




Modeling and Control of Small-Scale Underground Mine Ventilation Networks

Modelado y control de redes de ventilación de minas subterráneas de pequeña escala

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ABSTRACT

This paper presents a nonlinear model and a control strategy to regulate airflow in small-scale underground mine ventilation networks. In underground ventilation control systems, a sensor and an actuator for each branch are usually considered. However, in small-scale underground mines, it is too expensive to have automatic doors for controlling the airflow in each tunnel, as well as to install flow and gas sensors in each branch. In order to regulate airflow in small-scale underground ventilation networks, the number of sensors and actuators is regarded as a limitation for practical installation. This work presents an alternative modeling of the network, with direct control of the airflow in each tunnel, by varying the speed of the fans available on the external surface of the mine. A state space model is presented, and a linear quadratic controller with integral action and a state estimator is designed. To validate the model and the controller, a network ventilation system for a small-scale mine with nine branches (tunnels) and two actuators (fans) is presented.

Keywords: ventilation, modeling, control systems, mines

ABSTRACT

Este artículo presenta un modelado no lineal y una estrategia para el control del flujo de aire en redes de ventilación de minas subterráneas de pequeña escala. Para los sistemas de control de ventilación subterránea usualmente se consideran un sensor y un actuador en cada rama. Sin embargo, en las minas subterráneas de pequeña escala, es muy costoso colocar puertas automáticas para controlar el flujo de aire en cada túnel, así como instalar sensores de flujo y gas en cada rama. Con el fin de regular el flujo de aire en redes de ventilación subterránea de pequeña escala, se considera el número de sensores y actuadores como una limitante para la instalación práctica. Este trabajo presenta una alternativa para el modelado de la red, con control directo del flujo de aire en cada túnel, a través de la variación de velocidad de los ventiladores disponibles en la superficie externa de la mina. Se presenta un modelo de espacio de estado y se diseña un controlador cuadrático lineal con acción integral y un estimador de estado. Para validar el modelo y el controlador, se presenta un sistema de red de ventilación para una mina de pequeña escala con nueve ramas (túneles) y dos actuadores (ventiladores).

Palabras clave: ventilación, modelado, sistema de control, minas

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Introduction

The mining industry is an important contributor to the economies of many countries. Due to its characteristics, underground mining is considered to be a dangerous activity, with high rates of accidents reported each year [Knights and Scanlan \(2019\)](#); [Wang and Du \(2020\)](#). For this reason, substantial academic and industrial research has been focused on improving safety conditions in underground mining operations, focusing on the niche of knowledge in what is known as Industry 4.0. [Pałaka et al. \(2020\)](#) show the impact of this technology on the workers of underground coal mines. Ventilation systems play a key role in underground coal mine safety due to the presence of dangerous gases such as methane. The problems around modeling and controlling mine ventilation systems have been studied since the 1960s, [Tolmachev \(1966\)](#), [Petrov et al., 1992](#)). A nonlinear dynamic model for ventilation systems based on Kirchhoff's laws is presented by [Hu et al. \(2003\)](#), which allows modeling large-scale ventilation networks and airflow control in each tunnel. A similar model is proposed by [Zhu et al. \(2014\)](#), who include external perturbation in the model and design an H_1 optimal controller. In [Sui et al. \(2016\)](#), the analysis

of a large ventilation network with 11 nodes and 17 branches is proposed. In the aforementioned studies, a fully equipped network was considered in order to control the dynamic behavior of airflow in the tunnels, namely that there were sensors throughout the network and automatic doors working as actuators in every branch. An example for this type of instrumentation is shown in [Kozielski et al. \(2021\)](#),

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where data collected from 28 different sensors placed at various locations of an underground coal mine were made available for analysis or research.

In developing countries, it is common to find small-scale mining facilities whose complete measurements and network actuation are not available due to their high cost and lack of resources. It is therefore necessary to design control strategies for ventilation networks of underground mines while considering the availability of sensors and actuators. A typical scenario is a ventilation system with a reduced number of fans working as actuators.

From a control perspective, [Rodriguez-Diaz et al. \(2021\)](#) show the application of a classic control strategy in a prototype for underground mining built in GipsaLab (France). [Raji et al. \(2020\)](#) show control strategies to manipulate airflow and reduce energy costs. Moreover, [Rasool et al. \(2020\)](#) show the tuning of multiple controllers for an HVDC system-based optimization technique. Applications of modern control strategies for ventilation networks can be found throughout the literature [Jing et al. \(2020\)](#); [Ren and Cao \(2020\)](#); [Nardo and Yu \(2021\)](#), where the use of techniques such as artificial intelligence and on-demand ventilation are employed. In addition, in a study by [de Villiers et al. \(2019\)](#), the evaluation of auxiliary fan performance in underground mining highlights the engineering challenges involved in optimizing airflow.

