Methodology for Inventory Management in Neighborhood Stores Using Machine Learning and Integer Linear Programming

Metodología para la gestión de inventario en tiendas de barrio utilizando aprendizaje de máquina y programación lineal entera

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Abstract

Context: Nowadays, inventory management poses a challenge given the constant demands related to temporality, geographic location, price variability, and budget availability, among others. In neighborhood shops, this process is manually done based on experience (the data generated are ignored), which is sometimes not enough to respond to changes. This shows the need to develop new strategies and tools that use data analysis techniques.

Method: Our methodology predicts the weekly demand for 14 common products in neighborhood stores, which is later refined based on investment capital. The method is validated using a database built with synthetic information extracted from statistical sampling. For the prediction model, three supervised learning models are used: support vector machines (SVM), AutoRegressive models (Arx), and Gaussian processes (GP). This work proposes a restricted linear model given an inversion and the predicted quantity of products; the aim is to refine the prediction while maximizing the shopkeeper's profit. Finally, the problem is solved by applying an integer linear programming paradigm.

Results: Tests regarding the prediction and inventory adjustment stages are conducted, showing that the methodology can predict the temporal dynamics of the data by inferring the statistical moments of the distributions used. It is shown that it is possible to obtain a maximum profit with a lower investment.

Conclusions: Our method allows predicting and refining inventory management in a neighborhood store model where quantities are managed to maximize the shopkeeper's profits. This opens the way to explore this approach in a real scenario or to introduce new techniques that can improve its performance.

Keywords: machine learning, inventory, constrained optimization, demand estimation

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Resumen

Contexto: En la actualidad, la administración del inventario representa un reto dadas las constantes exigencias de temporalidad, ubicación geográfica, variabilidad en los precios, disponibilidad presupuestal, entre otros. En las tiendas de barrio, este proceso se realiza de forma manual y con base en la experiencia (se ignoran los datos generados), lo que en ocasiones no es suficiente para responder a los cambios. Esto muestra la necesidad de desarrollar nuevas estrategias y herramientas que utilicen técnicas de análisis de datos.

Método: Nuestra metodología predice la demanda semanal para 14 productos comunes en tiendas de barrio, la cual se refina posteriormente en función del capital de inversión para optimizar la ganancia. El método se valida a través de una base de datos construida con información sintética extraída a partir de muestreo estadístico. Para la predicción, se utilizan tres modelos de aprendizaje supervisado: máquinas de soporte vectorial (SVM), modelos AutoRegresivos (ARx) y procesos Gaussianos (GP). Luego, se plantea un modelo lineal restringido dada una inversión y las cantidades pronosticadas; el propósito es refinar la predicción maximizando la ganancia del tendero. Finalmente, el problema se soluciona aplicando un paradigma de programación lineal entera.

Resultados: Se realizan pruebas para las etapas de predicción y ajustes del inventario, donde se demuestra que la metodología logra predecir la dinámica temporal de los datos infiriendo los momentos estadísticos de las distribuciones utilizadas. Se muestra que es posible obtener una máxima ganancia con un monto menor de inversión.

Conclusiones: Nuestra metodología que permite predecir y refinar la gestión de inventario en un modelo de tienda de barrio en el que las cantidades se administran para maximizar las ganancias del tendero. Lo anterior abre el camino para explorar este enfoque en un escenario real o introducir nuevas técnicas que puedan mejorar su desempeño.

Palabras clave: aprendizaje de máquina, inventario, optimización restringida, estimación de la demanda

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1. Introduction

Over the years, the world economy has oriented its markets towards globalized supply and demand. In the case of Colombia, this process started in the 90s with the beginning of the economic opening, when new competitors were allowed to enter the market, many of which were foreign producers. Due to this situation, several economic sectors disappeared, as it was not possible to compete with the wide range of products and prices in offer (1,2).

In light of the above, a large number of international companies entered the Colombian market, especially in the retail sector, which represents 56 % of retail sales. Within this business paradigm, large sales chains such as Éxito, Carrefour (today Jumbo), Carulla, Olímpica, Super Inter, and Makro stand out. These companies assess the procurement of supplies based on efficiency indicators, aiming to maximize the return on investment. However, there is a second model that concerns the neighborhood convenience stores of the traditional channel. The most noticeable difference between these two models corresponds to administration, which is carried out by the owner or head of the family unit in the second case (1,2).

