



The Modified Quasi-geostrophic Barotropic Models Based on Unsteady Topography

Baojun Zhao ¹, Wenjin Sun ^{1,*}, Tianming Zhan ²

¹ College of Oceanography, HoHai University, Nanjing 210098, China

² School of Technology, Nanjing Audit University, Nanjing 211815, China

*Email of Corresponding Author: sunwenjin950925@126.com

ABSTRACT

New models using scale analysis and perturbation methods were derived starting from the shallow water equations based on barotropic fluids. In the paper, to discuss the irregular topography with different magnitudes, especially considering the condition of the vast terrain, some modified quasi-geostrophic barotropic models were obtained. The unsteady terrain is more suitable to describe the motion of the fluid state of the earth because of the change of global climate and environment, so the modified models are more rational potential vorticity equations. If we do not consider the influence of topography and other factors, the models degenerate to the general quasi-geostrophic barotropic equations in the previous studies.

Keywords: *Quasi-geostrophic; Potential vorticity; Scale analysis; Topography.*

Modelos Semigeostróficos Barotrópicos Modificados con Base en Topografía Inestable

RESUMEN

Este trabajo deduce nuevos modelos con el uso de los métodos de análisis a escala y de perturbación a partir de las ecuaciones de aguas poco profundas con base en fluidos barotrópicos. En este artículo se obtuvieron algunos modelos semigeostróficos barotrópicos para aplicar en zonas de topografía inestable con diferentes magnitudes y considerar especialmente la condición del extenso terreno. La topografía inestable es más propicia para describir el movimiento del estado fluido de la tierra debido al cambio del clima y ambiente, por lo tanto los modelos modificados son ecuaciones de vorticidad potenciales más razonables. Si no se considera la influencia de la topografía y otros factores, los modelos se reducirían a las ecuaciones generales semigeostróficas barotrópicas de estudios anteriores.

Palabras clave: *Semigeostrofia; vorticidad potencial; análisis a escala; topografía.*

Record

Manuscript received: 01/03/2017

Accepted for publication: 31/03/2017

How to cite item

Zhao, B., Sun, W., & Zhan, T. (2017). The Modified Quasi-geostrophic Barotropic Models Based on Unsteady Topography. *Earth Sciences Research Journal*, 21(1), 23-28. doi:<http://dx.doi.org/10.15446/esrj.v21n1.63007>

1. Introduction

In recent decades, many scholars have conducted extensive research on large-scale atmospheric and oceanic dynamics. Among them, the topographic effect has great influences on the dynamic mechanism of potential vorticity equations (Pedlosky, 1974; Collings, 1980; Pedlosky, 1980; Treguier, 1989). In the atmosphere, the topographic height plays an importance role in atmospheric cyclone and anticyclone changes. Luo (1990) pointed out the topographic effect was also an important factor of forming atmospheric blocking, and the global atmospheric circulation would even get affected by the topographic effect. Lu (1987) discussed the effects of topographic height and shaped on Rossby wave activation and the influences of topographic south-north and east-west slopes on waveform and energy propagation. Chen (1998) and Jiang (2000) derived the quasi-geostrophic potential vorticity equation with large-scale topography, friction, and heating under the barotropic model, and the large-scale effects of Qinghai-Tibet Plateau on atmosphere were discussed. In addition, the oceanic topography is very complex, such as The North West Shelf of Australia (Holloway, 1997), Portugal Shelf Sea Area (Sherwin, 2002), etc. The relationship between topography and ocean circulation was pointed out in the literature (Roslee et al., 2017b; Kamsani, 2017; La, 1990; Marshall, 1995; Alvarez, 1994; Sou, 1996). Cessi (1986) discussed the important role of topography in ocean circulation. Holloway (1992) introduced the interaction of eddies with seafloor topography and argued that ocean circulations would be significant interaction between turbulent vortices and topography rather than gravity wave drag. Then, general expressions for the eddy-topographic force, eddy viscosity, and stochastic backscatter, as well as a residual Jacobian term, are derived for barotropic flow over mean topography by Frederiksen (1999). All the above researches, Actually the topography height also changes with time in the earth fluid. Changes in topography can lead to tsunamis, floods and natural disasters (Abdullah, 2017; Elfithri, 2017; Rahim, 2017). Yang (2011, 2012) and Song (2012, 2013) discussed topography changes over time discussed topography changes over time t , the influence of the nonlinear long wave amplitude and waveform. Da (2013) discussed the shallow water equation forms when underlying surface slowly changes with time and obtained the vorticity equation with an underlying surface. This model considered the actual circumstances that the topography changes with time-space (Erfen et al., 2017; Roslee, 2017a). In this paper, we discuss different magnitude topography under spatial-temporal variable and obtain some modified models, which have important effects on the future discussion about the waveform changes of nonlinear long waves.