This study proposes another solution starting with a model based on the analysis provided by [Hu et al. \(2003\)](#), as well as a control design for ventilation systems in underground small-scale mines with reduced measurements and actuators. The control aim is to regulate the airflow in a work area (a branch of the ventilation network) by means of the control action provided only by the fans available in the mine. The designed controller is based on the optimization of sensors and actuators.

[Summers and Lygeros \(2014\)](#) have focused on the problem of optimal placement of sensors and actuators in complex dynamic networks. Moreover, control theory uses the representation in state space and the concepts of observability and controllability to reduce the number of devices needed in a control system ?.

The paper is organized as follows: Section 1 provides a brief introduction; Section 2 presents the modeling procedure; Section 3 presents a specific application where the ventilation system of a typical underground small-scale coal mine is considered, proposes a controller design, and presents the results obtained; and Section 4 outlines the conclusions and future research areas.

Modeling of ventilation networks in underground mines

Electric circuit theory can be used to analyze the dynamic behavior of mine ventilation systems, particularly the well known non-planar circuit analysis (topological analysis).

This work demonstrates a static analysis of the network under study. First, a representation of the ventilation network as a non-planar circuit is presented, identifying the elements in the system and organizing them according to the equivalent circuit. Then, the tree in the network must be defined. A tree is defined by considering a set of nodes and branch lines constituting a path without closing any mesh

and passing every node just once. Including the branches with air fans in the definition of trees is recommended for analyzing the system. The branches that are not part of the trees are called *links*. Thus, it is possible to state that

$$\begin{aligned} n_n &= \text{Amount of nodes in the network,} \\ n_r &= \text{Amount of branch lines in the network,} \\ n_{rs} &= \text{Amount of branch lines without a fan,} \\ l_1 &= n_r - (n_n - 1) = \text{Link number} \\ m &= \text{Amount of air fans} \end{aligned}$$

Static model of the network

The following conditions are assumed [Hu et al. \(2003\)](#):

1. The air is incompressible. Therefore, the air density is constant in all branches of the mine [Danko \(\(2017\)\)](#).
2. The temperature in all the branches of mine is constant.

These assumptions are made based on the typical conditions found in small-scale mines.

Airflow equations without direct fan action

Considering the connections in a ventilation network, air mass conservation at the nodes can be expressed in terms of airflow quantities, *i.e.*, the sum of the airflow entering the node is equal to the sum of the outgoing airflow. Kirchhoff's current law applied to the ventilation network can be expressed as follows:

$$\sum_{j=1}^{n_{rs}} E_{Qij} Q_j = 0, \quad i = 1, \dots, (n_{rs} - l_1), \quad (1)$$

or, equivalently,

$$E_Q Q = 0, \quad (2)$$

where Q represents the vector values of airflow quantities in the branches. The values of E_{Qij} are defined as follows:

$$E_{Qij} = \begin{cases} 1, & \text{if branch } \mathbf{j} \text{ is connected to the node } \mathbf{i} \\ & \text{and airflow leaves the node } \mathbf{i} \\ -1, & \text{if branch } \mathbf{j} \text{ is connected to the node } \mathbf{i} \\ & \text{and airflow enters the node } \mathbf{i} \\ 0, & \text{if branch } \mathbf{j} \text{ is not connected to the node } \mathbf{i} \end{cases}$$

The airflow in a ventilation network is associated with the current flow in an electric circuit. To explain the concept, a mathematical representation of the analysis of node i in matrix form is shown in Figure 1, as given by

$$\begin{bmatrix} -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = 0.$$

Then, for a system with $p = (n_{rs} - l_1)$, it is possible to express the following:

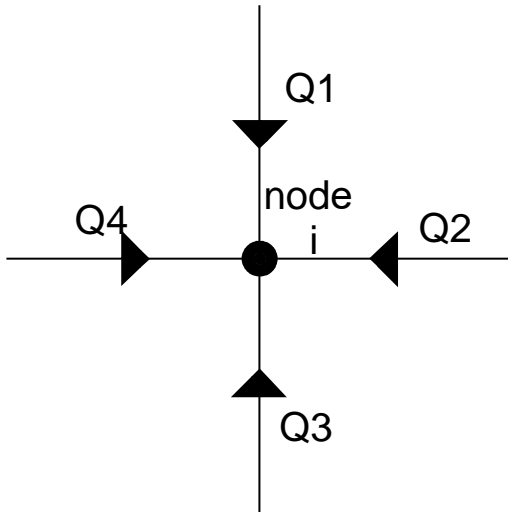


Figure 1. Example of bifurcation in an underground mine
Source: Authors

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n_{rs}} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n_{rs}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{p1} & a_{p2} & a_{p3} & \dots & a_{pn_{rs}} \end{bmatrix}}_{E_Q} \underbrace{\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{n_{rs}} \end{bmatrix}}_Q = 0.$$