Despite stiff competition, neighborhood stores have survived due to their closeness to the final consumer, their trust and empathy with customers, and the payment methods offered (interest-free credit). Neighborhood stores have been hit hard by the advent of large discount store chains, such as ARA, D1, and Justo & Bueno, which aim to get closer to the end customer and compete with low prices (3, 4). Therefore, the traditional channel in Colombia must improve, update, and transform its processes, specifically in making decisions on the projection of supply and demand, as well as in inventory rotation (5).

In general terms, the store administrator manages the investments in the purchase of the products offered according to their expertise and knowledge acquired from the market. It should be stressed that this is a subjective forecasting method based on experience. This strategy generates a high bias in facing the uncertainties presented by changes in the demand for certain items according to the season and the economic variations in prices, state policies, and social events (6).

Likewise, it has been identified that demand forecasting is a common problem in supply chains. Here, the objective is to achieve greater precision and less error. According to (7), given the diversity of demand patterns, it is unlikely that any single demand forecasting model can offer the highest accuracy across all items. On the other hand, managers frequently have more information that should be considered to improve forecast results. For example, they know customer habits and purchase intentions, among others (8). A better forecast should include all this information. However, this requires more advanced methods than those offered by the prediction techniques of machine learning.

Considering the above, artificial intelligence (AI) and machine learning techniques have been proposed in the literature, aiming to reduce bias and provide business units with decision-making tools under a data-based paradigm, largely because AI models allow them to carry out forecasting with

regard to the demand for items (9). In the state of the art, two major stages are identified in which these procedures are focused. The first involves collecting and characterizing the data, with the purpose of describing the behavior of the products and its variability over time. In the second stage, different machine learning techniques are used to estimate computational models that can predict and reproduce the dynamics of the data, so that the sales dynamics can be emulated over time for a set of specific items (10–13).

Usually, the forecast of pure units or goods follows a uniform or normal probability distribution (14). More specialized developments introduce planning aspects that drive the supply chain as customer expectations increase, shortening delivery times, and pressure for resource optimization (15). On the other hand, demand forecasting is generally carried out using machine learning techniques. An example of this is the use of neural networks, which emulate the forecast of items through a non-linear function in time. Historical sales data are included in the network design in combination with the effectiveness of advertising, promotions, and marketing events. However, this model is sensitive to the network architecture (*i.e.*, the number of hidden layers and the value of neurons per layer), which affects the generality of the method (16). Other methods also predict demand, such as the Auto Regressive Moving-Average (ARMA), Support Vector Machine (SVM), Multiple Linear Regression (MLR), and Random Forest models, showing competitive results adjusted to reality. Still, this approach predicts the quantities of products required without considering the investment and profit capabilities of the shopkeeper (17, 18). Other works demonstrate the application of chaotic methods, such as (19–21), achieving demand adjustment.

As of today, the sudden changes in the market have generated new paradigms of competition. These circumstances are experienced by emerging businesses such as neighborhood stores (22). Studies such as (23) show that more than 90 % of the data generated by these businesses are not exploited and are not used to make sense of the changing dynamics of the market. There are success stories where machine learning has been used to better predict and manage expenses (23). This optimizes business profits, thus allowing for small businesses' economic survival. Some sources claim that including these methodologies can add significant value to their demand forecasting processes, while more than half state that they can improve other eight critical supply chain capabilities (24).

The above-mentioned study added that retailers with the technology to predict these changes model contingency plans and options and quickly adapt their supply chain strategy to meet high demand and avoid excess inventory. Those who cannot do it risk falling behind with sales not meeting the goals, in addition to incurring losses through waste (23). This research describes and structures a methodology for forecasting the quantities needed to order certain products in neighborhood stores. The method proposes a linear model to maximize the shopkeeper's profits based on an available investment value that, combined with a machine learning procedure, allows inferring the amounts to be requested. This method constitutes an alternative detached from the traditional paradigm of neighborhood stores, which is built daily and considers the last order requested, ignoring the behavior in sales and the dynamics of customers (25).

