INGENIERÍA Y DESARROLLO

A multi-criteria application for an equipment replacement decision

Una aplicación multi-criterio para la decisión de reemplazar un equipo

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Abstract

One of the main problems, in the operations management, is the adequate planning of equipment replacement, not only because of its impact in the operations cost, but also, because of its effects on the service level. For such reason, the present article shows the construction of a procedure for equipment replacement, based on the use of multicriteria techniques and expert methods. The results of this application, in the replacement decision of two equipments for a sugar factory, are presented.

Key words: Decision criteria, Global economy, Multiple criteria analysis, Replacement Equipment.

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Resumen

Uno de los principales problemas, en la dirección de operaciones es la adecuada planificación para el reemplazo de los equipos, no solamente por su impacto sobre el costo de las operaciones, sino también por sus efectos sobre el nivel de servicios. Por esta razón, el presente artículo muestra la construcción de un procedimiento para el reemplazo de equipos, basado en el uso de técnicas multicriteriales y métodos expertos; los resultados de esta aplicación en la decisión de reemplazar dos equipos para una fábrica de azúcar se muestran a continuación.

Palabras clave: Análisis multicriterios, Criterios de decisión, Economía global, Sustitución de equipos.

1. INTRODUCTION

Machinery replacement consists of finding the adequate moment to change equipment in use, based on the analysis of a criterion or of a decision criteria group [1]; Equipment management and replacement is often one of the last options to maximize cost savings in a competitive global economy due to its intrinsic complexity. Equipment management and replacement in the process industry must consider the commissioning, operational and end-of-life phases of physical assets when starting a design and implementation project [2] As a field of knowledge, equipment replacement has been extensively studied in the specialized literature and its origins stem from the third decade of the 20th century. Since the first publications of Lotka (1933), [3] the contributions related to replacement decisions have been numerous [4],[5],[6],[7] An analysis of the publications of some authors [8] allows the establishment of the different methodologies developed, to support the replacement decision; they can be classified in three general categories: 1) according to its objective, 2) according to its application environment and 3) according to the methodology used for its solution. Mainly, the replacement analysis has been oriented to the development of economic models based on the mathematical optimization of a cost function or a utility function. In this group, there outstands: the opportunity cost model, [9] the equilibrium models between operation and maintenance costs, [10] the profitability models [11], [1] and the replacement cost models [12], [13], other economic models, based on the replacement decision by comparing machines in use, with available machines in the market, by means of the present net value analysis [14] or by means of the comparative analysis of the operative machine capacity [15], [16].

The replacement of economic models vary in dependence of the application context; in this sense, the main groups of models are: models that involve the guarantee service, [17] models that involve the replenishment time in the replacement process, [18],[19],[20] models based on the additional needs of capacity due to sales growth, [21], [22], [23] models that involve mechanical aspects, [24], [25], [26] models based on the impacts of the equipment failures [27], models that take into account the technological change [28] and models based on econometric methods and the evaluation of financial variables [29],[30],[31].

On the other hand, the operations research also presents an important group of contributions, such as: integer programming [32], dynamic programming, [33]decision trees [34] simulation techniques [35], Markov decision problems [36], [29], [37], [38], partially observable models [39] the use of Information Technology/software asset management - IT/SAM [40] and some special mathematical applications such as the Lorenz's transform approach, applied to determine the replacement moment that minimizes the machine cost [41] Some models include the operation conditions analysis [42] failures typology, [43] and multi-component systems analysis [44].

The revision of literature shows the tendency toward the construction of replacement models based on a single criterion, generally of economic type. However, in practical terms, the majority of the decisions involve different criteria and, in consequence, the existing contributions have a limited enforceability. In this case, models based on multiple criteria can generate better solutions. Nevertheless, regarding the equipment replacement, the multicriteria analysis has been used very little, and current contributions are scarce [45].

