on the return of the risky investment, \( C_i \).

16. The CB has the information needed to judge whether there is a risk that a commercial bank’s profits be insufficient to meet all its liabilities, in which case the bank may be supported with LOLR credit. Healthy banks are never supported by LOLR transactions, given that these are assumed to be only aimed at preventing uncertainty on the money multiplier.

17. This can be justified by supposing that depositors are identical in their financial wealth and preferences towards cash and deposits.

18. As we saw earlier, when a bank is closed the remaining funds of the bank are evenly distributed among its depositors base. For the remainder of the money deposited, depositors are also assumed to get it back from insolvent banks. As with Berlemann and Zeidler (2009), we abstract from other details and consequences concerning the deposit insurance scheme involved.

19. We shall later show how the number of banks receiving help, \( n_{\text{LOLR}} \), is determined. For \( n_{\text{LOLR}} \) to make sense, the number and size of the banks in trouble that are supported must keep some proportion with overall economic activity and the amount of CB financial resources available (both for assisting banks and for insuring deposits in the event of bankrupcies). Another exogenous parameter, \( n \), can only make sense under similar conditions.

20. As mentioned above, the additional capital provided by LOLR assistance is a fraction of the capital already invested in the project, \( C_i^* \). The policy weight is costly LOLR transactions (relative to macroeconomic instability).
2.5. Unions

Wages are assumed to be bargained collectively by unions. We split the economy into \( M \) symmetric sectors (indexed by \( k=1, \ldots, M \)). Workers of each sector are organised in a single union, so there are \( M \) unions in the economy (also indexed by \( k \)). The outcome of the wage bargaining is \( w_k \), the nominal wage (in logs). Sectoral output, \( y_k \), is given by

\[
y_k = -\alpha(w_k - \pi) - \eta(w_k - w)
\]

where \( \eta > 0 \). In (10), we follow Cukierman and Lippi (2001), who show this specification implies that (the absolute value of) the real wage elasticity is increasing in the number of unions, \( M \). Therefore, deviations of \( w_k \) from the economy-wide wage rate induce a loss in the sector's real activity that is proportional to \( \eta \). Aggregation over \( k \) in (10) gives (2).

We model the wage bargaining process explicitly. Instead of assuming that unions base their wage decisions solely on their inflation expectations (Berlemann and Zeidler, 2009), we set up an incomplete-information game between the CB and wage setters. Within the class of games where the monetary response to wages is assumed to be uncertain to unions, we deviate from the study of a monopoly union case (Grüner, 2002), allowing for many unions as in Spyromitros and Zimmer (2009) and Sánchez (2011 and 2013).

The unions’ views are formed conditional on a signal about the degree of monetary accommodation to wages. We label the expectations operator associated with the corresponding information set as \( E^{\omega} \). Each union \( k \) sets the nominal wage to minimise expected loss.

\[
E^{\omega} \Omega_k = -2E^{\omega}(w_k - \pi) + A E^{\omega} \lambda^2_k
\]

where \( A \) measures the relative concern for the stability in real activity.

As we have mentioned, the CB is allowed to be not fully transparent. In consequence, trade unions can only partly anticipate the monetary response to their wage claims. They only know two features about the degree of monetary accommodation \( \gamma \), namely, the mean \( E^{\omega}(\gamma) = \bar{\gamma} \) and the variance \( \sigma^2_{\gamma} = E^{\omega}[(\gamma - \bar{\gamma})^2] \). A lower \( \bar{\gamma} \) is interpreted as a stricter monetary policy, while a higher \( \sigma^2_{\gamma} \) represents a stronger degree of monetary policy uncertainty. The uncertainty about parameter \( \gamma \) should be interpreted as arising from lack of transparency about CB preferences.

2.6. Timing

The sequence of actions comprises the following six stages (sketched in Figure 1):

1. Commercial banks collect deposits from the private sector and choose investment portfolios consisting of two projects, a risky one and a safe one.
2. All unions independently and simultaneously formulate their wage demands, conditional on a signal about the CB reaction to the economy-wide wage rate. For simplicity, it is assumed that unions know how the CB trades off overall macroeconomic stability for LOLR-related costs.
3. Commercial banks receive signals (of either high or low success) on the status of the risky project. For simplicity, it is assumed that there is no information asymmetry between banks and the CB. The CB thus also observes the signals on the projects states. Neither the commercial bank nor the CB knows at this stage whether the project will end successfully. Given that banks invest all deposits in the two projects, illiquid banks getting a low-profitability signal request LOLR assistance from the CB.
4. The CB decides which banks (if any) it assists through the discount window.
5. The CB determines the monetary base via open market operations.
6. Bank returns on the two investment projects materialise. Banks turn out to be solvent or insolvent. In the latter case they are closed, with the public partially moving out of deposits into cash. As a result, there is a reduction in the money multiplier and thus in the money supply. Inflation and output are determined.