This proposal employs three models for sales forecasting, i.e., the performance of an SVM, a Gaussian process (GP), and an ARx model are examined using synthetic data generated via statistical sampling according to the Probability Distribution Function (PDF) that is accentuated to the sales of the products under analysis. The selection criteria for the PDFs of the products studied are in accordance with (14), which suggests using uniform or Gaussian PDFs for products offered in neighborhood stores. Additionally, the methodology proposes error metrics and statistical analysis to evaluate and validate each of the models selected in this research. The results show that our proposal allows maximizing the shopkeeper's profits by considering the dynamics in sales and the available investment. The main contributions of this research are the following:

- A methodology to forecast sales (demand) for a neighborhood store over a specific period which maximizes the shopkeeper's profits while considering the dynamics of demand and the number of products to invest in.

- A comparative study that evaluates the performance of forecast models applied to the prediction of demand in neighborhood stores.

- A validation procedure to measure the performance of each step in the proposed method.

This paper is organized as follows. Section 2 presents the basic nomenclature. Section 3 describes the proposed methodology, as well as the validation procedure, and the results obtained are shown in section 4. In the final section, the concluding remarks of our research are presented.

2. Basic nomenclature

Table I. Paper nomenclature

Symbol	Description
X_{Train}	Selected weeks for the learning model training
X_{Test}	Selected weeks for validating the learning model
Y_{Train}	Demands for the model training at X_{Train} instants
Y_{Test}	Demands for validating the model at X_{Test} instants
m	Analysis week in the [1-108] weeks window
n	n-th product under analysis $n \in Z^*[1 - 14]$
Z_{Test}	Validation matrix of the learning model
Z_{Train}	Training matrix for the learning model
W_n	Prediction model parameters for the n product
Z	Prediction of the trained model given the W_n vector parameters
A_G	Return gain
G_m	Investment to be made in the m week
ARMA	Auto Regressive Moving Average
SVM	Support Vector Machine
MLR	Multiple Linear Regression
GP	Gaussian Processes
PDF	Probability Distribution Function
ILP	Integer Linear Programming

3.   Methodology

Fig. 1 shows the implemented method in a block diagram. In short, it consists of three phases: firstly, a database with information on the demands per week for 14 everyday items offered in neighborhood stores; secondly, a regression model that emulates the prediction $Z = \{Z_1\ Z_2 \dots Z_n\}$, where Z_n is the prediction of the n-th product; and, finally, an integer linear programming model that refines the weekly orders to maximize the profit return A_G given a weekly investment rate. Traditionally, inventory management models include a systematic methodology to establish inventory control policies for products. This allows determining the security list, the reorder point, or when the order must be placed (26). This research aims to reduce the overall costs by predicting the demand for key products in neighborhood stores. It analyzes projected profits based on the model's predictions. It must be acknowledged that this study does not encompass all aspects of defining inventory control policies.
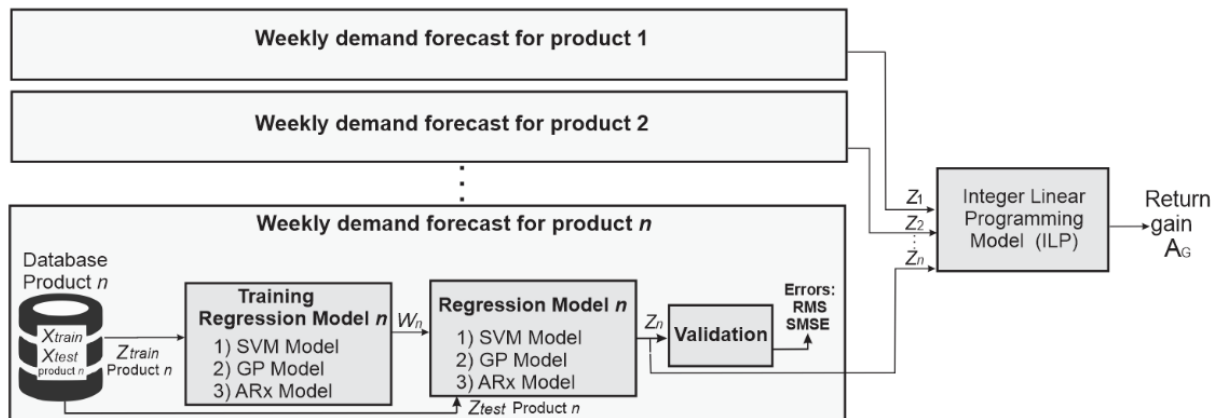
Figure 1. Implemented methodology