This paper is organized as follows: In Section 2, starting from the rotating shallow water equation set, we give the bottom topography which is not smooth boundary and simplify the equation set with unsteady topography. Section 3 is given scale analysis and perturbation methods; then we obtain new equation set by topographic conditions with different magnitudes. Later we derive the equations with different orders and get some modified models in Section 4. Finally, make the relevant conclusions in Section 5.

2. Basic equations

Assuming the static equilibrium condition, the fluid can be regarded as barotropic, incompressible, frictionless state. The upper boundary height is $h(x, y, t)$, and $w_b = w|_{z=h} = \frac{dh}{dt} = \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y}$, free surface pressure intensity $p_b = p|_{z=h} = p_0$ is constant. The basic equation set can be written as (Pedlosky, 1987)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (1a)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + fu = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (1b)$$

$$-g - \frac{1}{\rho} \frac{\partial p}{\partial z} = 0 \quad (1c)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1d)$$

especially, the bottom topography is about spatial-temporal variable $h_b = h_b(x, y, t)$, the coriolis parameter is

$$f(y) = 2\Omega[\sin \varphi_0 + \frac{\cos \varphi_0}{a} y - \frac{1}{2} \sin \varphi_0 (\frac{y}{a})^2 + \dots] \quad (2)$$

first of all, we integrate Eq. (1c)

$$\int_z^h \frac{\partial p}{\partial z} dz = - \int_z^h g \rho dz \quad (3)$$

and get

$$p = p_0 + \rho g(h - z) \quad (4)$$

so

$$\frac{\partial p}{\partial x} = \rho g \frac{\partial h}{\partial x} \quad (5)$$

$$\frac{\partial p}{\partial y} = \rho g \frac{\partial h}{\partial y} \quad (6)$$

Eqs. (5) and (6) indicate that the horizontal pressure gradient under the barotropic model can be expressed by the gradient of the free surface gravitational potential (Taharin & Roslee, 2017).

We assume that the preliminary horizontal velocity is independent of the z , Eqs. (1a) and (1b) can be rewritten as

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv = -g \frac{\partial h}{\partial x} \quad (7a)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu = -g \frac{\partial h}{\partial y} \quad (7b)$$

then integrate continuity Eq. (1d) from $z = h_b(x, y, t)$ to the free surface $z = h(x, y, t)$, we can get

$$\frac{\partial(h - h_b)}{\partial t} + u \frac{\partial(h - h_b)}{\partial x} + v \frac{\partial(h - h_b)}{\partial y} + (h - h_b) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad (7c)$$

assuming that the free surface height H is constant when the fluid is static (Pedlosky, 1987).

$$h(x, y, t) = H + \hat{h}, \quad \hat{h} < H \quad (8)$$

the barotropic equation set with unsteady topography can be written as

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv = -g \frac{\partial(H + \hat{h})}{\partial x} \quad (9a)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - f v = -g \frac{\partial(H + \hat{h})}{\partial x} \quad (9b)$$

$$\frac{\partial(H + \hat{h} - h_B)}{\partial t} + u \frac{\partial(H + h - h_B)}{\partial x} + v \frac{\partial(H + h - h_B)}{\partial y} + (H + \hat{h} - h_B) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad (9c)$$