Airflow equations with direct action of fans e_{Qm}

At nodes where an air fan actuator is located, the equivalent of Kirchhoff's current law can be expressed as

$$\sum_{j=1}^{n_{rs}} e_{Qij} Q_j = Q_{mi}, \quad i = 1, \dots, m, \quad (3)$$

or, equivalently

$$e_{Qm} Q = Q_m, \quad (4)$$

where e_{Qm} is a matrix with dimensions $m \times n_{rs}$, and m is the amount of the air fans in the network. e_{Qmij} is defined as

$$e_{Qmij} = \begin{cases} 1, & \text{if branch } j \text{ is connected to the node } i \\ & \text{and airflow leaves node } i \\ -1, & \text{if branch } j \text{ is connected to the node } i \\ & \text{and airflow enters node } i \\ 0, & \text{if branch } j \text{ is not connected to the node } i \end{cases}$$

For a ventilation network with m fans, Equation (4) can be written as

$$\underbrace{\begin{bmatrix} b_{11} & b_{12} & b_{13} & \dots & b_{1n_{rs}} \\ b_{21} & b_{22} & b_{23} & \dots & b_{2n_{rs}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & b_{m3} & \dots & b_{mn_{rs}} \end{bmatrix}}_{e_{Qm}} \underbrace{\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{n_{rs}} \end{bmatrix}}_Q = \underbrace{\begin{bmatrix} Q_{f1} \\ Q_{f2} \\ \vdots \\ Q_{fm} \end{bmatrix}}_{Q_m}.$$

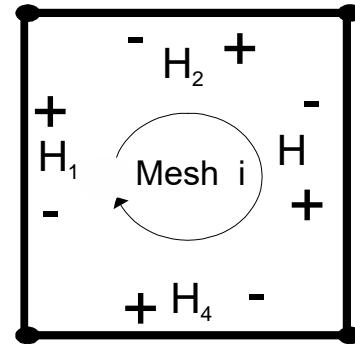


Figure 2. Pressure drops for a mesh in an underground mine
Source: Authors

Pressure drop equations without direct fan action

In an ventilation network, as in an electrical network, the Kirchhoff voltage law holds, i.e., the algebraic sum of pressure drops in a closed trajectory is zero, Hartman *et al.* (1997):

$$\sum_{j=1}^{n_{rs}} E_{Hij} H_j = 0, \quad i = 1, \dots, (l_1 - m), \quad (5)$$

or, equivalently

$$E_H H = 0, \quad (6)$$

where l_1 is the amount of links, namely the number of branches in the co-tree; and the column vector H contains the pressure drop variables in the branches where the air fans do not have direct action. The terms of the E_H matrix are defined as

$$E_{Hij} = \begin{cases} 1, & \text{if branch } j \text{ is connected to the mesh } i \\ & \text{and airflow is in same direction as the loop} \\ -1, & \text{if branch } j \text{ is connected to the mesh } i \\ & \text{and airflow is in the opposite direction to the loop} \\ 0, & \text{if branch } j \text{ is not connected to the mesh } i. \end{cases}$$

According to the example in Figure 2, Equation (6) can be written as follows:

$$\begin{bmatrix} -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = 0.$$

For a ventilation network with p meshes, where $p = l_1 - m$, Equation (6) can be written as

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n_{rs}} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n_{rs}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{p1} & a_{p2} & a_{p3} & \dots & a_{pn_{rs}} \end{bmatrix}}_{E_H} \underbrace{\begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{n_{rs}} \end{bmatrix}}_H = 0.$$

Pressure drop equations with direct fan action

For the loops of the ventilation systems where the actuators are working, the Kirchhoff voltage law can be expressed as follows:

$$\sum_{j=1}^{n_{rs}} e_{Hij} H_j = -H_{Vi}, i = 1, \dots, m, \quad (7)$$

or, equivalently

$$e_{Hm} H = -H_m, \quad (8)$$

where the values of e_{Hm} are obtained as

$$e_{Hmij} = \begin{cases} 1, & \text{if branch } j \text{ is connected to the mesh } i \text{ and} \\ & \text{airflow has the same direction of the loop} \\ -1, & \text{if branch } j \text{ is connected to the mesh } i \text{ and} \\ & \text{airflow has opposite direction of the loop} \\ 0, & \text{if branch } j \text{ is not connected to the mesh } i. \end{cases}$$