3.1.   Database

In this study, seven categories that cover 14 product varieties in neighborhood stores were selected, as summarized in Table II. Each product is independent, so all analyses are proposed and done for each one of them.

This selection was done by visiting shopkeepers in neighborhoods of Pereira (these items were selected since they were representative of the sales during the analyzed time horizon) and identifying common items such as those shown in Table II. Additionally, the variability in their demand was validated, finding that some exhibit a temporary behavior, as is the case of category 6 (products of great demand on celebration dates, *i.e.*, holidays, commemorative days, among others).

Conversely, other items show a stable behavior over time, as is the case of the products in categories 5 and 7. The literature states that the demand follows a certain PDF, which, in the case of basic products, can be adjusted to a normal or uniform behavior (27). Due to the above, it is possible to generate

Table II. Selected categories and products

	Product 1	Product 2
Category 1	Shampoo	Body soap
Category 2	Detergent	Bleach
Category 3	Milk	Yogurt
Category 4	Beer	Wine
Category 5	Chicken	Frozen food
Category 6	Rice	Salt
Category 7	Supplements	Vitamins

synthetic data from a statistical sample while assuming a PDF for each category, establishing the distribution parameters for each item (Table **III**).

Table III. Definition of the model for data generation

	PDF employed	Product 1	Product 2
Category 1	Gaussian	$\mu = 12$ and $\sigma = 5$	$\mu = 25$ and $\sigma = 8$
Category 2	Uniform	a = 10 and b = 24	a = 20 and b = 80
Category 3	Gaussian	$\mu = 60$ and $\sigma = 15$	$\mu = 40$ and $\sigma = 15$
Category 4	Gaussian	$\mu = 120$ and $\sigma = 40$	$\mu = 20$ and $\sigma = 10$
Category 5	Gaussian	$\mu = 40$ and $\sigma = 13$	$\mu = 30$ and $\sigma = 15$
Category 6	Uniform	a = 15 and b = 110	a = 20 and b = 120
Category 7	Gaussian	$\mu = 12$ and $\sigma = 3$	$\mu = 12$ and $\sigma = 2$

The expected values u_Y (where Y is the weekly demand) and the standard deviation σ_Y of the distribution given in Table **III**, are summarized in Table **IV**. The first moments for a normal and uniform distribution are $u_Y = \mu$ and $u_Y = \frac{b+a}{2}$, respectively. Similarly, the variance in the case of a Gaussian and uniform PDF is $\sigma_Y = \sigma$ and $\sigma_Y = \frac{b-a}{\sqrt{12}}$ (14).

Thus, the demand per week (for each product) was sampled over a time window of 78 weeks, *i.e.*, the data generated emulate the dynamics of the demands per product for this sales interval. Once the database was formed, it was segmented as follows: $X = \{X_{Train}, Y_{Train}, X_{Test}, Y_{Test}\}$. Then, a characteristic was added to the vectors X_{Train} and X_{Test} which models the increase or decrease in the number of requested units of a product with respect to the previous week. Eq. (1) shows the transformation for the Y_{Train} case, following a similar procedure for Y_{Test}.