3. Equilibrium Results

We solve for the equilibrium, which involves an incomplete-information game between the CB and wage setters. In stage 1, commercial banks choose a volume of risky investment \( C \). Conditional on the signal they receive in stage 2, unions decide their nominal wages, trying to anticipate the monetary policy reaction in stage 3. Moreover, in stage 4 the CB decides on the provision of LOLR services, based on information about expected bank returns. We resort to backward induction, starting out with the optimisation problem of the CB.

3.1. The CB optimisation problem

The CB problem can in turn be separated in two steps: i) the CB chooses the optimal monetary base, \( b \); and ii) the CB decides which illiquid banks it provides liquidity to.

In order to compute the optimal \( b \), let us insert Eqs. (3) and (4) for inflation and output, respectively, into the CB loss function (9), which yields:

\[
L = \left[ \frac{(1 - \beta) m + \beta w - \pi}{2} \right]^2 + Z\beta^2 (m - w)^2 + n^{\text{out}} FhC
\]

(12)
We can then derive the optimal money supply \( m^* \) by differentiating the expected loss function, \( EL \), with respect to \( m \) and solving for \( m^* \):

\[
m^* = \gamma w + (1 - \gamma) \bar{\pi}
\]

where \( \gamma = \beta'[Z\beta - (1 - \beta)]/(1 - \beta) + Z\beta^2 \) is the degree of accommodation of monetary policy. Eq. (13) is derived taking \( C^* \) and \( w \) as given.

In order to reach \( m^* \), the CB determines the monetary base. The optimal choice of \( b \) can be computed using Eq. (5) as

\[
b = m^* - E\sigma
\]

where we acknowledge that the money multiplier, \( \omega \), is uncertain. In (14), the CB forms an expectation on \( \omega \), which is \( \omega_i \), equals

\[
E\sigma = -\sum_{i=1}^{\infty} I(E_i \mid \varepsilon_i)
\]

We calculate the optimal monetary base by plugging Eqs. (13) and (15) into (14):

\[
b = \gamma w + (1 - \gamma) \bar{\pi} + \sum_{i=1}^{\infty} I(E_i \mid \varepsilon_i)
\]

Inserting Eqs. (8) and (16) into (5) gives the actual money supply:

\[
m = \gamma w + (1 - \gamma) \bar{\pi} + \sum_{i=1}^{\infty} I(E_i \mid \varepsilon_i - \varepsilon_i)
\]

Plugging (17) into Eqs. (3) and (4), we solve for actual inflation and output as

\[
\pi = \bar{\pi} = \left[ (1 - \beta) \gamma + \beta \right] (w - \bar{\pi}) + (1 - \beta) \sum_{i=1}^{\infty} I(E_i \mid \varepsilon_i - \varepsilon_i)
\]

\[
y = -\beta(1 - \gamma)(w - \bar{\pi}) + \sum_{i=1}^{\infty} I(E_i \mid \varepsilon_i - \varepsilon_i)
\]

Unexpected macroeconomic variability is here seen to be driven by bank closures, as captured by the deviations of \( \varepsilon_i \) from \( E_i \). The intensity of these deviations is reflected in the variance

\[
\sigma^2_{\varepsilon_i} = E\left[ \left( (E_i \mid \varepsilon_i - \varepsilon_i) \right)^2 \right]
\]

defined for each bank that is not only illiquid but also ultimately insolvent. The variance \( \sigma^2_{\varepsilon_i} \) is linked to bank size, increasing with the number of depositors, i.e.

\[d\sigma^2_{\varepsilon_i}/dh \geq 0\]

Having thus determined the monetary base in (16), let us turn to the other CB choice, namely, which illiquid banks (if any) to assist by means of LOLR transactions. The CB decides to provide liquidity assistance if the expected loss resulting from this action is lower than when abstaining from it. The simplest way to make this comparison is to consider the CB loss stemming from any given bank \( i \) applying for LOLR help, neglecting the role played by any other illiquid bank. LOLR transactions generate a trade-off between the opportunity cost of unsuccessful liquidity assistance, \( FhC^* \), and the gain from mitigating the effects of bank closures on macroeconomic volatility. The loss resulting from the closure of bank \( i \) receiving LOLR services can then be expressed as

\[
EL^{\text{LOLR}} = h^2 \left[ (1 - \beta)^2 + Z\beta^2 \right] \sigma^2_{\varepsilon_i} + \mu \left( w - \bar{\pi} \right)^2 + FhC^*
\]