Nonlinear dynamic model of the ventilation network

The previous section introduced a static mathematical analysis of the variables involved in a ventilation network. However, a dynamic model is necessary to design control strategies. For a single branch, the airflow dynamics [Sui et al. \(2016\)](#) are described as follows:

$$\frac{dQ_j}{dt} + K_j R_j |Q_j| Q_j = K_j H_j, \quad (9)$$

where

- Q_j : Airflow of branch j
- R_j : Aerodynamic resistance of branch j
- H_j : $P_{lj} - P_{lj0}$ Pressure drop of branch j
- P_{lj} : Absolute pressure at the end of branch j
- P_{lj0} : absolute pressure at the beginning of branch j
- K_j : $S_j / \rho L_j$ Inertial coefficient of branch j
- L_j : Length of branch j
- S_j : Cross section of branch j
- ρ : Air density

Parameter R_j can be obtained from the Atkinson equation [McPherson \(1993\)](#); [Hartman et al. \(1997\)](#):

$$R_j = \frac{\alpha_j L_j P_j}{S_j^3}, \quad (10)$$

with:

- α_j : Aerodynamic resistance constant of branch j
- P_j : Perimeter of branch j

[Hu et al. \(2003\)](#) propose a nonlinear generalization of ventilation systems in underground mining, which is expressed as

$$\frac{dQ}{dt} = A_v Q_D^2 R + B_v Q + C_v d, \quad (11)$$

$$H = Y_{RQ} Q_D^2 R + Y_Q Q + Y_d d, \quad (12)$$

with matrices $A_v, B_v, C_v, Y_{RQ}, Y_Q$, and Y_d of appropriate dimensions. Matrix A_v is defined as

$$A_v = -K(I - Y_{RQ}), B_v = KY_Q, C_v = KY_d. \quad (13)$$

The procedure to obtain the parameters of the dynamic model based on the static relations among the branches and nodes is presented in [Sui et al. \(2016\)](#) and is summarized as follows:

$$Y_{RQ} = \begin{bmatrix} S_{Ha} \zeta_{RQ} \\ \zeta_{RQ} \end{bmatrix}, \quad Y_Q = \begin{bmatrix} S_{Ha} \zeta_Q + S_Q \\ \zeta_Q \end{bmatrix},$$

$$Y_d = \begin{bmatrix} S_d + S_{Ha} \zeta_d \\ \zeta_d \end{bmatrix}, \quad S_{Ha} = - \begin{bmatrix} E_{Hc} \\ e_{Hmc} \end{bmatrix}^{-1} \begin{bmatrix} E_{Ha} \\ e_{Hma} \end{bmatrix},$$

$$S_Q = \begin{bmatrix} E_{Hc} \\ e_{Hmc} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} R_m e_{Qm}, \quad S_d = \begin{bmatrix} E_{Hc} \\ e_{Hmc} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -I \end{bmatrix},$$

$$\zeta_{RQ} = [E_{Qc} K_c S_{Ha} + K_a]^{-1} E_Q K, \quad (14)$$

$$\zeta_Q = -[E_{Qc} K_c S_{Ha} + K_a]^{-1} E_{Qc} K_c S_Q, \quad (15)$$

$$\zeta_d = -[E_{Qc} K_c S_{Ha} + K_a]^{-1} E_{Qc} K_c S_d. \quad (16)$$

In [Hu et al. \(2003\)](#), the existence of $[E_{Qc} K_c S_{Ha} + K_a]^{-1}$ is demonstrated. Considering the dynamics in the fan branch, it is possible to write the following:

$$H_m = d - R_m Q_m, \quad (17)$$

where d is the pressure drop generated by the action of the fan, R_m is the coefficient of resistance in the fan branch, and Q_m is the airflow in the fan branch.

Reduced nonlinear model of the ventilation network

In order to simplify the control design, a reduced model is proposed [Hu et al. \(2003\)](#). Considering the dependence of the co-tree variables, it is possible to analyze only the state variables related to the links. The state variables are arranged in ascending order in the matrix Q_c , starting with the airflows that correspond to the links. Then, the matrix A_c , which has direct action with the links, is decomposed. The matrix A_{ca} relates the complementary flows of the network, thus completing the dynamics.

