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Contract pricing evaluation of distributed generation: a game theory approach

Evaluación de precios de contrato de generación distribuida: una metodología basada en teoría de juegos

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Abstract

The main objective of this research is the contract pricing calculation of distributed generation (DG) considering its interaction with the distribution company. The proposed methodology consists in modeling the DG energy trading using game theory concepts and bilevel programming. To validate the proposed methodology several tests are carried out with a 34 bus distribution system, changing the number and size of the DG units. The contract prices calculated with this methodology represent an equilibrium between the profit maximization pursued by the DG owner and the minimization of payments procured by the distribution company. Consequently, it can be concluded that the proposed methodology provides efficient incentives for both agents, providing a solution in which the DG and the distribution company are mutually beneficial. Furthermore, the location of the DG units can also be considered in the model, so that not only the contract price but also the equilibrium location can be found.

Key words: Distributed generation, bilevel programming, game theory.

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Resumen

El objetivo principal de esta investigación consiste en calcular los precios de contrato de la generación distribuida (GD) considerando su interacción con la compañía distribuidora. La metodología propuesta consiste en modelar la dinámica de compra y venta de energía de GD usando conceptos de teoría de juegos y programación binivel. Para validar la metodología propuesta se realizaron varias pruebas en un sistema de distribución de 34 barras cambiando el número y el tamaño de las unidades de GD. Los precios de contrato calculados con esta metodología representan un equilibrio entre la maximización de utilidades que busca el propietario de la GD y la minimización de pagos que busca la empresa distribuidora. De esta forma, se concluye que la metodología propuesta en este artículo provee incentivos eficientes para ambos agentes, entregando una solución en la que la GD y la empresa distribuidora se benefician mutuamente. Adicionalmente, la ubicación de las unidades de GD también puede considerarse dentro del modelo y de esta manera encontrar, no solo el precio de contrato, sino también la ubicación de equilibrio.

Palabras clave: Generación distribuida, programación binivel, teoría de juegos.

1. INTRODUCTION

The electricity industry restructuring, along with advances in small scale generation technologies, and a higher awareness of environmental issues are the key factors that have motivated the development of distributed generation (DG) in the last decade [1]. In this scenario, several studies have been conducted to face the new challenges imposed by the DG integration in the electricity grid. Such studies include modifications on power flow techniques to account for DG in power systems [2]-[3]; the assessment of DG impacts in the distribution network [4]-[5] and the optimal sitting and sizing of DG units [6]-[7]. Economic issues have also been the focus of several studies and have been widely discussed in [8] and [9]. Most of the studies regarding the integration of DG are developed from the standpoint of the distribution company. These studies are conducted in order to maximize the potential benefits of DG. The approach presented in this paper considers both, the interest of the distribution company and the interest of the DG owners. In this sense, we have envisaged a scenario with a high penetration of DG, in which different DG owners might compete to sell energy to the distribution company. As a result of this competition, the DG contract prices can be calculated using a game theory approach. Thus, it is considered a set of DG owners that strive to sell energy to a distribution company. Such situation is modeled as a non cooperative game in which the strategies of the DG owners are represented by the contract prices at which they are willing to sell their energy. Given a set of energy price offers, the distribution company decides over the amount of energy to be purchased from the DG units and the wholesale energy market. Such decision process is performed trough an Optimal Power Flow (OPF)-based dispatch, and has been developed under a bilevel programming framework.

A bilevel programming problem (BPP) is composed by an inner and an outer optimization problem. In this case, the outer problem corresponds to the optimization performed by the DG owners. Every DG owner considers the contract prices of the other DG owners as fixed, and solves his own optimization problem aiming to maximize his profits. The inner optimization problem corresponds to the distribution company, which given the set of contract price offers, procures the minimization of the energy payments. A BPP is equivalent to a Stackelberg game defined

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by two agents: the leader and the followers. Initially, the leader makes his move first, anticipating the reaction of the followers, and then, the followers move sequentially reacting to the leader`s strategy [10]. In this case, the leader is the DG owner who makes his move first providing a contract price offer, and the distribution company is the follower.