where \( \mu = Z\beta^2 / \left( (1 - \beta)^2 + Z\beta^2 \right) \in (0, 1) \). When the CB decides not to provide LOLR to illiquid bank \( i \) (a scenario labelled as NLOLR), the loss function does not include the term \( FhC^* \). This potential saving comes at the expense of no attempt being made by the CB at dampening macroeconomic uncertainty (i.e. bank \( i \) is closed with certainty instead of with probability \( h \)). In this case, the CB loss function thus amounts to

\[
EL^{\text{NLOLR}} = \frac{1}{2} \left[ (1 - \beta)^2 + Z\beta^2 \right] \sigma^2_{\varepsilon_i} + \mu \left( w - \bar{\pi} \right)^2
\]

The CB provides LOLR assistance if and only if doing so leads to a total expected loss that is lower than in the case where no LOLR services are provided, that is, if and only if \( EL^{\text{LOLR}} < EL^{\text{NLOLR}} \).

\[
\sigma^2_{\varepsilon_i} > \frac{2FhC^*}{(1 - h^2) \left( (1 - \beta)^2 + Z\beta^2 \right)}
\]

The decision to provide LOLR assistance depends on the size of the bank \( i \) via \( \sigma^2_{\varepsilon_i} \), which—as we have seen—rises with bank size. Intuitively, a larger depositor base causes greater uncertainty in how depositors react to the failure of bank \( i \), thereby raising the chances of LOLR support. In contrast, small banks get no access to the discount window and are closed as soon as they face liquidity problems. This result is in line with the so-called too-big-to-fail doctrine.

Let \( \sigma^2_{\varepsilon_i} \) denote the variance which turns inequality (21) into an equality. This variance is associated with the critical bank size, i.e. the minimum size for which a bank is provided LOLR services. It is useful to sort the \( n \) illiquid banks by size such that \( \sigma^2_{\varepsilon_i} = \sigma^2_{\varepsilon_i} < \ldots < \sigma^2_{\varepsilon_i} \). Thus, we end up with two different types of banks within the group of banks in liquidity troubles: small banks with \( \sigma^2_{\varepsilon_i} \leq \sigma^2_{\varepsilon_i} \) and large banks with \( \sigma^2_{\varepsilon_i} > \sigma^2_{\varepsilon_i} \). Let \( n_{\text{crit}} \) and \( n - n_{\text{crit}} \) be the number of such small and large banks, respectively. A larger critical bank size is associated with a larger number of banks not being assisted by LOLR transactions (a larger \( n_{\text{crit}} \)). We can then rewrite the expressions for actual inflation and output in (18) and (19), respectively, as

\[
\pi = \bar{\pi} + [1 - (1 - \beta)\gamma + \beta] (w - \bar{\pi}) + (1 - \beta) \sum_{i=1}^{n_{\text{crit}}} I(E_i \mid \varepsilon_i - \varepsilon_i) + h \sum_{i=n_{\text{crit}+1}}^{n} I(E_i \mid \varepsilon_i - \varepsilon_i)
\]

\[
y = -\beta(1 - \gamma)(w - \bar{\pi}) + \beta \sum_{i=1}^{n_{\text{crit}}} I(E_i \mid \varepsilon_i - \varepsilon_i) + h \sum_{i=n_{\text{crit}+1}}^{n} I(E_i \mid \varepsilon_i - \varepsilon_i)
\]

In these two expressions, macroeconomic variability arises from unexpected fluctuations in the money multiplier. These are eliminated when there are no bank closures, in which case \( h = 0 \). (Technically, this would amount to having the critical bank size \( n_{\text{crit}} \) equal to zero.) For \( h \in (0, 1) \), macroeconomic fluctuations are dampened, with some banks avoiding bankruptcy thanks to having access to LOLR services. This CB action comes on top of standard monetary policy actions via the injection of base money.

Expected CB losses can be found to equal

\[
EL = \frac{1}{2} \left[ (1 - \beta)^2 + Z\beta^2 \right] \sum_{i=1}^{\infty} \sigma^2_{\varepsilon_i} + \frac{h^2}{2} \sum_{i=n_{\text{crit}+1}}^{n} \sigma^2_{\varepsilon_i} + \mu \left( w - \bar{\pi} \right)^2 + (n - n_{\text{crit}}) FhC^*
\]

23. Technically, the relevant expected CB losses can be computed by substituting the impact exerted by a given bank \( i \) on objective function (12) via Eqs. (18) and (19). Stochastic macroeconomic volatility is driven by the last term in both Eqs. (18) and (19).