Table IV. Expected values and distributions from Table **III**

	u_Y		σ_Y	
	Product 1	**Product 2**	**Product 1**	**Product 2**
Category 1	12	25	5	8
Category 2	18	50	4	17.3
Category 3	60	40	15	15
Category 4	120	20	40	10
Category 5	40	30	13	15
Category 6	62.5	70	27.4	28.9
Category 7	12	12	3	2

$$\nabla M_{Train}^{m,n} \begin{cases} 1 & \text{if } Y_{\text{Train}}^{m,n} - Y_{\text{Train}}^{m-1,n} > 0 \\ -1 & \text{if } Y_{\text{Train}}^{m,n} - Y_{\text{Train}}^{m-1,n} < 0 \text{ and } m > 1 \text{ and } \nabla M_{\text{Train}}^{1,n} = 0 \\ 0 & \text{if } Y_{\text{Train}}^{m,n} - Y_{\text{Train}}^{m-1,n} = 0 \end{cases} \tag{1}$$

The sub-index m refers to the analyzed week for product n (defined in Table **II**). Eq. (1) indicates that, if the value of $\nabla M_{Train}^{m,n}$, there was an increase when ordering product n in week m regarding the $m-1$ period. On the other hand, a value of $\nabla M_{Train}^{m,n}$ implies a stable behavior in week m regarding the moment $m-1$. This descriptor seeks to guide the learning model at the moments when changes occur in the demands of the n products. For more details, the reader should consult (28). With these changes, the terms are defined as follows: $Z_{\text{train}} = \{[X_{\text{train}}, \nabla M_{\text{train}}], Y_{\text{train}}\}$ where $\nabla M_{\text{Train}} = \{\nabla M_{\text{Train}}^{1,1}, \nabla M_{\text{Train}}^{2,1}, \dots, \nabla M_{\text{Train}}^{m,1}, \nabla M_{\text{Train}}^{m,2}, \dots, \nabla M_{\text{Train}}^{m,n}\}$, similarly $Z_{\text{test}} = \{[X_{\text{Test}}, \nabla M_{\text{Test}}], Y_{\text{test}}\}$.

3.2. Regression model

This phase is divided into two processes: model training and simulation. The first process consists of computing the set of parameters W_n given the demands per week for an item n of a specific category. On the other hand, the simulation process consists of predicting the behavior regarding the future demands of product n once the parameters W_n are found.

3.3. Model training

Fig. **2** shows each of the stages that make up the training phase. In short, it consists of a database that integrates synthetic data of demands per week for a family of products, as well as a supervised learning model to emulate the dynamics of weekly demands. The purpose is to transfer the data content to the model through a learning rule that allows finding the set of parameters W_n.
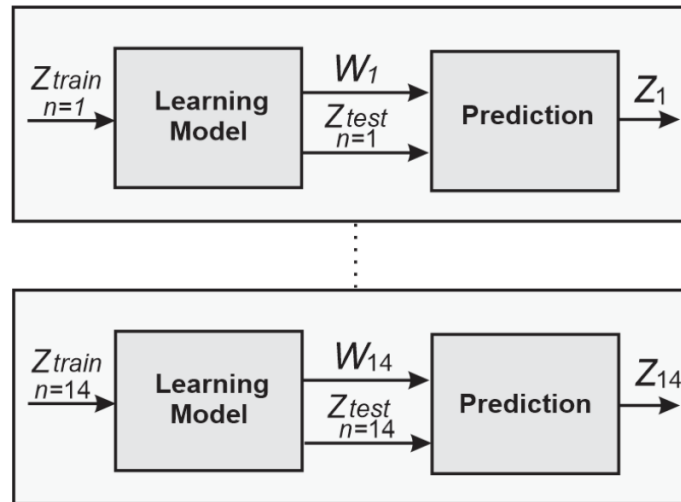
Figure 2. Prediction model diagram

3.3.1.   Supervised learning model

In this paper, three supervised options for state-of-the-art training are examined. Each of these is described below.

SVM

The use of an SVM as a prediction model is mainly due to its robustness against data with noise. It is a widely studied method, and it is easy to implement. This method is characterized by being a linear model in the W_n parameters, which are tuned by minimizing a regularized error function subject to a set of linear constraints.

Generally, the problem is usually transformed into an equivalent dual one by applying the KKT (Karush-Kuhn-Tucker) conditions, obtaining a quadratic representation that allows the value W_n to be inferred when it is solved. Once the parameter values W_n are known, the model is trained. However, the equivalent objective function will depend on some variables assumed to be known, such as the Z_{Train} data set, as well as the Kernel structure (covariance function) (29, 30).