Thereupon, a reduced model using only the airflows of the links is proposed as

$$\dot{Q}_c = A_c Q_{cD}^2 R_c + A_{ca} Q_{aD}^2 R_a + B_c Q_c + C_c d, \quad (18)$$

$$H_c = \zeta_{RQc} Q_{cD}^2 R_c + \zeta_{RQa} Q_{aD}^2 (Q_c) R_a + \zeta_{Qc} Q_c + \zeta_d d, \quad (19)$$

where Q_c is the reduced state vector. This work considers the fan pressure drop d as the control input. The matrices of the system are defined as

$$A_c = -K_c + K_c S_{Ha} \zeta_{RQc}, \quad (20)$$

$$A_{ca} = K_c S_{Ha} \zeta_{RQa}, \quad (21)$$

$$B_c = K_c (S_{Ha} \zeta_{Qc} + R_m S_{Qc}), \quad (22)$$

$$C_c = K_c (S_d + S_{Ha} \zeta_d). \quad (23)$$

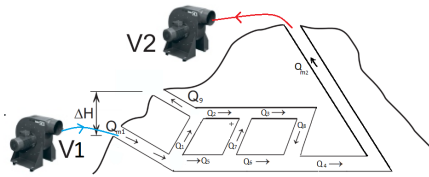


Figure 3. Physical view of the underground coal mine

Source: Authors

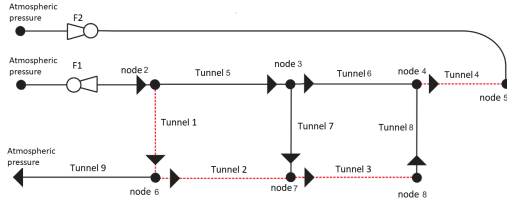


Figure 4. Schematic network diagram (tree and links)

Source: Authors

Application example

This section presents a case study of an underground coal mine in Colombia. A physical model of the mine is shown in Figure 3. The ventilation system consists of a couple of fans working as actuators, a forcing fan at the entrance of the mine (V_1), and an auxiliary extractor fan (V_2) strategically placed to improve the conditions of the working area. Usually, in small mining facilities, fans working with 440 V and a nominal power in the range of 10 to 30 HP are installed. The parameters in Table 1 are drawn from real data of the small-scale mine. The tunnel resistance is defined as $R = (\alpha_j * L_j * P_j / S_j^3)$, where α is the aerodynamic resistance constant, P is the perimeter of the tunnel, S is the cross-section of the tunnel, and L is the length of the tunnel.

A schematic diagram of the ventilation system is shown in Figure 4. In order to simplify the network analysis, the tree and co-tree must be identified. The tree of the network is indicated by the black solid line, and the links are represented by means of the red dashed line.

According to the network analysis for the system shown in Figure 4, the following elements can be defined

n_r	= 11	Number of branches in the network
n_{rs}	= 9	Number of branches without fans in the network
n_n	= 8	Number of nodes
l_1	= 4	Number of links = $n_r - (n_n - 1)$
m	= 2	Number of fans

Note that the branches forming the tree are V_1 , V_2 , Tunnel 5, Tunnel 6, Tunnel 7, Tunnel 8, and Tunnel 9. The links are formed by branches Tunnel 1, Tunnel 2, Tunnel 3, and Tunnel 4. The conservation equations for the flow in each node are defined as

$$\begin{aligned} -Q_4 + Q_{m2} &= 0, \\ -Q_{m1} + Q_1 + Q_5 &= 0, & -Q_1 + Q_2 + Q_9 &= 0, \\ -Q_5 + Q_6 + Q_7 &= 0, & -Q_7 - Q_2 + Q_3 &= 0, \\ -Q_6 - Q_8 + Q_4 &= 0, & -Q_3 + Q_8 &= 0. \end{aligned}$$

The mesh equations can be written as

$$H_{m1} + H_1 + H_9 = 0,$$

$$H_{m1} + H_5 + H_7 - H_2 + H_9 = 0,$$

$$H_6 - H_8 - H_3 - H_7 = 0,$$

$$H_{m1} + H_5 + H_6 + H_4 + H_{m2} = 0.$$

By reorganizing the equations in a structured matrix form, it is possible to write

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{E_Q} \underbrace{\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_9 \end{bmatrix}}_Q = 0,$$