24. See derivation in Appendix A.

25. See derivation in Appendix A.
In equilibrium, the critical bank size depends on the following factors:

**Proposition 1.** The critical bank size is larger (smaller) and thus the number of banks not being assisted by LOLR transactions, ncrit, is larger (smaller)

i) the larger (smaller) the volume of the risky investment, C;

ii) the larger (smaller) the probability that the risky investment fails, h;

iii) the flatter the aggregate supply curve when initially this curve is steep (flat) and the CB displays a rather strong preference for price (output) stability; or the steeper the aggregate supply curve when initially this curve is flat (steep) and the CB displays a rather strong preference for output (price) stability.

**Proof.** (i)-(ii) It is straightforward to see that the RHS of the inequality in (21) is increasing in both C and h. Concerning the LHS of (21), we have shown that it is increasing in bank size. Therefore, the critical bank size —obtained when (21) holds at equality— is increasing in both C and h. This establishes items (i) and (ii).

(iii) As we have just mentioned, the critical bank size is obtained when (21) holds at equality— is increasing in both C and h. This establishes items (i) and (ii).

Therefore, if initially the supply curve is steep (flat) and the CB displays a stronger preference for output (price) stability —a scenario that occurs when both β and Z are low (high)— then it is likely that 

\[\frac{d}{dβ}(1-β)^2 + Zβ^2\] 

is affected by β (which is an inverse measure of the aggregate supply slope) as follows:

\[\frac{d}{dβ}(1-β)^2 + Zβ^2\] = \[2Zβ - (1-β) > (0) ⇔ β(1+Z) > (1)\]

3.2. The Unions’ Problem

Let us turn to the wage setting problem. All unions independently and simultaneously set their nominal wage, trying to minimise (11) taking into account the expressions for inflation in (18) and sectoral output in (10), as well as the expected CB reaction from (17). Optimisation implies that, in the symmetric equilibrium, the aggregate nominal wage equals

\[w - \pi = \frac{(M-1)+β(1-γ)}{Aβ(1-γ)^{1.5} + (M-1)(1+ηM)(1-γ) + α(1-β)σγ^2}\]

which is positive when unions are non-atomistic (M∞,∞), but vanishes (i.e. wages approach their competitive level) as unions become atomistic (M→∞). Use of (23) leads to the following result:

**Proposition 2.** If unions are non-atomistic (M∞,∞), the aggregate nominal wage

i) decreases with monetary policy uncertainty, σγ;  

ii) decreases (increases) with the degree of monetary policy strictness—as given by a smaller γ—when CB policies are initially seen as rather strict (accommodating) and transparent (opaque);

iii) decreases (increases) with the number of unions, M, and thus increases (decreases) with the degree of wage bargaining centralisation, when CB policies are initially seen as rather strict (accommodating) and transparent (opaque).

As unions become atomistic (M→∞) all of these wage effects vanish.

**Proof.** (i) Differentiation of (23) with respect to σγ yields

\[dw = \frac{α(1-β)(w - \pi)}{[α(1-β)β(1-γ)^{1.5} + (M-1)(1+ηM)(1-γ) + α(1-β)σγ^2]}\]

If M∞, the nominal wage clearly falls as a result of larger monetary uncertainty. Intuitively, a larger value of σγ renders monetary policies more unpredictable, inducing unions to moderate their wage claims.

(ii) Differentiation of (23) with respect to γ gives

\[dγ = \frac{2α(1-β)^2 + (M-1)(1+ηM)(1-γ) - α(1-β)σγ}{[α(1-β)β(1-γ)^{1.5} + (M-1)(1+ηM)(1-γ) + α(1-β)σγ^2]}\]

If M∞, by inspecting the numerator, dw/dγ is more likely to be positive (negative) under two initial conditions: i) when γ is small (large) and thus \((1 - γ)^{1.5}\) large (small), and ii) when \(σγ^2\) is small (large). This establishes (ii), as it implies that \(dw/dγ(n < 0)\), i.e. w decreases (increases) with a smaller γ—which characterises a tighter policy rule— when the CB is initially known to be rather strict (accommodating) and transparent (opaque). Intuitively, when the CB is seen as non-accommodating and monetary uncertainty is low (i.e. for low values of γ and \(σγ^2\)), a stricter monetary regime elicits wage restraint on the part of unions. The opposite effect is induced by a fall in γ when monetary policies are known to be rather accommodating and opaque (i.e. for high values of γ and \(σγ^2\)). Under these initial conditions the deterrence exerted on unions is undermined, with tighter monetary policymaking ultimately raising \(w\).