The solution of the game is achieved by finding the Nash equilibrium. Such equilibrium is defined as a combination of strategies in which no player can benefit from changing his strategy unilaterally, provided that the other players keep their strategies unchanged. In this case, the Nash equilibrium is found using the specialized software GAMBIT [11], for this, a payoff matrix with the profits of the DG owners for each combination of contract price offers is built.

The contributions of this paper are threefold:

- It provides a game theory approach for the contract pricing evaluation of dispatchable distributed generation.

- A bilevel programming framework is developed to explicitly consider the reaction of the distribution company.

- Besides contract pricing, the proposed approach can be easily expanded to consider the location of the DG units as part of the DG owner's set of strategies.

2. MARKET STRUCTURE

With the unbundled operation of electric power systems, retailers emerged to fill the gap between the wholesale energy market and small consumers. In some markets the distribution company can also play the role of a retailer. To meet the expected demand, the distribution company purchases energy from the point of interconnection with the transmission system, known as substation. Figure 1 illustrates the market structure considered in this paper. Note that, apart from the wholesale energy market, the distribution company can also buy energy from the DG units (DG1 and DG2) located within its network and owned by independent producers.

Depending on specific market rules, wholesale market prices might be time and space varying. In this case, the distribution company receives the contract price offers of DG1 and DG2 and also knows the wholesale market prices at substations A and B. With this information, the distribution company must decide on the amount of energy to be purchased from the wholesale electricity market and from the DG units. This decision making process is not trivial, since it must consider not only the price signals but also the impact of the DG units in the network. For example, if the power injected by a DG unit has a negative impact in the voltage profile or increases power losses significantly; then, such DG unit would not be dispatched, even if its contract price offer is lower than the wholesale market price. Conversely, if the power injected by a DG unit contributes to the enforcement of a voltage constraint and/or has a positive impact reducing power losses, then, even if the DG contract price offer is slightly higher than the wholesale market price, the DG unit is likely to be dispatched. This decision making process is performed by means of an AC OPF-based dispatch.

On the other hand, DG owners are concerned with finding the contract price offers that would render maximum profits. For this, every DG owner must consider, not only the reasoning performed by the distribution company, but also the most likely contract price offers of the other DG owners.

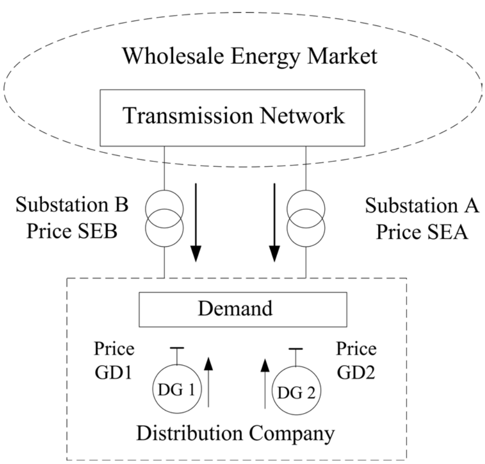

Figure 1. Market structure

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3. METHODOLOGY

This section describes the mathematical model of the bilevel programming formulation, some game theory basis, and an illustrative example.

Bilevel programming formulation

The mathematical formulation of the bilevel programming problem is given by the set of equations (1) to (9). The objective function given by equation (1) represents the maximization of profits procured by the DG owners, where where λ_{DGj} is the contract price of DG unit j in $/MWh; c_{DGj} is the production cost of DG unit j in $/MWh; $P_{DGj}(t)$ is the active power supplied by the DG unit j in period t in MW; Δ_t is the length of the time interval t in hours; T is the set of time intervals and J is the set of indices of DG units. The objective function, given by equation (2), represents the minimization of energy payments procured by the distribution company, where K is the set of substations and $\rho_{SEk}(t)$ is the wholesale energy price at substation k in period t in $/MWh. Such minimization is in turn, subject to constraints (3)-(9). Equations (3) and (4) represent the active and reactive power balance constraints, respectively. In this case, $P_{Gn}(t)$ and $Q_{Gn}(t)$ are the active and reactive power generated in bus n in period t, respectively; $P_{Dn}(t)$ and $Q_{Dn}(t)$ are the active and reactive power demand in bus n in period t, respectively; $P_n(t)$ $Q_n(t)$ are the active and reactive power injections calculated in bus n in period t, respectively; and N is the set of indices of network nodes.