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{e_{Qm}} \underbrace{\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_9 \end{bmatrix}}_Q = \underbrace{\begin{bmatrix} Q_{m1} \\ Q_{m2} \end{bmatrix}}_{Q_{Gm}},$$

$$\underbrace{\begin{bmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix}}_{E_H} \underbrace{\begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_9 \end{bmatrix}}_H = 0,$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & -1 \end{bmatrix}}_{e_{Hm}} \underbrace{\begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_9 \end{bmatrix}}_H = \underbrace{\begin{bmatrix} -H_{m1} \\ -H_{m2} \end{bmatrix}}_{H_{Gm}}.$$

Considering the network's static analysis, a dynamic model can be found based on Equations (11) and (12). Matrices A_v , B_v , and C_v are obtained from Equation (13), which yields a non-linear system with nine state variables. The simulation parameters of the system are presented in Table 1.

Reduced model for small-scale mining

In order to simplify the analysis and control design, a reduced model is used. The states of the reduction are the airflows in the links, namely Q_1 , Q_2 , Q_3 , and Q_4 . The airflows Q_5 , Q_6 , Q_7 , Q_8 , and Q_9 can be expressed as a linear combination of the airflow in the links.

$$\begin{aligned} Q_5 &= Q_4 - Q_2, & Q_6 &= Q_4 - Q_3, & Q_8 &= Q_3, \\ Q_7 &= Q_3 - Q_2, & Q_9 &= Q_1 - Q_2. \end{aligned}$$

The reduced model is characterized via Equation (18)

$$\dot{Q}_c = A_c Q_{cD}^2 R_c + A_{ca} Q_{aD}^2 R_a + B_c Q_c + C_c d, \quad (24)$$

$$H_c = \zeta_{RQc} Q_{cD}^2 R_c + \zeta_{RQa} Q_{aD}^2 (Q_c) R_a + \zeta_{Qc} Q_c + \zeta_d d, \quad (25)$$

Table 1. Parameters of an underground coal mine

Tunnel	Resistance [Ns ² /m ⁸]	K [Ns ² /m ⁴]	Length [m]	operation point [m ³ /h]
1	0,20	0,11	25	43,75
2	0,12	0,14	25	19,16
3	0,13	0,13	25	31,31
4	0,13	0,138	25	75,34
5	0,13	0,138	25	56,17
6	0,16	0,13	25	44,03
7	0,14	0,138	25	12,14
8	0,16	0,13	25	31,31
9	0,55	0,04	80	24,58

Source: Authors

where

$$A_c = \begin{bmatrix} -0,0546 & -0,0334 & -0,0118 & -0,0150 \\ -0,0334 & -0,0450 & -0,0159 & -0,0202 \\ -0,0118 & -0,0159 & -0,0418 & -0,0197 \\ -0,0150 & -0,0202 & -0,0197 & -0,0585 \end{bmatrix},$$

$$A_{ca} = \begin{bmatrix} 0,0184 & -0,0032 & 0,0216 & -0,0118 & -0,0212 \\ 0,0248 & -0,0043 & 0,0291 & -0,0159 & 0,0115 \\ -0,0038 & 0,0221 & -0,0259 & -0,0418 & 0,0041 \\ -0,0383 & -0,0388 & 0,0005 & -0,0197 & 0,0052 \end{bmatrix},$$

$$B_c = 0_{(4 \times 4)}, \quad C_c = \begin{bmatrix} -0,0362 & -0,0150 \\ -0,0087 & -0,0202 \\ -0,0156 & -0,0197 \\ -0,0533 & -0,0585 \end{bmatrix},$$

where $R_{c(4 \times 4)}$, $R_{a(4 \times 5)}$. Figure 5 shows the behavior of the airflow at the operating point. A 15% variation in the signal applied to fans 1 and 2 is taken as input at $t = 800$ s.

Controller design

An operation point given for a ventilation network with a constant input given by both fans is considered. Thus, by linearizing the nonlinear model, a linear state space representation is obtained:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (26)$$

$$Y(t) = Cx(t) + Du(t), \quad (27)$$

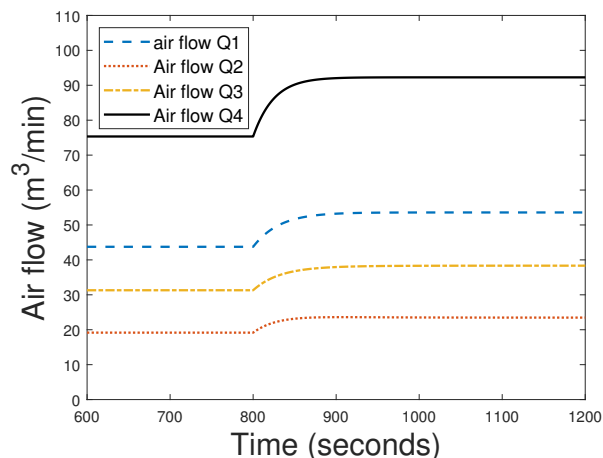
where the Jacobian matrices A, B, C, and D are obtained by applying the partial derivatives Egeland and Gravdahl (2002), evaluated at the point of operation as