26. An effect similar to those of C and h would arise if the distribution of the σγ’s shifted upwards, i.e. if households were to consider it riskier to keep deposits at their banks in the event the latter have to be closed.

27. Under asymmetric CB objective functions, the range of macroeconomic factors weighing on the critical bank size will be expanded to include monetary policy parameters.

28. See derivation in Appendix A.
competition among unions. This makes means that the denominator of (21) rises because of the increased As a result of this free riding, wages tend to increase, as given by the impact that individual wage claims exert on the macroeconomy. Here. First, more unions means that each of them internalises less of (opaque). This establishes (iii). Intuitively, there are two forces at play here. First, more unions means that each of them internalises less of the impact that individual wage claims exert on the macroeconomy. As a result of this free riding, wages tend to increase, as given by an increase in the numerator of (21). Second, more unions also means that the denominator of (21) rises because of the increased competition among unions. This makes w go down, as each union becomes more concerned consequences of individual wage demands for real activity. For instance, one gets that dw/dM < 0 when the denominator of (21) rises by more than the numerator, which occurs when the increased competition arising from a larger M outweights the free riding effect also involved. From (26), this turns out to be the case when monetary policies are perceived as non-accommodating and predictable (i.e. for low values of ᵃ and ᵃ₂). Finally, as unions become atomistic (M → ∞) the wage effects in Eqs. (24) through (26) go to zero. Atomic unions are not at all concerned about the consequences of their individual wage demands on the macroeconomy, irrespective of the values adopted by monetary policymaking parameters ᵃ and ᵃ₂, QED.

With regard to item (iii) in Proposition 2, the present model can be seen as encompassing two earlier studies, namely, those of Cukierman and Lippi (2001) and Grüner et al. (2009). Under full CB transparency (i.e. ² = 0), we reproduce Cukierman and Lippi’s (2001) result that a less centralised wage bargaining (higher M) reduce nominal wages owing to unions’ fear that high wages will lead to increased unemployment owing to greater competition among workers. Grüner et al. (2009) instead allow for imperfect CB transparency, while abstracting from labour competition (i.e. ² = 0). We reproduce the two possible cases considered by Grüner et al. (2009): i) under full CB transparency (i.e. ² = 0), decentralisation in wage bargaining has no effect on w;² and ii) under incomplete CB transparency (i.e. ² > 0), decentralisation in wage bargaining raises nominal wages as each (now smaller) union internalises to a lesser extent the adverse consequences that higher wage claims exert on sectoral output. Compared with these two earlier studies, our derivations present the advantage of a general formulation, also including the case when CB transparency is imperfect (i.e. ² > 0) and there is competition between different unions’ workers (i.e. ² > 0).

In addition, the result in item (iii) indicates that the effect of wage bargaining centralisation on wages depends on the nature of monetary institutions. When unions perceive CB policies as rather strict and transparent, wages are lower with less centralised wage bargaining (higher M). In contrast, when CB policies are seen rather accommodating and opaque, a fall in w is induced not by higher competition among unions but by more centralised wage bargaining (lower M). The latter is associated with more cautious behaviour on the part of unions, each of which internalises to a greater extent the macroeconomic consequences of its wage claims.

Finally, it is worth elaborating on one aspect of items (ii) and (iii) in Proposition 2. These items refer to two initial conditions, namely, the monetary authority’ conservativeness and transparency. While there is no clear upper-bound on CB’s unpredictability, it has been argued that there may be a lower bound on ². This is the case of Cukierman’s (2009) discussion about the so-called “limits of transparency”. These are supposed to reflect constraints on how much the monetary authority knows about the actual level of the output gap and about the impact of policy on inflationary expectations. If the lower bound for ² is large enough (i.e. if CB transparency cannot be too high), then Proposition 2 would be relevant only in case greater monetary strictness (lower ᵇ) and less centralised wage bargaining (higher M) raise nominal wages. Otherwise, there would be scope for the fully non-linear (threshold) results contained in Proposition 2.

### 3.3. Enhancing Macroeconomic and Financial Stability

We assume that the loss function of the CB is formally the same as that of society. Therefore, (22) equals expected losses for both the CB and society. We thus abstract from aspects that would imply a difference between the two. Such as may include optimal delegation à la Rogoff (1985), whereby the CB should be more conservative than society, or the type of CB approach advocated by Blinder (1997) to eliminate the inflation bias, which involves a CB target for unemployment above its socially optimal level.