ARx Model

The acronym ARx refers to an autoregressive model of exogenous terms. It involves an input signal in its structure. Eq. 2 shows this model. The meaning of each term is shown in Table **V**.

$$A(q)y(t) = B(q)u(t - l) + e(t) \tag{2}$$

The set of parameters to infer W_n corresponds to the set of values of the coefficients a_{n_a} and b_{n_b}. These are tuned via least squares through the Z_{Train} dataset. Then, once the values of W_n are known, the model is determined for any value of t, assuming that W_n is known. For more details, the reader can refer to (31).

Table V. Nomenclature associated with the ARx model

Parameter	Definition
$y(t)$	Output in instant t
q	Delay operator
$u(t)$	Input in instant t
$e(t)$	Gaussian white noise disturbance value
$A(q)$	$A(q) = 1 + a_1 q^{-1} + \ldots + a_{n_a} q^{-n_a}$
$b(q)$	$B(q) = 1 + b_1 q^{-1} + \ldots + b_{n_b} q^{-n_b}$
n_a	Number of poles
n_b	Number of zeros
l	The system's dead time

Gaussian process (GP)

Contrary to previous methods, the GP introduces a probabilistic non-linear prediction function, preventing a fixed structure in the model. From its base, the model assumes that the nature of the data follows a Gaussian distribution function established by a mean function $m(x)$ and a covariance $cov(x, x)$. In the literature, $cov(x, x)$ is also known as *Kernel*, so the model training consists of tuning the parameters of $m(x)$ and $cov(x, x)$ (*i.e.*, W_n) given the input X_{Train} and output $Y_{Train}(Z_{Train})$ dataset. The prediction process (once the model has been trained) is performed from the mean predictive function, as shown in Eq. (3).

$$Z = m'(X_{Test}) = cov(X_{Test}, X_{Train}) \cdot [cov(X_{\text{Train}}, X_{\text{Train}})]^{-1} \cdot Y_{\text{Train}} \tag{3}$$

where $m'(x_1)$ is the prediction in space x_1 given the input x and the corresponding output y. An interesting feature of the GP allows computing the evolution of uncertainty (Eq. (4)) in the prediction using the predictive standard value (Eq. (3)), where $\sigma(m'(x_1))$ evaluates the uncertainty (variance) of $m'(x_1)$. The GP implicitly assumes that the function $m'_1(x_1)$ obeys – as does $m(x)$ – a Gaussian behavior (32).

$$\sigma(m'(X_{Test})) = cov(X_{Test}, X_{Test}) - cov(X_{Test}, X_{Train}) \cdot [cov(X_{Train}, X_{Train})]^{-1} \cdot cov(X_{Train}, X_{Test}) \tag{4}$$

3.3.2. Validation

In this process, the performance of the trained models is evaluated through the dataset $[X_{Test}, \nabla M_{Test}]$ and the corresponding prediction Z (Fig. 2). In the first place, the effective error is used as a performance metric (Eq. (5)) where E is the RMS error between the synthetic data validation vector and the prediction of the model (30). Here, N refers to the size of the vector Y_{Test}.

$$E = \sqrt{\frac{\|Z - Y_{Test}\|^2}{N}} \tag{5}$$

This metric allows evaluating the fidelity of the trained model to emulate the prediction process of product demands. However, the Y_{Test} and Y_{Train} data must be normalized to avoid bias regarding

strong changes. In this work, they are normalized with respect to the mean value of the data as follows: $Y_{Test} \rightarrow \frac{T_{Test} - \mu_Y}{\sigma_Y}$. A similar transformation applies to Y_{Train}. Thus, Eq. (5) is transformed into the following expression:

$$E = \sigma_Y \sqrt{\frac{\|Z - Y_{Test}\|^2}{N}} \tag{6}$$

On the other hand, Eq. (7) defines the form of the standardized mean squared error ($SMSE$) (33):

$$\text{SMSE} = \frac{1}{\sigma_{Train}^2 + \mu_{Train}^2} \|Z - Y_{Test}\|^2 \tag{7}$$

where σ_{Train}^2 and μ_{Train} are the variance and arithmetic mean of the Y_{Test} data. The metric given in Eq. (6) has the purpose of evaluating the stability of the prediction model Z when considering changes in the training data Z_{Train}.