$$A_{(4 \times 4)} = \left. \frac{\partial f}{\partial x} \right|_{x_o(t), u_o(t)}, \quad B_{(4 \times 2)} = \left. \frac{\partial f}{\partial u} \right|_{x_o(t), u_o(t)},$$

$$C_{(4 \times 4)} = \left. \frac{\partial h}{\partial x} \right|_{x_o(t), u_o(t)}, \quad D_{(4 \times 2)} = \left. \frac{\partial h}{\partial u} \right|_{x_o(t), u_o(t)}.$$

Then, the matrix values are

$$A = \begin{bmatrix} -0,0256 & 0,0014 & -0,0016 & -0,0012 \\ -0,0045 & -0,0164 & -0,0021 & -0,0015 \\ -0,0016 & -0,0007 & -0,0192 & -0,0022 \\ -0,0020 & 0,0053 & 0,0030 & -0,0371 \end{bmatrix},$$


Figure 5. Airflow in the links (open loop)

Source: Authors

$$B = \begin{bmatrix} -0,0362 & -0,0150 \\ -0,0087 & -0,0202 \\ -0,0156 & -0,0197 \\ -0,0533 & -0,0585 \end{bmatrix}, \quad C^T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad D = 0.$$

Given the limited number of sensors in the ventilation network, an observer-based controller is shown in Figure 6. The reduced set of airflow variables $Q_c(t)$ is considered as the state variables. The set of measured variables $Q_{meas}(t)$ is selected for the pair (A, C) . Thus, with the estimated states \hat{Q}_c , an LQI (Linear-Quadratic-Integral) control is implemented. The aim is to compute an optimal state-feedback control law in the form

$$u(t) = F\hat{Q}_c(t) + K_i \int e(t)dt, \quad (28)$$

where $e(t) = Q_{ref} - Q_{meas}(t)$, with a constant set-point Q_{ref} . A constraint of the system is defined by the airflow through the work front, as it can not exceed the maximum and minimum limits allowed Kirk (1998), namely

$$Q_{i_{min}} \leq Q_i \leq Q_{i_{max}}. \quad (29)$$

The control signal must also be restricted, i.e., the pressure drop generated by the fans must be limited, avoiding mechanical or electrical stresses in the fans:

$$U_{i_{min}} \leq U_i \leq U_{i_{max}}. \quad (30)$$

The objective of ventilation network optimization is to reduce energy consumption and to regulate airflow in the work front. Therefore, the cost function can be defined as

$$J(t) = \int_{t_0}^{t_f} \{U_i(t)^2 + Q_i(t)^2\} dt \quad (31)$$

For the issue of optimal control, the airflow of branch 1 will not be less than a minimum flow. Similarly, the function of cost is to minimize the energy applied to the ventilation system, and it is therefore necessary to find an admissible

control U^* that causes the system to follow a permissible trajectory X^* to minimize the cost function. The control law $u(t)$ for the ventilation system is defined as

$$u(t) = -Kx(t) - K_1v(t) = -\begin{bmatrix} K & K_1 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} (t), \quad (32)$$

where $[K \ K_1]$ are the constants of the controller, and $v(t)$ is the state variable of error. Based on Equation (32), the controller design problem focuses on solving the Riccati equation Kirk (1998), obtaining the values in the matrix K_T for

$$0 = -K_TA - A^TK_T - Q_o + K_TBR^{-1}B^TK_T, \quad (33)$$

where $K_T = [K \ K_1]$, $Q_o = 100 * I_{4 \times 4}$ and $R = 0.5 * I_{2 \times 2}$. The constants of the LQI controller are defined as

$$K = \begin{bmatrix} -29,5700 & 1,1614 & -2,7243 & -11,9784 \\ 9,3227 & -15,9746 & -10,0604 & -22,8246 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 0,6765 \\ -0,2059 \end{bmatrix}.$$