As we have seen, social losses are expected to rise in the event of adverse exogenous financial developments, as given by a rise in financial assistance costs (higher C and h), or a reassessment by households of deposit risks leading to an upward shift in the distribution of the ²’s. In response to these developments, the CB would reduce the scope of its LOLR services. Put differently, the critical bank size used to decide LOLR assistance would go up (i.e. a higher ncrít). Although this action reduces the cost associated with LOLR transactions, overall social losses would normally rise.

If the initial equilibrium were fully efficient, there would simply be no scope for improvement. In our model the equilibrium reflects noncooperative actions concerning the interaction between unions and the CB. Labour market outcomes thus deviate from the efficient solution. In the case of CB transparency, it could the case that it is set at an inefficiently high level owing to international benchmarking or domestic regulation (e.g. because inflation targeting involves this as a requirement). Initial deviations from efficiency motivate our interest in the role played by policy actions in dampening social losses.

In this regard, let us look at the inefficiency in the labour market. Inspection of (22) indicates that the wage premium over its competitive level is socially costly. Therefore, the factors pushing wages down in Proposition 2 contribute to a more efficient outcome, helping offset to a certain extent the loss arising from the financial shock. From this particular angle, we can derive the following conclusions:

**Proposition 3. Macroeconomic stability is higher**

- **i)** the more opaque the CB (higher ²);
- **ii)** the greater (smaller) the degree of monetary policy accommodation, ᵇ, when CB policies are initially seen as rather strict (accommodating) and transparent (opaque);
- **iii)** the smaller (greater) the number of unions, M, and thus the greater (smaller) the degree of wage bargaining centralisation when CB policies are initially seen as rather strict (accommodating) and transparent (opaque).

As unions become atomistic (M → ∞) all of these wage effects vanish.
Proof. From (22), the wage premium over its competitive level is socially costly, i.e. $dE/dw>0$. Based on this together with the results in Proposition 2 concerning the parameters of interest (namely, $\sigma_i^2$, $\gamma$ and $M$), it is straightforward to derive the results in Proposition 3.

QED.

Proposition 3 refers to macroeconomic stability in terms of an ex-ante performance evaluation. As such, it concerns the unconditional expectation of variability arising from money demand disturbances. This Proposition need not hold ex-post, i.e. for the actual realisation of shocks at a given point in time.

In addition to wage developments, it is also worth considering the role of the supply curve slope (which may be influenced by trade openness) for macroeconomic and financial stability. It is found that the impact of $\beta$ on $EL$ is relatively complex. A change in the supply curve slope affects wages in complicated ways —see Eq. (23)—, on top of raising the coefficient $\mu$ premultiplying $(w-\pi)^2$ in (22). Moreover, $\beta$ has ambiguous consequences for stability via the number of banks not assisted by LOLR transactions, $ncrit$. This is so both because $\beta$ influences $ncrit$ in a way that is parameter-dependent (see Proposition 1) and because $ncrit$ itself has two effects which are overall ambiguous. First, it lowers the financial costs relating to LOLR transactions, given by $(\alpha-ncrit)FIRC$. Second, as long as LOLR transactions need not be successful (i.e. for $h=(0,1)$), $ncrit$ raises $\left(\sum_{i=1}^{n} \sigma_i^2 + h^2 \sum_{i=1}^{\nu} \sigma_i^2 \right)$ in (22), as fewer banks receive LOLR assistance. All in all, it is not possible to derive an unequivocal role of $\beta$ for macroeconomic and financial stability, as the effects described in this paragraph point in various possible directions. Given the complexity involved in the effects of the supply curve slope (which may be affected by trade openness) on $EL$, policymakers might want to abstain from influencing $\beta$ unless they have a clear idea of the structural parameters of the model.

3.5. Role of Moral Hazard

In the absence of moral hazard, we have seen that only the largest banks receive LOLR assistance. LOLR services are often seen as leading to moral hazard behaviour on the part of commercial banks. In the present modelling context, Berlemann and Zeidler (2009) analyse the effects of moral hazard behaviour, considering the roles of both the expected profit of banks and the optimal amount of capital they invest into the risky project. These authors show that the largest banks react to the provision of LOLR services by increasing their risky exposures. Berlemann and Zeidler (2009) also show that, concerning the remaining banks (i.e. the “small” and “very small”), those in the former group have an incentive to decrease their investments into risky projects in order to get access to LOLR transactions. It is only the “very small” banks that do not alter their behaviour when LOLR is provided by the CB and thus display no moral hazard behaviour. In equilibrium banks’ responses to (a larger) LOLR provision contribute to lower inflation and output variability but also higher costs associated with financial assistance. The overall effect on $EL$ is thus ambiguous, with macroeconomic stability pushing CB losses down as financial costs push them up.