3.4. Integer linear programming model (ILP)

Z is an expected value of the demand. However, the shopkeeper will not always have sufficient resources to meet the quota established by the prediction model, which is why a strategy that refines the suggested Z values through a linear optimization model is adopted. The objective function to be minimized is presented in Eq. (8).

$$f(Z^*) = p_1 Z_1^* + p_2 Z_2^* + \ldots + p_{14} Z_{14}^* = PZ^{*T} \tag{8}$$

where $Z^* = [Z_1^*, Z_2^*, \ldots, Z_{14}^*]$ are the quantities demanded weekly to be refined for each of the items defined in Table **II**, and $P = [p_1, p_2, p_3, \ldots, p_{14}]$ denotes the corresponding percentages of profit from the sale of each product. The restrictions associated with the maximum number of items allowed are as follows:

$$Z_n^* - Z^{m,n} = g(Z) \leq 0$$
$$Z_n^* = r(Z) \geq 0 \tag{9}$$

where $Z^{m,n}$ refers to the predictions of item $n(n : 1 : 14)$ for week m. Therefore, Eq. (9) yields 28 linear constraints given a value of m. Finally, the equality associated with the investment in week m is

$$G_m - c_1 Z_1^* - c_2 Z_2^* - \ldots - c_{14} Z_{14}^* = G_m - CZ^{*T} = 0 = h(Z^*) \tag{10}$$

where $C = [c_1, c_2, c_3 \ldots, c_{14}]$ are the purchase costs per item. Now, the refined quantities Z^* are found by solving the following ILP problem (Eq. (11)) for an investment G associated with week m.

$$\max_{Z^*} f(Z^*)$$
$$\text{Subject to:}$$
$$g(Z^*) \leq 0$$
$$r(Z^*) \geq 0$$
$$h(Z^*) = 0$$

Note that the presumption of solving the problem stated in Eq. (11) is due to the fact that the quantities to be refined have a discrete characteristic. The values of P and C have been extracted from the portal www.megatiendas.co, and they are condensed in Table **VI**.

Table VI. Definition of the values for the percentage gain vector P and the cost per product C

	Cost ($)		Profit (%)	
Category	**Product 1**	**Product 2**	**Product 1**	**Product 2**
1	$10.550 \rightarrow c_1$	8,000	$25 \rightarrow p_1$	25
2	4,700	4,400	25	25
3	3,100	6,250	15	15
4	2,500	14,700	20	20
5	10,650	12,950	15	15
6	3,900	1,250	10	10
7	4,550	$11.320 \rightarrow c_{14}$	25	$25 \rightarrow p_{14}$

3.5. Experiments

The test procedures evaluated the performance of the learning rules and the ILP. For the first case, the performance of the three strategies was evaluated by computing the values of E and $SMSE$ (Eqs. (6) and (7)) for the prediction Z produced by the model and the test dataset Y_{Test}. Out of the 78 weeks, the first 55 were selected with their corresponding demands to build the vectors X_{Train}, Y_{Train}, and ∇M_{Train}. Then, with the other weeks (56 to 78), the validation array X_{Test}, Y_{Test}, and ∇M_{Test} was made. The test in Algorithm 1 shows the process that summarizes the above.

Algorithm 1 Training and validation algorithm

Input: $Z_{\text{train}}, Z_{\text{test}} \leftarrow$ to interaction i

Output: Prediction Z, RmsError E, Standardized Square Error, SMSE $\leftarrow$ to interaction i, Average $Z(\mu)$, standard deviation of $E(\sigma)$

1: **for** $i = 1$ **to** j **do**

2:      $\mu_{\text{train}} \leftarrow \text{Mean}(Y_{\text{train}}[i, :])$

3:      $\sigma_{\text{train}}^2 \leftarrow \text{Var}(Y_{\text{train}}[i, :])$

4:      $Y_{\text{train}}[i, :] \leftarrow \frac{Y_{\text{train}}[i, :] - \mu_{\text{train}}}{\sqrt{\sigma_{\text{train}}^2}}$