If

$$\phi = gh \quad \phi_0 = gH = c_0^2 \quad \hat{\phi} = g\hat{h} \quad \phi_B = gh_B \quad (10)$$

the equation set can be written as

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - f v = -\frac{\partial \hat{\phi}}{\partial x} \quad (11a)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + f u = -\frac{\partial \hat{\phi}}{\partial y} \quad (11b)$$

$$\frac{\partial(\hat{\phi} - \phi_B)}{\partial t} + u \frac{\partial(\hat{\phi} - \phi_B)}{\partial x} + v \frac{\partial(\hat{\phi} - \phi_B)}{\partial y} + (c_0^2 + \hat{\phi} - \phi_B) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad (11c)$$

3. Scale analysis, perturbation methods

Making dimensionless analysis on the Eqs. (11a), (11b) and (11c), for the large-scale motions

$$(x, y) = L(x_1, y_1) \quad t = \left(\frac{L}{U}\right)t_1 \quad (u, v) = U(u_1, v_1) \quad \hat{\phi} = f_0 U L \phi_1$$

$$f = f_0 f_1 \quad \phi_B = \lambda f_0 U L \phi_{B1} \quad (12)$$

where λ is an undetermined parameter.

Substituting Eq. (12) into Eqs. (11a), (11b) and (11c), then

$$\frac{U^2}{L} \left(\frac{\partial u_1}{\partial t_1} + u_1 \frac{\partial u_1}{\partial x_1} + v_1 \frac{\partial u_1}{\partial y_1} \right) - f_0 U (f_1 v_1) = f_0 U \left(-\frac{\partial \hat{\phi}_1}{\partial x_1} \right) \quad (13a)$$

$$\frac{U^2}{L} \left(\frac{\partial v_1}{\partial t_1} + u_1 \frac{\partial v_1}{\partial x_1} + v_1 \frac{\partial v_1}{\partial y_1} \right) + f_0 U (f_1 u_1) = f_0 U \left(-\frac{\partial \hat{\phi}_1}{\partial y_1} \right) \quad (13b)$$

$$f_0 U^2 \frac{d(\hat{\phi}_1 - \lambda \phi_{B1})}{dt_1} + [c_0^2 + f_0 U L (\hat{\phi}_1 - \lambda \phi_{B1})] \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} \right) = 0 \quad (13c)$$

Eqs. (13a) and (13b) divided by $f_0 U$, (13c) divided by $\frac{c_0^2 U}{L} = \frac{g H U}{L}$, we get

$$Ro \frac{du_1}{dt_1} - f_1 v_1 = -\frac{\partial \hat{\phi}_1}{\partial x_1} \quad (14a)$$

$$Ro \frac{dv_1}{dt_1} + f_1 u_1 = -\frac{\partial \hat{\phi}_1}{\partial y_1} \quad (14b)$$

$$Ro \mu_0^2 \frac{d(\hat{\phi}_1 - \lambda \phi_{B1})}{dt_1} + [1 + Ro \mu_0^2 (\hat{\phi}_1 - \lambda \phi_{B1})] \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} \right) = 0 \quad (14c)$$

where

$$Ro = \frac{U}{f_0 L}, \mu_0^2 = \left(\frac{f_0 L}{c_0} \right)^2, \frac{d}{dt_1} = \frac{\partial}{\partial t_1} + u_1 \frac{\partial}{\partial x_1} + v_1 \frac{\partial}{\partial y_1} \quad (15)$$

Let

$$\begin{aligned} u_1 &= u_1^{(0)} + Ro u_1^{(1)} + \dots \\ v_1 &= v_1^{(0)} + Ro v_1^{(1)} + \dots \\ \hat{f}_1 &= f_1^{(0)} + Ro f_1^{(1)} + \dots \end{aligned} \quad (16)$$

meantime $f = f_0 f_1 = f_0 + \beta_0 y$, $\beta_1 = \frac{L^2}{U} \beta_0$, so

$$f_1 = 1 + Ro \beta_1 y_1 \quad (17)$$

then we get

$$\begin{aligned} Ro \frac{d}{dt_1} (u_1^{(0)} + Ro u_1^{(1)} + \dots) - (1 + Ro \beta_1 y_1) (v_1^{(0)} + Ro v_1^{(1)} + \dots) \\ = -\frac{\partial}{\partial x_1} (\hat{\phi}_1^{(0)} + Ro \hat{\phi}_1^{(1)} + \dots) \end{aligned} \quad (18a)$$