Our introduction of unionised wage setting into the analysis does not affect the previous results on moral hazard. But wage setting considerations are worth considering with regard to the implications of moral hazard for CB losses. First of all, given that the overall effect of moral hazard on $EL$ is ambiguous, in case CB losses rise the authorities could consider institutional changes that lead to wage discipline in an attempt to keep macroeconomic performance under check. Second, the results described in the previous paragraph assume that the CB has access to the additional financial resources derived from moral hazard. If the CB lacks the funds needed to dampen macroeconomic instability, this would be another reason for the policymakers to adjust economic institutions so as to reduce the premium of wages over their competitive level.

Going beyond the present modelling environment, the literature has studied cases where LOLR services may have different implications for moral hazard. As we have seen, Berlemann and Zeidler (2009) show that, under moral hazard behaviour, relatively large banks react by increasing their risky investments. In contrast, Perotti and Suárez (2002) argue that moral hazard implies that it is smaller banks that would be willing to take greater risks, while larger banks would proceed more cautiously. Among other studies, Repullo (2011) shows that LOLR does not increase the incentives to take risk, while insufficient capital requirements and penalty interest rates charged for liquidity assistance do. Freixas et al. (2004) find that LOLR has different implications for the efficiency of an unsecured interbank market depending on the source of moral hazard.

4. Conclusions

We introduce imperfect monetary policy transparency and strategic wage setting into a macro model where the CB provides LOLR services to banks on top of its standard stabilisation policy. The CB provides liquidity assistance to illiquid commercial banks with the aim that these banks avoid bankruptcy. Whenever banks are closed there are wider fluctuations in the money multiplier (and thus in inflation and output), which makes monetary policy more difficult and causes welfare decreasing fluctuations of inflation and output around their target levels.

At a given point in time, the equilibrium of a given economy may be characterised by coordination failure. The occurrence of major shocks (such as those behind the latest financial crisis) may offer the opportunity for institutional reform that tackles both the latest disturbance and the existing coordination problem. In addition, policymakers’ behaviour, which may in “normal” times face binding constraints, e.g. from regulation or benchmarking, might change in the light of exceptional circumstances. Against this background, the present paper studies how the trade-off between financial instability and macroeconomic variability can be improved by adjustments in the economic structure (possibly influenced by trade openness) and institutions (as given by the monetary policy setup, wage bargaining and the non-cooperative game involved between the CB and wage setters). This comes on top of the CB policy decisions concerning standard monetary policy and LOLR transactions, also considered in this paper. It is also different from the roles of the CB in bank supervision and macroprudential regulation, which could be seen as attempting to tackle the trade-off between moral hazard and bailout uncertainty (Cukierman, 2013).

The main results obtained here are the following. In a context of costly LOLR transactions, the CB has an incentive to save only large banks, a well-documented empirical phenomenon known as “too-big-to-fail doctrine”. CB opaqueness can help improve macroeconomic stability by making wages closer to their competitive levels. In contrast to this role for opaqueness about CB...
preferences, it is best for the CB to reveal its LOLR transactions so as to avoid excessive macroeconomic instability. Turning to monetary policy accommodation and wage bargaining centralisation, these factors may help discipline wages and thus improve stability when CB policies are initially seen as rather strict and transparent. The role of the supply curve slope (which may be affected by trade openness) is relatively complex, so policymakers might want to abstain from influencing it unless they have a clear idea of the structural parameters of the model. This means that, even if the economy falls in a recession amidst financial uncertainty, we advocate a cautious approach regarding international trade. The reasons given here differ from the position that, in the absence of international coordination, protectionism may lead to retaliation and risk bringing global trade down.

Concerning the role of wages in the model, two clarifications are in order. First, the result that lower wages are more efficient in part reflects the fact that the model used here represents the supply side in much more detail than the demand side. In a recessionary context, it may be realistic to allow wage restraint to play some role in pushing aggregate demand down. Second, wages are found not to influence the critical bank size involved in LOLR decisions. This could be seen as arising from our use of double-quadratic preferences for the CB. It may be worth examining if wages could not to influence the critical bank size involved in LOLR decisions.

viour on the side of commercial banks, it is not clear from the literature how smaller

to acknowledge a number of limitations. For instance, the returns factors may help discipline wages and thus improve stability when

to have a different information set from that available to individual banks; monitoring issues and information disclosure could then be discussed in relation with LOLR operations.

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References


The lack of detail concerning demand also means that the study is unable to assess the role of product market competition (and thus the effects of product market reform).


Appendix

This Appendix contains the derivations of Eqs. (20a), (22) and (23) in the text.