Equations (5) and (6) represent the active and reactive power limits of the substations, where P_{SEK}^{Min} and P_{SEK}^{Max} represent the minimum and maximum active power limits of substation k, respectively; Q_{SEK}^{Min} and Q_{SEK}^{Max} represent the minimum and maximum reactive power limits of substation k, respectively; finally, $P_{SEK}(t)$ and $Q_{SEK}(t)$ represent the active and reactive power provided by substation k in period t, respectively. Equation (7) represents the active power limits of the DG units, where P_{GDj}^{Min} and P_{GDj}^{Max} are the minimum and maximum active power limits of DG unit j. Equation (8) represents voltage limits, where V_n^{Min} and V_n^{Max} are the minimum and maximum voltage limits in bus n; and $V_n(t)$ is the voltage of bus n

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in period *t*. Equation (9) accounts for power flow limits, where S_{lmn} is the power flow in the line connecting nodes *n, m* in period *t*, and S_{lmn}^{Max} is the maximum power flow limit in the same line.

$$\underset{\lambda_{DGj}}{Max} \sum_{t \in T} \sum_{j \in J} \Delta_t \left(\lambda_{DGj} - c_{DGj} \right) P_{DGj}(t) \qquad (1)$$

Subject to:

$$\underset{P_{DGj}(t), P_{SEK}(t)}{Min} \sum_{t \in T} \sum_{k \in K} \Delta_t \rho_{SEK}(t) P_{SEK}(t) + \sum_{t \in T} \sum_{j \in J} \Delta_t \lambda_{DGj} P_{DGj}(t) \qquad (2)$$

Subject to:

$$P_{Gn}(t) - P_{Dn}(t) - P_n(t) = 0;\ \forall n \in N, \forall t \in T \qquad (3)$$

$$Q_{Gn}(t) - Q_{Dn}(t) - Q_n(t) = 0;\ \forall n \in N, \forall t \in T \qquad (4)$$

$$P_{SEk}^{Min} \leq P_{SEk}(t) \leq P_{SEk}^{Max}; \forall k \in K, \forall t \in T \qquad (5)$$

$$Q_{SEk}^{Min} \leq Q_{SEk}(t) \leq Q_{SEk}^{Max}; \forall k \in K, \forall t \in T \qquad (6)$$

$$P_{DGj}^{Min} \leq P_{GDj}(t) \leq P_{DGj}^{Max}; \forall j \in J, \forall t \in T \qquad (7)$$

$$V_n^{Min} \leq V_n(t) \leq V_N^{Max}; \forall n \in N, \forall t \in T \qquad (8)$$

$$-S_{lmn}^{Max} \leq S_{lmn}(t) \leq S_{lmn}^{Max}; \forall l_{mn} \in L, \forall t \in T \qquad (9)$$

Power injections in equations (3) and (4) are given by equations (10)-(11) as shown below. In this case and are the real and imaginary parts of element *m, n* of the admittance matrix, respectively; and is the angle between nodes *m, n*.

$$P_n = V_n \sum_{n \in N} V_m \left[q_{nm} \cos(\theta_{nm}) + b_{nm} \sin(\theta_{nm}) \right] \qquad (10)$$

$$Q_n = V_n \sum_{n \in N} V_m \left[q_{nm} \cos(\theta_{nm}) + b_{nm} \sin(\theta_{nm}) \right] \qquad (11)$$

Apparent power is given by its active and reactive components as shown in equations (12)-(14), where and are the active and reactive power flows in line connecting nodes *n, m*.