In this sense and due to the multiple goals that are sought in replacement problems, (cost minimization, obsolescence reduction, productivity maximization, among others), the current tendencies in the decision making process, have been utilizing the multi-criteria techniques. These techniques permit, in a simultaneous way, the consideration of all the goals or prominent criteria, achieving a better approach to reality. According to Barba-Romero & Pomerol (1993), [46] the multicriteria paradigm, in the current business context, *"... is an effective aid in decision making and businesses management; nevertheless, many executives and practitioners don't know these important tools"*.

The multi-criteria modeling intends to find "the best solutions" among "the possible solutions", in situations in which the decisions complexity level is greater, due to the commitment among diverse aspirations that are impossible to satisfy fully. In order to develop a multi-criteria analysis, in an adequate way, it is necessary to apply the following procedure: 1) to define the problem, 2) to establish the prominent criteria, 3) to formulate the model, 4) to identify and to evaluate the alternatives until finding the best, and 5) to apply the decision [46].

Generally, the prominent criteria present different importance levels in the decision; this situation forces the establishment of hierarchical organization of the criteria, in function of the type of decision, by doing a weighting process. The hierarchical organization of the criteria can be done through subjective methods using the expertsí judgment, who, based on their knowledge and experience, organize the criteria according to their importance. In this process, it is important to control the degree of agreement among the experts, due to its impact on the reliable results [47].

Considering the previous approaches and the need to expand the multi-criteria theory contributions in the replacement problems, in the present article a methodology is exposed, that allows the construction of an integral indicator of replacement based on nonlinear continuous algebraic functions. The procedure makes allowance for knowledge in detail, for the application of a series of phases in order to find the adequate time for the equipment replacement. Finally, and in order to show its enforceability, its application in two cases of study in a sugar factory is presented.

2. PROCEDURE

The general procedure used for the construction of the Integral Replacement Indicator (IIR) is shown in Figure 1. The main objective of the IIR, is to establish the adequate time for the machine replacement, based on a group of prominent criteria. This procedure can be applied in any industrial sector. Nevertheless, the results depend on the characteristics of each industrial sector, technology and type of machines. For some authors, the pattern of replacement of equipment can be best explained by non-linear functions. So, in order to facilitate the solutions, the non linear programming supposes that all functions are differentiable in all parts. That is to say, they are linear functions by parts [48]. For this reason, in this paper a similar scheme is used. [49],[50] A description of each step is as follows:

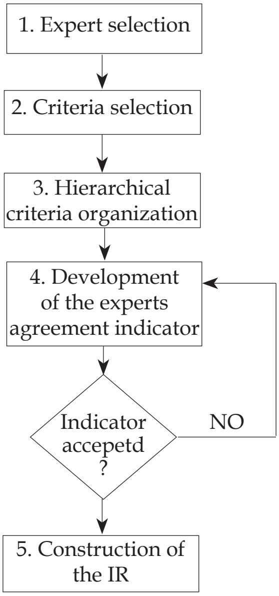

Figura 1. General procedure for the IIR construction

2.1. Experts selection (E_j): This phase consists of selecting a group of experts in equipment replacement. The number of experts must be calculated using equation 1. This expression was calculated using a mathematical equation, based on the probabilistic method that assumes a binomial probability law, and it was defined by Calves and Calderon. [51]

$$m = \frac{p \times (1 - p) \times k}{i^2} \tag{1}$$

Where: m: Number of experts, p: Percentage of error tolerated, k: Constant value that depends on the confidence level, i: Precision Level.

2.2 Criteria selection (C_i): This phase consists in determining the group of criteria that are important in the indicator. The methodology requires the development of successive sessions of work, until achieving certain agreement level in the list of criteria. A first session of work with the experts, should be carried out with the objective of establishing the list of preliminary criteria. Initially, it is necessary to select a first group of criteria. However, taking into account the principles of importance, redundancy and relevance, in successive sessions of work, the list should be reduced until establishing the prominent criteria. Although there are various methodologies applicable, such as Delphi or Nominal Group Technique, in this work, the methodology proposed by Poveda (2002) is considered [52] because it is necessary to describe the behavior using real time quantitative variables, to establish aspects of the operation equipment in the form of algebraic functions with respect to time.