$$\begin{aligned} Ro \frac{d}{dt_1} (v_1^{(0)} + Ro v_1^{(1)} + \dots) + (1 + Ro \beta_1 y_1) (u_1^{(0)} + Ro u_1^{(1)} + \dots) \\ = -\frac{\partial}{\partial y_1} (\hat{\phi}_1^{(0)} + Ro \hat{\phi}_1^{(1)} + \dots) \end{aligned} \quad (18b)$$

$$\begin{aligned} Ro \mu_0^2 \frac{d(\hat{\phi}_1 - \lambda \phi_{B1})}{dt_1} + \left[\frac{\partial(u_1^{(0)} + Ro u_1^{(1)} + \dots)}{\partial x_1} + \frac{\partial(v_1^{(0)} + Ro v_1^{(1)} + \dots)}{\partial y_1} \right] \\ + Ro \mu_0^2 (\hat{\phi}_1 - \lambda \phi_{B1}) \left(\frac{\partial(u_1^{(0)} + Ro u_1^{(1)} + \dots)}{\partial x_1} + \frac{\partial(v_1^{(0)} + Ro v_1^{(1)} + \dots)}{\partial y_1} \right) = 0 \end{aligned} \quad (18c)$$

where

$$\frac{d}{dt_1} = \frac{\partial}{\partial t_1} + (u_1^{(0)} + Ro u_1^{(1)} + \dots) \frac{\partial}{\partial x_1} + (v_1^{(0)} + Ro v_1^{(1)} + \dots) \frac{\partial}{\partial y_1} \quad (19)$$

4. Derive the barotropic models

Making classified discussion on the magnitude of λ .

4.1. $\lambda \sim 1$ magnitude

The scale of the ϕ_B is consistent with the small amplitude function ϕ , most of the topography parameters following with this situation in the real world.

Ro^0 order approximation

$$u_1^{(0)} = -\frac{\partial \hat{\phi}_1^{(0)}}{\partial y_1}, v_1^{(0)} = \frac{\partial \hat{\phi}_1^{(0)}}{\partial x_1}, \frac{\partial v_1^{(0)}}{\partial y_1} + \frac{\partial u_1^{(0)}}{\partial x_1} \quad (20)$$

or a dimensional form

$$u_1^{(0)} = -\frac{\partial \hat{\phi}_1^{(0)}}{\partial y_1}, v_1^{(0)} = \frac{\partial \hat{\phi}_1^{(0)}}{\partial x_1}, \frac{\partial v_1^{(0)}}{\partial y_1} + \frac{\partial u_1^{(0)}}{\partial x_1} \quad (21)$$

Ro¹ order approximation

$$\left(\frac{\partial}{\partial t_1} + u_1^{(0)} \frac{\partial}{\partial x_1} + v_1^{(0)} \frac{\partial}{\partial y_1}\right) u_1^{(0)} - \beta_1 v_1^{(0)} - v_1^{(1)} = -\frac{\partial \hat{\phi}_1^{(1)}}{\partial x_1} \quad (22a)$$

$$\left(\frac{\partial}{\partial t_1} + u_1^{(0)} \frac{\partial}{\partial x_1} + v_1^{(0)} \frac{\partial}{\partial y_1}\right) v_1^{(0)} + \beta_1 u_1^{(0)} + u_1^{(1)} = -\frac{\partial \hat{\phi}_1^{(1)}}{\partial y_1} \quad (22b)$$

$$\mu_0^2 \left(\frac{\partial}{\partial t_1} + u_1^{(0)} \frac{\partial}{\partial x_1} + v_1^{(0)} \frac{\partial}{\partial y_1}\right) (\hat{\phi}_1^{(0)} - \lambda \phi_{B1}) + \left(\frac{\partial u_1^{(1)}}{\partial x_1} + \frac{\partial v_1^{(1)}}{\partial y_1}\right) = 0 \quad (22c)$$