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$$S_{lmn} = P_{lmn} + jQ_{lmn} \tag{12}$$

$$P_{lmn} = V_n^2 q_{nm} - V_n V_m q_{nm} \cos(\theta_{nm}) - V_n V_m b_{nm} \sin(\theta_{nm}) \tag{13}$$

$$Q_{lmn} = V_n^2 b_{nm} + V_n V_m b_{nm} \cos(\theta_{nm}) - V_n V_m q_{nm} \sin(\theta_{nm}) \tag{14}$$

Game theory basis

Game theory is a branch of applied mathematics that studies strategic situations involving decision-making among individuals. This theory was initially developed as a tool to understand economic behavior [10]. However, since the 1970s game theory has been applied in various fields such as political science, war strategy, auctions, ethics, philosophy, and computer science. In a game, the welfare of a player depends not only upon his own actions, but also on the actions of the other participants. In normal form, an *n* person game can be defined as the three-tuple given by (15).

$$\{N,(X_i),(\varphi_i),i \in N\} \tag{15}$$

Where, $N = \{1, 2, 3\ldots n\}$ is the set of players (in this case the DG owners); X_i is the set of strategies of player *i*, (contract price offers); and $\boldsymbol{\varphi}_i$ is the payoff function of player *i* that assigns a real number to each element of the Cartesian product of the strategy space $X1 \times X2 \times X3 \ldots \times Xn$. In this case, the Cartesian product represents all possible contract price offer combinations. For each of them, the payoff function, (that is, the profits of the DG units given by (1)), is obtained by solving the OPF-based economic dispatch described by (2)-(14). Subsequently, a payoff matrix is built and the specialized software GAMBIT [11] is used to find the Nash Equilibrium, in this point, no player can improve his individual payoff by unilaterally changing his current strategy.

Illustrative example

Consider the 10 bus distribution system depicted in Figure 2. This distribution system has two distributed generation units labeled as DG1 and DG2 located at buses 2 and 10, respectively. Every DG unit has a capacity of 2 MW and an operation cost of 60 $/MWh. Suppose that the

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wholesale market price is 60 $/MWh. For the sake of simplicity, without loss of generality, a single time interval will be considered. The demand of the distribution system, for every node is 1 MW with a lagging power factor of 0.948. The impedance is considered to be (0.001 +0.001j) ohm.

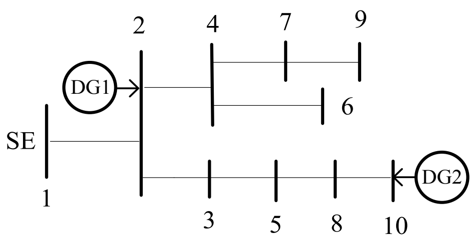

Figure 2. 10-bus distribution system

Figure 3 illustrates the Locational Marginal Prices (LMP) of the distribution system without distributed generation. Such prices (given in $/h) correspond to the dual variables of the active power balance constraint (equation (3)) and represent the cost of providing an additional Megawatt to a particular bus. Note that despite of the fact that the wholesale market price is 60 $/MWh, the marginal prices of buses 8 to 10 are above 68$/h. This means that buying energy from these buses at any price lower than 68$/MWh represents savings to the distribution company.

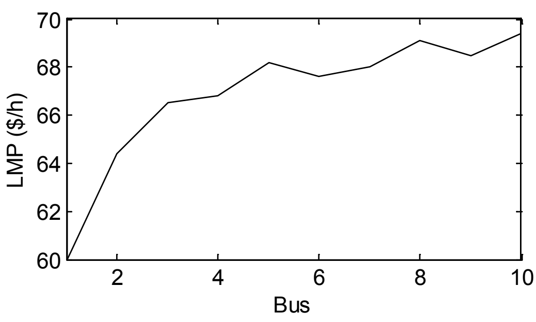

Figure 3. Locational marginal prices

The contract price offer strategies of DG1 are: 62, 64 and 66 $/MWh; while those of DG2 are: 62, 64, 66, 68, and 70 $/MWh. For each combination of contract price offers a payoff is calculated (solving (1)-(9)) and the payoff matrix shown in Table 1 is obtained. The profits of DG1 and DG2 are given

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in pairs. For example, the entry (4 – 8) means that the profits of DG1 and DG2 are $ 4 and $ 8, respectively. It can be observed that when DG1 and DG2 offer their energy at 62$/MWh the profits for each DG unit is $4. That is, every unit sells 2 MW at a price given by 62 $/MWh with operating costs of 60 $/MWh. Note that if DG1 and DG2 offer their energy at 66 and 70 $/MWh, respectively, their profits are zero.