2.3 Hierarchical criteria organization (W_i): In this phase, the experts should establish the criteria's importance order. Diverse methods for the hierarchical criteria organization exist, such as: simple arrangement method, weighed sum model (WSM), weighed product model (WPM), entropy method, Churchman and Ackoff method and Analytic Hierarchy Process method (AHP).

2.4 Development of the expertsí agreement indicator: In order to establish the agreement level among the experts in hierarchical criteria organization, it is necessary to apply the Kendall Agreement Indicator (W), by using equation 2. According to Siegel (1994) [47] if W ≥ 0.5, the agreement level is accepted; if not, the procedure must be repeated.

$$W = \frac{12\sum D^2}{M^2\left(C^3 - C\right)} \tag{2}$$

Where: W: Kendall Agreement Indicator, M: Number of Experts, C: Number of criteria. D: Medium deviation value of the emitted judgments. (Average differences in absolute values of the averages.)

2.5 Construction of the IIR: The IIR integrates all criteria in a single mathematical function, which allows the establishment of the adequate time for equipment replacement. Likewise, for each criterion, it is necessary to create a mathematical function that describes its behaviour through time. This function can arise from the primary estimations or it can be created from the analysis of historical regression data, complemented with projections of the experts; the IRR goal is to obtain a complete vision of the equipment behaviour during its planned life cycle.

3. RESULTS

In order to explain the application of the procedure shown in Figure 1, in this section the construction of the IIR in a sugar factory, and its application in two machines is presented. The results are the following:

3.1 Experts selection (E_j): The following values were used: p = 1%, k = 6.6564 (confidence level = 95%), i = 0.10. Applicable equation 1, seven experts were required.

3.2 Criteria selection (C_i): Initially, a group of 15 criteria were selected. However, in the successive work sessions, the list was reduced to four criteria. These were: maintenance cost (C), equipment efficacy (E), quality of product processed by the equipment (Q) and equipment obsolescence (O). The equipment efficacy was defined as the Equipment Capability to contribute to the fulfilment of the manufacture objectives, according to the equipment importance; it can be expressed by the productivity, availability or reliability. In the case of the equipment obsolescence, the relative position of the equipment, depends on the technological progress of the industrial sector. In other words, the level of obsolescence obeys to the behaviour of the equipment life cycle. For the case of study, and according to Figure 2, in the axis Y, four modernity levels are proposed, where each level covers 25 points in a total modernity scale of 100 points. The variables C, E, Q, O show a high correlation with the time and it can measure the actual behavior of the machine. Other external factors as people, materials, environmental conditions among others, provide the random behavior of the measured variable and are not taken into account because this model is only relevant for the trend test by each criteria.

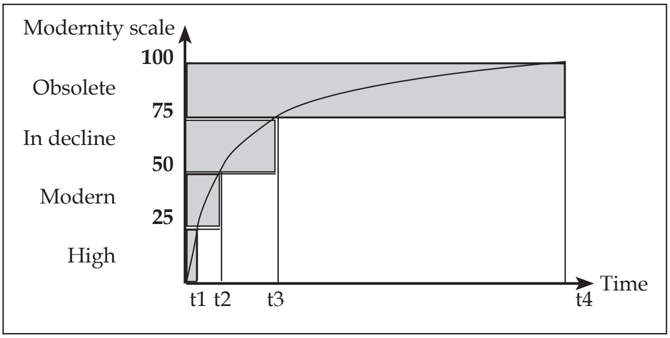

Figura 2. Behaviour of the level of equipment obsolescence

On the other hand, the time length of each modernity level is not equal and depends on the innovative dynamics of the industry (See Axis X). For example, the equipment can be in the high technology level during some months and in the modern technology level during a couple of years. In order to establish the equipment qualification, it is necessary to locate its age on axis X. Subsequently, with the aid of the curve, the obsolescence level can be established.