The algorithm for estimating the state vector $\hat{x}(t)$ is

$$\begin{aligned} \frac{d}{dt}\hat{x} &= A\hat{x}(t) + Bu(t) + L[y(t) - C\hat{x}(t)], \\ \hat{y}(t) &= C\hat{x}(t), \end{aligned} \quad (34)$$

and the gain L of the estimator is given by

$$L = [4,446 \quad 0,1925 \quad -0,2003 \quad -0,0616]^T.$$

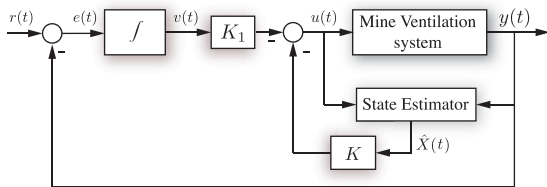


Figure 6. Observer-based LQI controller
Source: Authors

The dynamics of the nonlinear model and the controller were simulated in Matlab. The nonlinear model of the coal mine was implemented in an s -function, as shown in Figure 7. First, the system was set to its operation point, with $Q_1 = 43,75 \text{ m}^3/\text{h}$. At instant $t = 900 \text{ s}$, a change in the set point is required, so the airflow in the front work was adjusted to $Q_1 = 61,15 \text{ m}^3/\text{s}$. A couple of disturbances were introduced to the system at instants $t = 1100 \text{ s}$ and $t = 1150 \text{ s}$ in order to visualize the disturbance rejection provided by the integral term. The dotted line in the aforementioned Figure shows the response of the system while assuming the measurement of all the variables of the state vector $Q_c(t)$. The dashed line corresponds to the closed-loop behavior of the system considering the estimator-based controller proposed in Figure 6, with $Q_{meas}(t) = Q_1(t)$.

Figure 7 shows the comparison between the reference signal Q_1 , the airflow measured by sensor 1 and the estimated value of Q_1 . The objective of the control system is to keep the airflow at the reference level and reject disturbances. The characteristic closed-loop time for the proposed model is $1,5 \text{ min}$, an adequate time for a small-scale mine. In

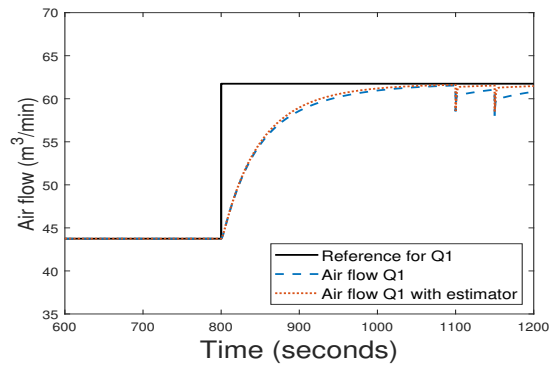


Figure 7. Controlled system output with reduced model
Source: Authors

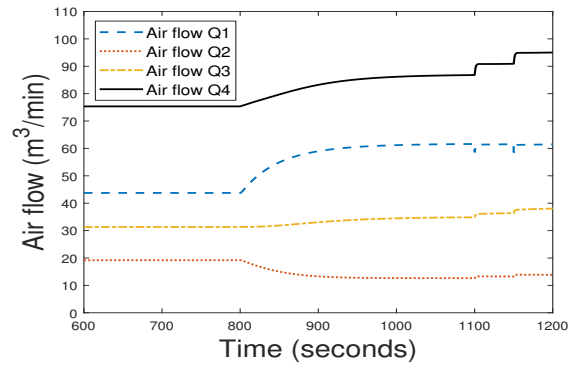


Figure 8. Airflows of the state variables with the estimator
Source: Authors

addition, the disturbance at 1100 s with a magnitude of 10% is attenuated by the controller in 7 s , showing the response speed of the closed loop system.

Figure 8 shows the behavior of the airflow in the four branches. In addition, the Figure shows how the airflow in branches 2, 3, and 4 is adjusted to ensure the airflow in branch 1 when there are disturbances at $t = 1100 \text{ s}$. The dynamics of all the branches can also be observed; the fastest dynamic corresponds to branch 1, where the airflow for the control loop is measured.

Conclusions

A control strategy to regulate air flow in small-scale underground mine ventilation networks was designed in this work. In order to design a controller, a nonlinear model was obtained which has direct control of the airflow in each tunnel by varying the speeds of the fans available on the external surface of the mine. Based on this model, a linear quadratic controller with integral action and a state estimator was designed. The results were obtained from simulations of a typical condition of an underground coal mine using two fans: the first fan injects air into the mine, and the other one extracts it. The reduction of sensors and actuators helps reduce the implementation costs of underground coal mining. Future work in this field could include conducting an analysis to determine the minimum number of sensors and actuators in the different topologies of mine ventilation networks.

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