In this case, the Nash equilibrium can be found using the concept of dominance. A given strategy is said to be dominant if, no matter the strategies of the other players, the welfare obtained by choosing such strategy is always the highest. In this case, the dominant strategy for DG1 is 64 $/MWh, with a minimum profit (in the worst case) of 4.8 $. A higher price offer of 66 $/MWh would result in zero profits (see last column of Table 1), while a lower one of 62 $/MWh would always result in profits of $ 4. Similarly, the dominant strategy for DG2 is 68 $/MWh. The intersection of the dominant strategies is the Nash equilibrium, marked with an asterisk in Table 1. In this point there is no incentive for any of the players to unilaterally change its strategy. It is worth to mention at this point that, finding a Nash equilibrium is not an easy task, since in many cases it is not always possible to identify dominant strategies. Consequently, in most cases, in order to find a Nash equilibrium it is necessary to solve a set of polynomial equations or use numerical methods [12]. Several factors might be responsible for the problem not having dominant strategies. Such factors include the number of players and strategies. The more strategies (or players, or both) the more difficult would be to find, for a given player, a strategy that would render maximum benefits no matter what the other players do.

Table 1

Payoff matrix for different contract price offers of DG1 and DG2

	DG1 offer: 62 $/MWh	**DG1 offer: 64 $/MWh**	**DG1 offer: 66 $/MWh**
DG2 offer: 62 $/MWh	$ (4 - 4)	$ (4.8-4)	$ (0-4)
DG2 offer: 64 $/MWh	$ (4 - 8)	$ (4.8-8)	$ (0-8)
DG2 offer: 66 $/MWh	$(4 - 12)	$ (4.8-12)	$ (0-12)
DG2 offer: 68 $/MWh	$ (4-12.8)	$ **(6.1-13.7)***	$ (0-15.8)
DG2 offer: 70 $/MWh	$ (4 - 0)	$ (8 - 0)	$ (0 - 0)

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In the following examples we proceed as follows:

First, a set of strategies is defined for each DG owner. Such strategies are the contract price offers at which DG owners are willing to sell their energy. Then, for each combination of these strategies, the reaction of the distribution company is computed by solving the inner optimization problem (optimal power flow given by equations (2)-(9)). Once the inner optimization problem is solved, the profits of the DG owners are computed considering their production cost and the amount of energy sold. With the profits for each combination of strategies a payoff matrix, similar to the one presented in Table 1, is built. Such matrix is introduced to the software GAMBIT that computes the Nash equilibrium.

4. TEST AND RESULTS

In order to show the applicability of the proposed approach, several tests were carried out with the 34 bus distribution system shown in Figure 4. The line data of this system can be consulted in [13]. Figure 5 depicts the load distribution of the system. Note that most of the load is concentrated in buses far away from the substation. The contract pricing of the DG units has been considered for one year. Figure 6 depicts the load duration curve of the expected demand and Figure 7 shows the wholesale market prices. Note that Figures 6 and 7 exhibit a similar shape. That is because higher prices on the wholesale market are expected to take place precisely during peak hours; conversely, lower prices are expected during off-peak hours.