3.3 Hierarchical criteria organization (Wi): In this case study, the simple arrangement method was applied; the results are shown in Table 1.

Table 1
Criteria Weights

Experts (E_j)	**CRITERIA (i)**				**TOTAL**
	C1	**C2**	**C3**	**C4**	
1	4	2	3	1	10
2	3,5	3,5	2	1	10
3	3	4	2	1	10
4	4	3	2	1	10
5	4	2,5	2,5	1	10
6	4	3	2	1	10
7	4	2	3	1	10
Σ	26,5	20,0	16,5	7,0	Average = 17.5
Weights (Wi)	**0.3786**	**0.2857**	**0.2357**	**0.1000**	Σ Wi =1
D^2	81,0	6,3	1,0	110,3	Σ D^2=198,5

3.4 Development of the experts' agreement indicator: Applying equation 2, the Kendall Agreement Indicator was calculated. The result obtained was of 0.8102, which means that agreement level among experts is high, and therefore, the results of Table 1 are accepted.

3.5 Construction of the IIR: The proposed general functions, in each one of the criteria, were the following:

- Cost of Maintenance: $C = F_1(t)$
- Equipment efficacy: $E = F_2(t)$
- Quality of the product processed by the equipment: $Q = F_3(t)$
- Equipment obsolescence : $O = F_4(t)$

Where Fi(t) is a mathematical function that describes the behavior of the criterion *i* during time *t*. The purpose is to find the best point (time of replacement), that permits the optimization of the equation 3.

$$IIR = W_1 \frac{F_1(t) - Z_{1MIN}}{R_1} + W_2 \frac{F_2(t) - Z_{2MIN}}{R2} + W_3 \frac{F_3(t) - Z_{3MIN}}{R_3} + W_4 \frac{F_4(t) - Z_{4MIN}}{R_4} \qquad (3)$$

Where: Wi: Weight of the criterion i. (i = 1, 2,3,4). Ri: Difference between the "non-ideal point" and the "ideal point" in the associate mathematical function, to the *i* criterion. Zi_{min}: Minimum value of the associate mathematical function to the *i* criterion.

The "ideal point" of an objective function is its optimum value and is represented with Z*; on the contrary, the "non-ideal" point is the most distant point of the optimun value, and it represents a not attractive solution, its representation is Z_*. Including Ri and Zi_{min} in the IIR is the way to normalize and homogenize the functions that compose it. IIR is a mathematical function of non-linear character which should be minimized, taking into account that two criteria must be minimized (F_1 (t), F_4 (t)), the other criteria can be minimized or maximized; this problem is solved, by changing the sign in the above formula.

For the present case, the objective function (IIR) is defined by mean of dependent functions of time. Therefore, the complete mathematical model is the following:

Minimizing:

$$IIR = 0.3786\ \frac{F_1(t) - Z_{1M'n}}{Z_{1*} - Z_1^*} + 0.2857\ \frac{F_2(t) - Z_{2M'n}}{Z_{2*} - Z_2^*} + 0.2357\ \frac{F_3(t) - Z_{3M'n}}{Z_{3*} - Z_3^*} + 0.1\ \frac{F_4(t) - Z_{4M'n}}{Z_{4*} - Z_4^*} \tag{4}$$

Subject to: $t \leq P$ y $t \geq 0$, Where P is the life cycle planned by the equipment supplier.