after calculation, the dimensionless vorticity equation is derived

$$\frac{f_0 L}{U} \left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) \zeta^{(0)} + \beta_0 v^{(0)} = -f_0 \left(\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y}\right) \quad (23a)$$

$$\frac{f_0 L}{U} \left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) (\phi^{(0)} - \phi_B) + c_0^2 \left(\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y}\right) = 0 \quad (23b)$$

eliminating $\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y}$ from the Eqs. (23a), (23b), we get

$$\left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) (f + \zeta^{(0)}) = \frac{f_0^2}{c_0^2} \left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) (\psi - \psi_B) \quad (24)$$

namely

$$\left(\nabla^2 - \frac{1}{\lambda_R^2}\right) \frac{\partial \psi}{\partial t} + \frac{f_0}{H} \frac{\partial h_B(x, y, t)}{\partial t} + J(\psi, \beta_0 y + \nabla^2 \psi - \frac{1}{\lambda_R^2} \psi + \frac{f_0}{H} h_B(x, y, t)) = 0 \quad (25)$$

where

$$u^{(0)} = -\frac{\partial \psi}{\partial y}, \quad v^{(0)} = \frac{\partial \psi}{\partial x}, \quad \zeta^{(0)} = \nabla^2 \psi, \quad \psi = \frac{\hat{\phi}^{(0)}}{f_0}, \quad \psi_B = \frac{g h_B}{f_0}, \quad \lambda_R = \frac{\sqrt{gH}}{f_0} \quad (26)$$

λ_R is the Rossby radius of deformation, Eq. (25) is a modified model.

4.2. $\lambda \sim 10$ magnitude

Large-scale atmospheric motion of large topography is suitable for such conditions ($L \sim 10^6 m$, $U \sim 10 m/s$, $f_0 \sim 10^{-4} s^{-1}$). For example, a case study of Tibetan Plateau topography, the height is $3.4 \times 10^3 m$ approximately which is suit for the situation.

Ro⁰ order approximation

$$u_1^{(0)} = -\frac{\partial \hat{\phi}_1^{(0)}}{\partial y_1}, \quad v_1^{(0)} = \frac{\partial \hat{\phi}_1^{(0)}}{\partial x_1}, \quad \frac{\partial v_1^{(0)}}{\partial y_1} + \frac{\partial u_1^{(0)}}{\partial x_1} = 0 \quad (27)$$

or a dimensional form

$$f_0 u^{(0)} = -\frac{\partial \hat{\phi}^{(0)}}{\partial y}, \quad f_0 v^{(0)} = \frac{\partial \hat{\phi}^{(0)}}{\partial x}, \quad \frac{\partial v^{(0)}}{\partial y} + \frac{\partial u^{(0)}}{\partial x} = 0 \quad (28)$$

in order to get the results, we should consider the magnitude of μ_0^2
 $\mu_0^2 = \frac{(f_0 L)^2}{gH} \sim 10^{-1}$, according to the Eq. (18c), μ_0^2 can be considered as a parameter.

Ro¹ order approximation

$$\left(\frac{\partial}{\partial t_1} + u_1^{(0)} \frac{\partial}{\partial x_1} + v_1^{(0)} \frac{\partial}{\partial y_1}\right) u_1^{(0)} - \beta_1 v_1^{(0)} - v_1^{(1)} = -\frac{\partial \hat{\phi}_1^{(1)}}{\partial x_1} \quad (29a)$$

$$\left(\frac{\partial}{\partial t_1} + u_1^{(0)} \frac{\partial}{\partial x_1} + v_1^{(0)} \frac{\partial}{\partial y_1}\right) v_1^{(0)} + \beta_1 u_1^{(0)} + u_1^{(1)} = -\frac{\partial \hat{\phi}_1^{(1)}}{\partial y_1} \quad (29b)$$