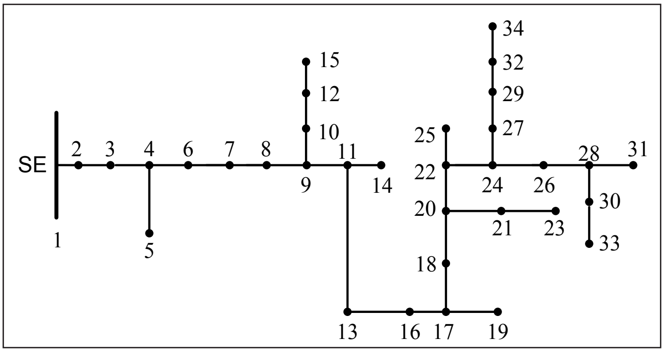

Figure 4. 34-bus distribution system

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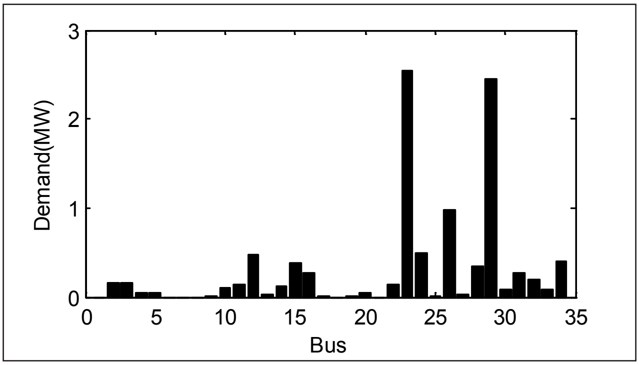

Figure 5. Load distribution.

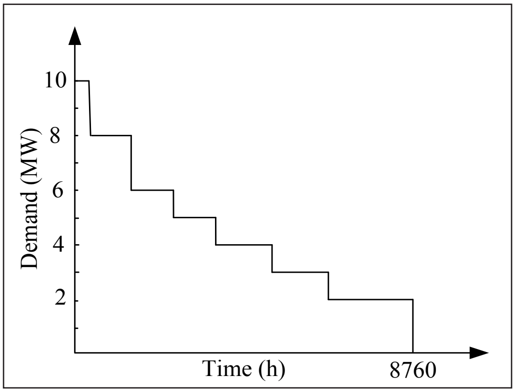

Figure 6. Load duration curve

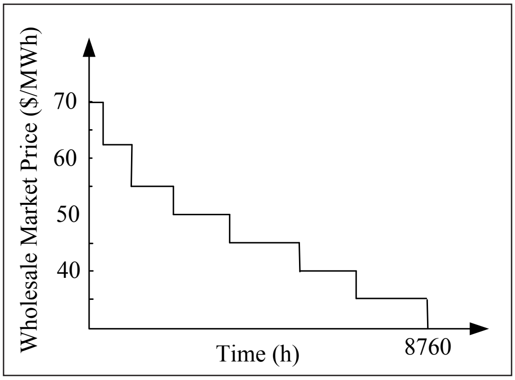

Figure 7. Wholesale market prices

In order to illustrate the effect of competition on equilibrium prices, several tests were performed by changing the quantity and capacity of DG units.

Case 1. Two DG units

Initially, two DG units (labeled as DG1 and DG2) with operation costs of 60 $/MWh, located in buses 19 and 34, respectively, are considered. To illustrate the effect of DG size in equilibrium prices, four cases were analyzed with different DG sizes ranging from 0.5 MW to 2.0 MW. For each test, strategies ranging from 65 to 75 $/MWh, at intervals of 0.1 $/MWh, were considered.

Tables 2 and 3 show the equilibrium contract prices and profits of the DG units obtained with the proposed approach. It can be observed in Table 2 that, as the size of the DG units increase, the contract equilibrium prices decrease. Also note that the contract price of DG2 (located in bus 34) is always higher than the one of DG1. Consequently, DG2 always gets higher profits as can be observed in Table 3. This situation occurs because DG2 is located farther away from the substation than DG1, and given the load distribution of the network (see Figure 5), its contribution to power loss reduction and improvement of voltage profile is greater than the one provided by DG1. As a consequence, the distribution company has preference for DG2 when deciding over the dispatch of the DG units.