By using the previous general mathematical models, the specific results obtained in two machines, are presented. These machines are part of the sugar factory production system. The results were the following:

4. RESULTS ANALYSIS: CENTRIFUGAL MACHINE

The useful life time (P) planned by the machine supplier is 20 years. The mathematical functions necessary for each criterion, were built using the regression analysis of historic data, between the years 1997 to 2005, and by using projections between the years 2006 to 2015. Subsequently identified the functions that best described the approach, with respect to time, from a set of models determined by the Startgraphics; a series of ideal functions were obtained with a reliability of 87.02% for the Maintenance cost criterion (Max=23.2, Min=3.12), 94.4% for the criterion of Equipment efficacy (Max=0.99, Min=0.75), 78.75% for the criterion quality of the product processed by the equipment (Max=85, Min=64) and 99.92% for the test equipment obsolescence (Max=0.99, Min=0.1). The maximum and minimum for functions IRR were 0.2622 and 0.063, respectively. The year 1997 corresponds to the moment of the machine purchase; the study was carried out in the year 2005. The equations were the following:

Maintenance cost. In this criterion, the purchase cost was considered (deferred along the useful life of the machine) and the maintenance cost. The model is shown in equation 5.

$$C = F_1(t) = 2.3399 \times 10^6 + \frac{1.0073479 \times 10^7 \times 1.075^t}{-1 + 1.075^t} + 124614 \times t + 41548.1 \times t^2 \tag{5}$$

Equipment efficacy. According to the definition given for this criterion, the model was built in function of the availability, due to its importance for the productive process. Equation 6 represents the obtained model.

$$E = F_2(t) = 0.997517 = 0.00113293t = 0.000514675t^2 \qquad (6)$$

Quality of the product processed by the equipment. This criterion was established in function of the percentage of products rejected per day; the model is in equation 7.

$$Q = F_3(t) = 0.0858737 = 0.00382884t - 0.000168148t^2 \qquad (7)$$

Equipment obsolescence: Using the scale of measure established, the model that describes the centrifuge's life cycle is in equation 8.

$$O = F_4(t) = 0.188403 + 0.06061156t - 0.000993684t^2 \qquad (8)$$

The "non-ideal point" and the "ideal point" for the four equations is presented in Table 2.

Table 2

"non-ideal point" and "ideal point" for the criteria

CRITERION	$F_i(t)$	NON-IDEAL POINT	IDEAL POINT
		Z_{i*}	Z_i^*
C1	$F_1(t)$	146768045	27093300
C2	$F_2(t)$	0.7689884	0.9958
C3	$F_3(t)$	0.082213	0.0640
C4	$F_4(t)$	1	0.2480

Introducing in equation 4 the information in Table 2 and the equations 5, 6, 7 and 8, the function of the IIR, for the centrifuge, is obtained (Seeing the equation 9). Figure 3, shows the plot of this function.

$$IRR = \frac{1}{-1+1.075^t}\left(0.2887 * 1.075^t - 0.2979\right) + \left(0.042 - 0.0042 * 1.075^t\right) * t + (0.002 + 0.002 * 1.075^t) * t^2$$
$$+\left(-4.43 * 10^{-7} + 4.43 * 10^{-7} * 1.075^t\right) * t^3 + \left(-8.52 * 10^{-8} + 852 * 10^{-8} * 1.075^t\right) * t^4 \qquad (9)$$