$$\mu_0^2 \left(\frac{\partial}{\partial t_1} + u_1^{(0)} \frac{\partial}{\partial x_1} + v_1^{(0)} \frac{\partial}{\partial y_1}\right) (\hat{\phi}_1^{(0)} - \lambda \phi_{B1}) \quad (29c)$$

$$+ (1 - \mu_0^2 Ro \lambda \phi_{B1}) \left(\frac{\partial u_1^{(1)}}{\partial x_1} + \frac{\partial v_1^{(1)}}{\partial y_1}\right) = 0$$

or dimensional form

$$\frac{f_0 L}{U} \left[\left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) u^{(0)} - \beta_0 v^{(0)}\right] - f_0 v^{(1)} = -\frac{\partial \hat{\phi}^{(1)}}{\partial x} \quad (30a)$$

$$\frac{f_0 L}{U} \left[\left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) v^{(0)} + \beta_0 u^{(0)}\right] + f_0 u^{(1)} = -\frac{\partial \hat{\phi}^{(1)}}{\partial y} \quad (30b)$$

$$\frac{f_0 L}{U} \left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) (\hat{\phi}^{(0)} - \phi_B) + (c_0^2 - \phi_B) \left(\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y}\right) = 0 \quad (30c)$$

Eqs. (30a), (30b) and (30c) can be transformed into the vorticity equations

$$\frac{f_0 L}{U} \left[\left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) \zeta^{(0)} + \beta_0 v^{(0)}\right] = -f_0 \left(\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y}\right) \quad (31a)$$

$$\frac{f_0 L}{U} \left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) (\hat{\phi}^{(0)} - \phi_B) + (c_0^2 - \phi_B) \left(\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y}\right) = 0 \quad (31b)$$

Eqs. (31a), (31b) are quasi-geostrophic barotropic models, so

$$\frac{f_0 L}{U} \left[\left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) (f + \zeta^{(0)})\right] = -f_0 \left(\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y}\right) \quad (32a)$$

$$\frac{f_0 L}{U} \left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right) [f_0 (\psi - \psi_B)] + (c_0^2 - f_0 \psi_B) \left(\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y}\right) = 0 \quad (32b)$$

where

$$u^{(0)} = -\frac{\partial \psi}{\partial y}, \quad v^{(0)} = \frac{\partial \psi}{\partial x}, \quad \zeta^{(0)} = \nabla^2 \psi, \quad \psi = \frac{\hat{\phi}^{(0)}}{f_0}, \quad \psi_B = \frac{g h_B}{f_0} \quad (33)$$

assuming $c_0^2 - f_0 \psi_B \neq 0$, we eliminate $\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y}$ from Eqs. (32a) and (32b), then

$$\left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right)(f + \zeta^{(0)}) = \frac{f_0^2}{c_0^2 - f_0 \psi_B} \left(\frac{\partial}{\partial t} + u^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y}\right)(\psi - \psi_B) \quad (34)$$

finally

$$\left(\frac{\partial}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y}\right)[\beta_0 y + \nabla^2 \psi - \frac{f_0^2}{c_0^2 - g h_B} \psi + \frac{f_0}{c_0^2 - g h_B} g h_B] = 0 \quad (35)$$

$$\text{where } u^{(0)} = -\frac{\partial \psi}{\partial y}, v^{(0)} = \frac{\partial \psi}{\partial x}, \zeta^{(0)} = \nabla^2 \psi, \psi = \frac{g \hat{h}}{f_0}, \psi_B = \frac{\phi_B}{f_0} = \frac{g h_B}{f_0}$$

Eq. (35) is a modified model.

4.3. $\lambda \leq 10^{-1}$ magnitude

According to (18c), it is observed that the magnitude of topography expression form λ is too small, which approximately ignores the influences of topographic effect.

Now we can observe that the Eqs. (25), (35) are two quasi-geostrophic barotropic models under variable topographic conditions with time t . If the topographic effect is independent of time, equations degrade into the following forms.