Table 2

Equilibrium contract prices for case 1

	0.5 MW	1.0 MW	1.5 MW	2.0 MW
DG1	71.6 $/MWh	70.6 $/MWh	69.6 $/MWh	68.7 $/MWh
DG2	73.0 $/MWh	71.8 $/MWh	70.6 $/MWh	69.5 $/MWh

Table 3

Profits of the DG units for case 1

	0.5 MW	1.0 MW	1.5 MW	2.0 MW
DG1	$ 12702.0	$ 23214.0	$ 31536.0	$ 37821.4
DG2	$ 14235.0	$ 25842.0	$ 34821.0	$ 41161.7

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Case 2. Three DG units

In this case we introduce a third DG unit in bus 14, labeled as DG3, and with the same production cost as the other two units. The new equilibrium contract prices and profits are shown in Tables 4 and 5, respectively. Note that, due to competition among the DG units, equilibrium prices, in all cases, are lower as compared with those shown in Table 2, consequently the profits of DG1 and DG2 also decrease as shown in Table 5.

Table 4
Equilibrium contract prices for case 2

	0.5 MW	1.0 MW	1.5 MW	2.0 MW
DG1	71.0 $/MWh	69.8 $/MWh	68.6 $/MWh	67.3 $/MWh
DG2	72.5 $/MWh	71.0 $/MWh	69.5 $/MWh	68.2 $/MWh
DG3	70.0 $/MWh	69.0 $/MWh	68.0 $/MWh	67.1 $/MWh

Table 5
Profits of the DG units for case 2

	0.5 MW	1.0 MW	1.5 MW	2.0 MW
DG1	$ 12045.0	$ 21462.0	$ 28251.0	$ 31974.0
DG2	$ 13687.5	$ 24090.0	$ 31207.5	$ 35916.0
DG3	$ 10950.0	$ 18898.2	$ 24811.2	$ 30673.5

Note that, in all cases, DG3 always gets the lowest contract price equilibrium and the lowest profits. That is because this unit has not been strategically located in the network. If DG3 had been positioned in a different bus, it might have gotten higher profits.

Case 3. Location and contract price equilibrium

Cases 1 and 2 showed the importance of DG location in contract prices and profits. DG units located in strategic buses are able to obtain greater profits than DG units located in nonstrategic buses. Bearing this in mind, the location of the DG units has also been considered as part of the set of strategies. In this case, DG units compete among them to find the locations and contract prices that would render them maximum profits.

Including the location in the set of strategies significantly increases the combinations of strategies to be evaluated. Consequently, in order to limit the search space, only nodes from 20 to 34 are considered to be suitable for DG location. Also, the set of contract price offers is considered to vary from 65 to 70 $/MWh at intervals of 1 $/MWh. Such considerations reduce significantly the combination of strategies to be evaluated. Note that it is not reasonable to evaluate locations of DG near the substation, since their energy is more valuable when they contribute to the reduction of power losses. Such reduction is higher when DG units are located far from the substation or at the end of heavily loaded feeders.

Tables 6 and 7 show the equilibrium contract prices and locations for two and three DG units, respectively. In both cases the capacity of the DG units is 2.0 MW and a production cost of 60 $/MWh is considered. Note that when the number of DG units increase from 2 to 3, the equilibrium contract prices reduce from 68 to 67 $/MWh. Such reduction is due to competition.

Table 6

Equilibrium contract prices and locations for two DG units

	Bus	Price ($/MWh)	Profits ($)
GD1	24	68	39216.7
GD2	27	68	40327.4

Table 7

Equilibrium contract prices and locations for three DG units.

	Bus	Price ($/MWh)	Profits ($)
GD1	24	67	32916.3
GD2	28	67	32617.8
GD3	29	67	32145.6

5. CONCLUSIONS

A game theory approach for the contract pricing evaluation of DG is presented. The proposed approach is envisaged under a market structure in which the distribution company can purchase energy either from the wholesale market or from the DG units located within its network. The

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main contribution of the paper consists in the modeling of the DG energy trading using game theory concepts and a bilevel optimization framework. From a regulatory point of view, the proposed model provides efficient incentives to both, the distribution company and DG owner. This is because their objective functions have been explicitly considered in the model.

It was observed that when the size and/or number of DG units increases, the corresponding equilibrium contract prices reduce. It can be concluded that competition among DG units benefits the distribution company which can purchase energy at lower prices. Besides contract prices, it was shown that the proposed approach is suitable for considering also the location of the DG units. Future work will include other market structures as well as the active participation of demand.

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