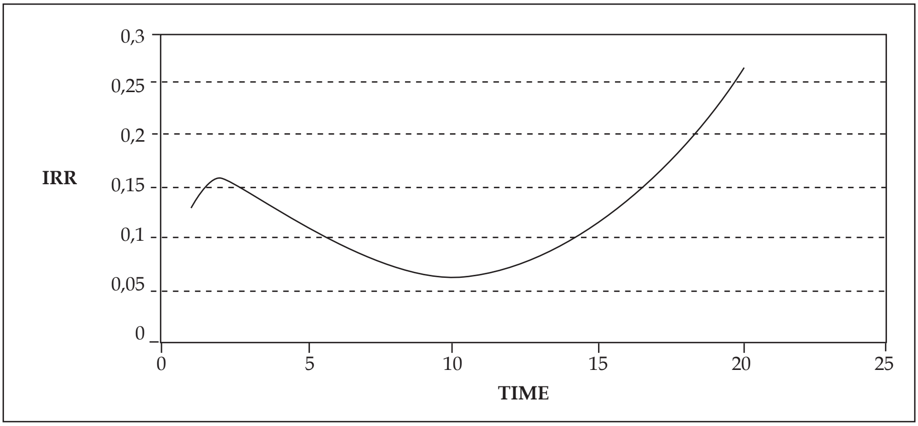

Figura 3. Graphic of the centrifuge IIR

The obtained objective function is the following: Minimizing: IIR Subject to: t ≤ 20, t ≤ 0 By solving the mathematical model, the minimum IIR is 0,063241 when t =10.104. This indicates, that the optimum centrifuge useful life is obtained at 10 years, 1 month and 8 days.

5. RESULTS ANALYSIS: TURBINE

Taking into account the characteristics of the equipment and its role in the productive process, it was established that "The quality of the product processed by the equipment" was not a prominent criterion; in this case, the turbine does not participate directly in the production process and only acts as a service equipment. For this reason, the procedure of Figure 1 was repeated with the three remaining criteria. Table 3 presents the new hierarchical organization of the criteria. By a similar process, (See case 1) the ideal functions, that best described the approach, were identified with respect to time. From a set of models determined by the Startgraphics; a series of functions with a reliability of 99.43% were obtained, for the Maintenance cost criterion (Max=0.8, Min=0.1), 99.81% for the criterion of equipment efficacy (Max=29.1, Min=3.1), and 99.3% for the test equipment obsolescence (Max=0.99, Min=0.1). The maximum and minimum for IRR functions were 0.2287 and 0.059, respectively.

Table 3
Prominent criteria in the construction of the IIR for the turbine

CRITERIA	W_i
Maintenance cost (C1)	0.4954
Equipment efficacy (C2)	0.3738
Equipment obsolescence (C4)	0.1308

The Kendall Agreement Indicator was 0.92 and therefore, the results of Table 3 were accepted. The new general equation for the IIR is the following:

$$IIR = 0.4954\frac{F_1(t)-Z_{1M'n}}{Z_{1*}-Z_1^*} + 0.3738\frac{F_2(t)-Z_{2M'n}}{Z_{2*}-Z_2^*} + 0.1308\frac{F_4(t)-Z_{4M'n}}{Z_{4*}-Z_4^*} \quad (10)$$

Just like the centrifuge, the useful life time (P) planned by the turbine's supplier is 20 years. The mathematical functions for each criterion, was built using the regression analysis of historic data, between the years 1995 to 2005 and by using projections between the years 2006 to 2015. The year 1995 corresponds to the time of the machine purchase; the study was carried out in the year 2005. The equations were the following:

Maintenance cost. In the same way as the centrifuge, in this criterion, the differed purchase cost and the maintenance cost were considered. The model is shown in equation 11.

$$C = F_1(t) = 79893.3 + \frac{2.97363*10^6*1.075^t}{-1+1.075^t} + 72741.6*t + 16557.5*t^2 \quad (11)$$

Equipment efficacy. For the turbine, the reliability was selected as the most important attribute. The obtained mathematical model, is the following:

$$E = F_2(t) = e^{-3.65*t+0.213585*t^2-0.0285373*t^3} \quad (12)$$

Equipment obsolescence. According to the scale, the mathematical function that better describes the turbine life cycle is the following.

$$O = F_4(t) = 0{,}7975 + 0{,}01125*t - 0{,}00015625*t^2 \quad (13)$$

The “non-ideal point” and the “ideal point”, for the three equations, are presented in the Table 4.