Under the stable topography, Eq. (35) degenerates into

$$\left(\frac{\partial}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y}\right)(f + \nabla^2 \psi - \frac{f_0^2}{c_0^2 - g h_B} \psi) + \frac{f_0 g}{c_0^2 - g h_B} \left(\frac{\partial \psi}{\partial x} \frac{\partial h_B}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial h_B}{\partial x}\right) = 0 \quad (36)$$

where

$$u^{(0)} = -\frac{\partial \psi}{\partial y}, v^{(0)} = \frac{\partial \psi}{\partial x}, \zeta^{(0)} = \nabla^2 \psi, \psi = \frac{\hat{\phi}^{(0)}}{f_0}, \psi_B = \frac{g h_B}{f_0} \quad (37)$$

the model (25) under the stable topography can be written as (Pedlosky, 1987)

$$\left(\nabla^2 - \frac{1}{\lambda_R^2}\right) \frac{\partial \psi}{\partial t} + J[\psi, \beta_0 y + \nabla^2 \psi - \frac{1}{\lambda_R^2} \psi + \frac{f_0}{H} h_B(x, y)] = 0 \quad (38)$$

where

$$u^{(0)} = -\frac{\partial \psi}{\partial y}, v^{(0)} = \frac{\partial \psi}{\partial x}, \zeta^{(0)} = \nabla^2 \psi, \psi = \frac{\hat{\phi}^{(0)}}{f_0}, \psi_B = \frac{g h_B}{f_0}, \lambda_R = \frac{\sqrt{g H}}{f_0} \quad (39)$$

otherwise, if we don't consider the topographic effect and the divergence of the fluid, Eqs. (36) and (38) degenerate into (Liu, 1991)

$$\nabla^2 \frac{\partial \psi}{\partial t} + J(\psi, \beta_0 y + \nabla^2 \psi) = 0 \quad (40)$$

Eq. (40) is a classics dynamics model used by the large-scale atmospheric and oceanic motions.

5. Conclusion

(a). Under the unsteady topography, some new modified models (25), (35) are derived. These models meet the general rule that the topography changes with time in reality. When the topography has nothing to do with the time, Eq. (35) degenerates into a dynamics model (36), Eq. (25) degenerates into Eq. (38) which is a quasi-geostrophic barotropic

model under the spatial topography.

(b). The modified models under the topographic effect with different magnitudes are presented, we can see the pattern under the condition of large terrain, which is the improvement of the model. After the above-modified models are given, we will also derive the mathematical model for Rossby wave in the further study, and the further exploration of the large-scale factual influences of topography on atmosphere and ocean are required.

Acknowledgements

This work was supported by The National Nature Science Foundation of China under Grant (No. 61502206), Key projects of water conservancy science and technology of Jiangsu Province, China (No. 2010500312), and The Research Innovation Program for College Graduates of Jiangsu Province (No. CXZZ12_0224).