A.1 Derivation of Eq. (20a)

Starting from (18) and (19), if we abstract from all banks but individual bank \(i\) (which is assumed to be given -for the sake of the present derivation— to be given LOLR assistance) then the macroeconomic variables would adopt the values

\[
\pi = \hat{\pi} + (1 - \beta)\gamma + \beta \hat{\pi} + (1 - \beta)\eta_i (E e_i - e_i) \tag{A.1}
\]

\[
y = -\beta(1 - \gamma)(w - \hat{\pi}) + \beta \eta_i (E e_i - e_i) \tag{A.2}
\]

Plugging these expressions into CB loss (9), we get for bank \(i\)

\[
E_{i,LOLR} = \frac{E\left[(1 - \beta)\gamma + \beta \hat{\pi} + (1 - \beta)\eta_i (E e_i - e_i)\right]}{2} + \frac{\beta E\left[(1 - \gamma)(w - \hat{\pi}) + \beta \eta_i (E e_i - e_i)\right]}{2} + F_{hi}^C
\]

\[
= \frac{\left[(1 - \beta)\gamma + \beta \hat{\pi} + (1 - \beta)\eta_i (E e_i - e_i)\right]}{2} + \frac{\beta \eta_i (E e_i - e_i)}{2} + F_{hi}^C
\]

Using the expression for \(y = \beta[Z\hat{\pi} - (1 - \beta)] + (1 - \beta)\gamma\) given right below Eq. (13), the curly bracket premultiplying the wage term in (A.3) can be found to equal

\[
\left[(1 - \beta)\gamma + \beta \hat{\pi} + (1 - \beta)\eta_i (E e_i - e_i)\right] = \frac{(Z\hat{\pi})^2}{(1 - \beta)\gamma + \beta \hat{\pi} + (1 - \beta)\eta_i (E e_i - e_i)} + \frac{Z\hat{\pi}^2(1 - \beta)}{(1 - \beta)\gamma + \beta \hat{\pi} + (1 - \beta)\eta_i (E e_i - e_i)} = \mu \tag{A.4}
\]

where \(\mu = Z\hat{\pi}^2(1 - \beta)\gamma + \beta \hat{\pi} + (1 - \beta)\eta_i (E e_i - e_i)\). Plugging (A.4) into (A.3) yields Eq. (20a).

A.2 Derivation of Eq. (22)

Eq. (22) can be derived from expressions \(E_{i,LOLR}^1\) in (20a) and \(E_{i,NLOLR}^1\) in (20b), by noticing that there are \(n-ncrit\) banks being assisted by the CB (and thus each contributing to overall CB losses, \(EL\), with \(E_{i,LOLR}^1\), and \(ncrit\) banks not being assisted (each contributing to \(EL\) with \(E_{i,NLOLR}^1\)).

A.3 Derivation of Eq. (23)

All unions independently and simultaneously set their nominal wage. Each of them minimises (11) taking into consideration the expressions for inflation in (18) and sectoral output in (10), as well as the expected CB reaction from (17). The first-order condition for an interior sectoral wage, \(w_{Ei}\), is

\[
-2E^* \left[1 - \frac{(1 - \beta)\gamma + \beta \hat{\pi}}{M}\right] - 2AE^* \left\{y_i G = 0 \right\} = 0 \tag{A.5}
\]

where \(G = \alpha \left[1 - (1 - \beta)\gamma + \beta \hat{\pi}\right] / M + \eta (M - 1) / M\). Under a symmetric equilibrium (i.e. \(w_{Ei}=w_0\) for all \(k\), from (10) one gets \(y_{Ei} = -\alpha (w - \pi)\). Therefore, one is able to derive

\[
y_i G = \frac{\alpha(w - \pi)}{M} \left[\frac{\alpha \left[(M - 1) + (1 - \gamma)(1 - \beta)\right]}{M} + \eta (M - 1)\right] \tag{A.6}
\]

Also taking into account the following expression for real wages, where inflation is substituted from (18):

\[
w - \pi = (1 - \beta)(1 - \gamma)(w - \hat{\pi}) - (1 - \beta) \sum_{i=1}^{M} \eta_i (E e_i - e_i) \tag{A.7}
\]

we can take expectation on (A.6) to obtain

\[
E^* \left[y_i G\right] = \frac{\alpha(1 - \beta)(w - \hat{\pi})}{M} \left[\frac{\alpha \eta (M - 1)(1 - \gamma)(1 - \beta)}{M} + \alpha(1 - \beta)(1 - \gamma)^2 + \alpha \beta \gamma (1 - \beta)\right] \tag{A.8}
\]

Plugging (A.8) into (A.5) and making some algebraic manipulations, it is straightforward to derive Eq. (23).