Table 4

“non-ideal point” and “ideal point” for the criteria

CRITERION	$F_i(t)$	NON-IDEAL POINT	IDEAL POINT
		Z_{i*}	Z_i^*
C1	$F_1(t)$	42791222,36	8238836,58
C2	$F_2(t)$	0	0.0313
C4	$F_4(t)$	0.96	0.8116

Equation 14 is the mathematical function that describes the IIR for the turbine. Figure 4, shows the graphic for this function:

$$\begin{aligned} \mathrm{IIR} &= 0.4957\left(79893.3+\frac{2.97363*10^6*1.075^t}{-1+1.075^t}+72741.6*t+16557.5*t^2\right) \\ &+0.3738\left(e^{-3.65*t+0.2135*t^2-0.0285*t^3}\right)+0.1308\left(0{,}7975\ +\ 1{,}125*10^2*t-1{,}5625*10^4*t2\right) \end{aligned} \tag{14}$$

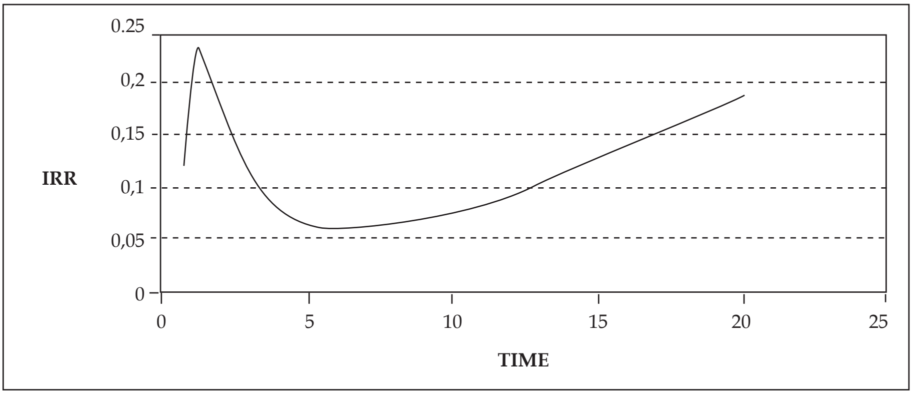

Figura 4. Graphic of the turbine IIR

The obtained objective function is the following: Minimizing: IIR, Subject to: t ≤ 20, t ≤ 0

By solving the mathematical model, the minimum IIR is 0,0594686, when t = 6.85.

6. CONCLUSIONS

In spite of the fact that, related to time, contributions of diverse authors in the theme of equipment replacement have evolved and have achieved important advances, the majority of the existing replacement models, generally, make the decision based on a single criterion, excluding other variables of non-technological and technological type, which describe the real behaviour of the equipment; the models based on a single criterion, do not permit to reach a more realistic and holistic vision of the machine operation conditions. Nevertheless, the present article shows that, applying multi-criteria techniques, it is possible to establish the adequate equipment replacement moment, involving in the decision a set of prominent criteria. The multi-criteria techniques are an important tool of the operation research that allows the solution of the incompatibility problem among decision variables that are expressed in different units of measure, facilitating the use of different criteria, which can be arranged hierarchically according to the expertsí judgment. The proposed Integral Replacement Indicator (IIR), is a decision tool that allows to group a set of weighed, normalized and homogenized functions, which represent the criteria behavior related to time. This indicator allows the establishment of the equipment replacement time in technological and economic terms, based on various prominent criteria. In the specific case of the centrifuge and the turbine, by using the IIR, it was possible, to establish an approximate period for their replacement, which becomes an important information to facility management. In the case of the studied enterprise (sugar factory), the criteria which, according to the expertsí judgment, should be taken into account for the replacement decision, are the following: maintenance cost, equipment efficacy, quality of the product processed by the equipment and equipment obsolescence. In other cases, according to the equipment characteristics, other criteria should be considered, such as: the operator security restrictions, the environmental impact and the useful life time planned by the machine supplier. In fact, the results shown in the present article are valid only for the studied enterprise. However, the procedure can be applied and adapted in other industrial sectors, by using the criteria that is prominent in each case, but the results reliability depends, on the one hand, on the expertsí judgment quality and, on the other hand, on the reliability of the utilized data to establish the mathematical functions.

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