References

- Abdullah, M. P. (2017). Stakeholders' response and perspectives on flood disaster of Pahang river basin. *Malaysian Journal of Geosciences*, 1(1) 43-49.
- Alvarez, A. (1994). Effect of topographic stress on circulation in the western Mediterranean. *Journal of Geophysical Research Atmospheres*, 99(C8), 16053-16064.
- Cessi, P., & Pedlosky, J. (1986). On the role of topography in the ocean circulation. *Journal of Marine Research*, 44 (3), 445-471.
- Chen, J., & Liu, S. K. (1998). The Solitary waves of the Barotropic Quasi-Geostrophic model with large-scale orography. *Advances in Atmospheric Sciences*, 15 (03), 404-41.
- Collings, I. L., & Grimshaw, R. (1980). The effect of topography on the stability of a barotropic coastal current. *Dynamics of Atmospheres and Oceans*, 5(2), 83-106.
- Da, C. J. (2013). Dynamic modification of the shallow water equation on the slowly changing underlying surface condition. *Acta Physica Sinica*, 62(3), 039202.
- Elfithri, R. (2017). Pahang flood disaster: The potential flood drivers. *Malaysian Journal of Geosciences*, 1(1) 34-37.
- Erfen, H. F. W. S. (2017). Geochemical characterization of sediments around Nukakatan valley, Tambunan, Sabah. *Geological Behavior*, 1(1)13-15.
- Frederiksen, J. S. (1999). Subgrid-scale parameterizations of eddy-topographic force, eddy viscosity, and stochastic backscatter for flow over topography. *Journal of the atmospheric sciences*, 56(11), 1481-1494.
- Holloway, G. (1992). Representing topographic stress for large-scale ocean models. *Journal of Physical Oceanography*, 22(9), 1033-1046.
- Holloway, P. E., et al., 1997. A nonlinear model of internal tide transformation on the Australian North West Shelf. *Journal of Physical Oceanography*, 27(6), 871-896.
- Jiang, X. (2000). A Barotropic Quasi-geostrophic model with large-scale topography, Friction and Heating. *Journal of Tropical Meteorology*, 6(01), 65-74.
- La, V., & Paul, E. (1990). The western Mediterranean circulation experiment (WMCE): introduction. *Journal of Geophysical Research: Oceans*, 95(C2), 1511-1514.
- Liu, S. K., & Liu, S. D. (1991). *Atmospheric Dynamics*. Beijing, BJ: Peking University Press.
- Lu, K. L. (1987). The effects of orography on the solitary Rossby waves in a barotropic atmosphere. *Acta Meteorologica Sinica*, 45, 267-273.
- Luo, D. H. (1990). Topographically forced Rossby wave instability and the development of blocking in the atmosphere. *Advances in atmospheric sciences*, 7(4), 433-440.
- Marshall, D. (1995). Influence of topography on the large-scale ocean circulation. *Journal of physical oceanography*, 25(7), 1622-1635.

- Pedlosky, J. (1974). Longshore currents, upwelling and bottom topography. *Journal of Physical Oceanography*, 4(2), 214-226.
- Pedlosky, J. (1980). The destabilization of shear flow by topography. *Journal of Physical Oceanography*, 10(11), 1877-1880.
- Pedlosky, J. (1987). *Geophysical Fluid Dynamics* second edition. Springer, New York, 57-246.
- Rahim, I. A. (2017). Kampung Mesilou landslide: The controlling factors. *Geological Behavior*, 1(1) 19-21.
- Roslee, R. (2017). GIS application for comprehensive spatial soil erosion analysis with MUSLE Model in Sandakan Town area, Sabah, Malaysia. *Geological Behavior*, 1(1) 01-05.
- Roslee, R. (2017). Flood Potential Analysis (FPAn) using Geo-Spatial data in Penampang area, Sabah. *Malaysian Journal of Geosciences*, 1(1) 01-06.
- Sherwin T. J., Vlasenko V. I., & Stashchuk, N. (2002). Along-slope generation as an explanation for some unusually large internal tides. *Deep Sea Research Part I: Oceanographic Research Papers*, 49(10), 1787-1799.
- Song, J., et al., 2012. Beta effect and slowly changing topography Rossby waves in a shear flow. *Acta Physica Sinica*, 61(21), 210510, <http://dx.doi.org/10.7498/aps.61.210510>.
- Song, J. (2014). Nonlinear Rossby waves excited slowly changing underlying surface and dissipation. *Acta Physica Sinica*, 63(6), 060401, <http://dx.doi.org/10.7498/aps.63.060401>.
- Sou, T. (1996). Effects of topographic stress on Caribbean Sea circulation. *Journal of Geophysical Research: Oceans*, 101(C7), 16449-16453.
- Taharin, M. R., & Roslee, R. (2017). Comparison of cohesion (c'), and angle of internal friction (Φ') distribution in highland area of Kundasang by using ordinary Kriging and simple Kriging. *Geological Behavior*, 1(1) 16-18.
- Treguier, A. M. (1989). Topographically generated steady currents in barotropic turbulence. *Geophysical and Astrophysical Fluid Dynamics*, 47, 43-68.
- Yang, H. W. (2011). Forced solitary Rossby waves under the influence of slowly varying topography with time. *Chinese Physics B*, 20(11), 120203.
- Yang, H. W. (2012). Generation of solitary Rossby waves by unstable topography. *Communications in Theoretical Physics*, 57(3), 473-476.