5:      $Z_{\text{train}} \leftarrow [Y_{\text{test}}, \nabla M_{\text{test}}, Y_{\text{train}}]$

6:      $Y_{\text{test}}[i, :] \leftarrow \frac{Y_{\text{test}}[i, :] - \mu_{\text{train}}}{\sqrt{\sigma_{\text{train}}^2}}$

7:      $W_n \leftarrow \text{Train}(Z_{\text{train}}, \text{Model})$ $\triangleright \text{Model} \leftarrow \{ARx, SVM, GP\}$

8:      $Z[i] \leftarrow \text{Predicr}(X_{\text{test}}, \nabla M_{\text{test}}, W_n, \text{Model})$

9:      $E[i] \leftarrow \sqrt{\frac{\sigma_{\text{train}}^2 \times \|Z - Y_{\text{test}}\|^2}{N}}$

10:     $\text{SMSE}_{[i]} \leftarrow \frac{1}{\sigma_{\text{train}}^2 + \mu_{\text{train}}^2} \cdot \|Z - Y_{\text{test}}\|^2$

11: **end for**

12: $\mu \leftarrow \text{Mean}(Z)$

13: $\sigma \leftarrow \text{Var}(E)$

For the case of GP and SVM, a radial basis covariance function (RBF) with scale factor ρ^2 was assumed (32). This parameter was tuned given the Z_{Train} dataset. The GPML (34) toolbox was used to train (find W_n) and validate the GP (the value of ρ^2 was also tuned with this toolbox). For the SVM, MATLAB's regression toolbox in MATLAB was employed. In this case, a value of $\tau^2 = 0,001$ (length scale) was set for the tests. The results are presented by means of tables and figures depicting the behavior of each model. In the case of the GP, the results are based on Eqs. (3) and (4). Finally, based on the results, the best prediction model was selected in order to evaluate the performance of the ILP.

The experiments carried out in the ILP consist of validating the problem stated in (11). For this purpose, weekly test investments G_m, are defined. Thus, the refined quantities Z^* are calculated given the weekly investment G_m and $Z^{m,n}$. Finally, the expected return earning rate A_G is calculated according to the suggested amount Z^* by the model in week m (Eq. (**11**)). Algorithm 2 shows a summary of the refinement process of the quantities Z and the profits for each of the weeks m under analysis.

$$A_G^m = c_1 p_1 Z_1^{*m} + c_2 p_2 Z_2^{*m} + \ldots + c_{14} p_{14} Z_{14}^{*m} \tag{11}$$

Algorithm 2 Algorithm for calculating A_G^m profits given vector Z^*

Input: $G_m, C, Z^{m,n}, P \leftarrow$ to week m

Output: $Z^*, A_G^m \leftarrow$ to week m

1: **for** $i = m$ **to** j **and** $j > m$ **do**
2:    $Z^*[i] \leftarrow \text{Linprog}(Z^n, C, P, G_i)$
3:    $A_G^{m*} \leftarrow c_1 p_1 Z_1^* + \ldots + c_n p_n Z_n^*$
4: **end for**

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4.   Results

This section presents the analysis and discussion of the results obtained by implementing the methodology and performing the validation tests proposed in the methodology section. Fig. 3 shows the demand estimation made via the proposed prediction methods for product 14. In 3c, it can be seen that the ARx is the predictor that manages to capture the trend of the data and the demand variability for the item in the time window.

Fig. 3b shows GP's low performance, as it fails to predict the variability of the demands when estimating an average value. Although the SVM manages to improve the prediction concerning the GP, this variation is not enough to emulate the dynamics of the weekly demands of the products (Fig. 3a). The quantitative results are shown in Fig. 3b. The results regarding the effective errors are condensed (Eq. (5)) once the Monte Carlo experiment has been simulated (Algorithm 1). As per Fig. 4b, the ARX shows a lower error when compared to the SVM (Fig. 4d) and the GP (Fig. 4f), which corroborates the results shown in Fig. 3. It is noteworthy that the minimum error for the training demands is reached by GP (Fig. 4e). This suggests that the model is overtrained, which explains the result in Fig. 3b. On the
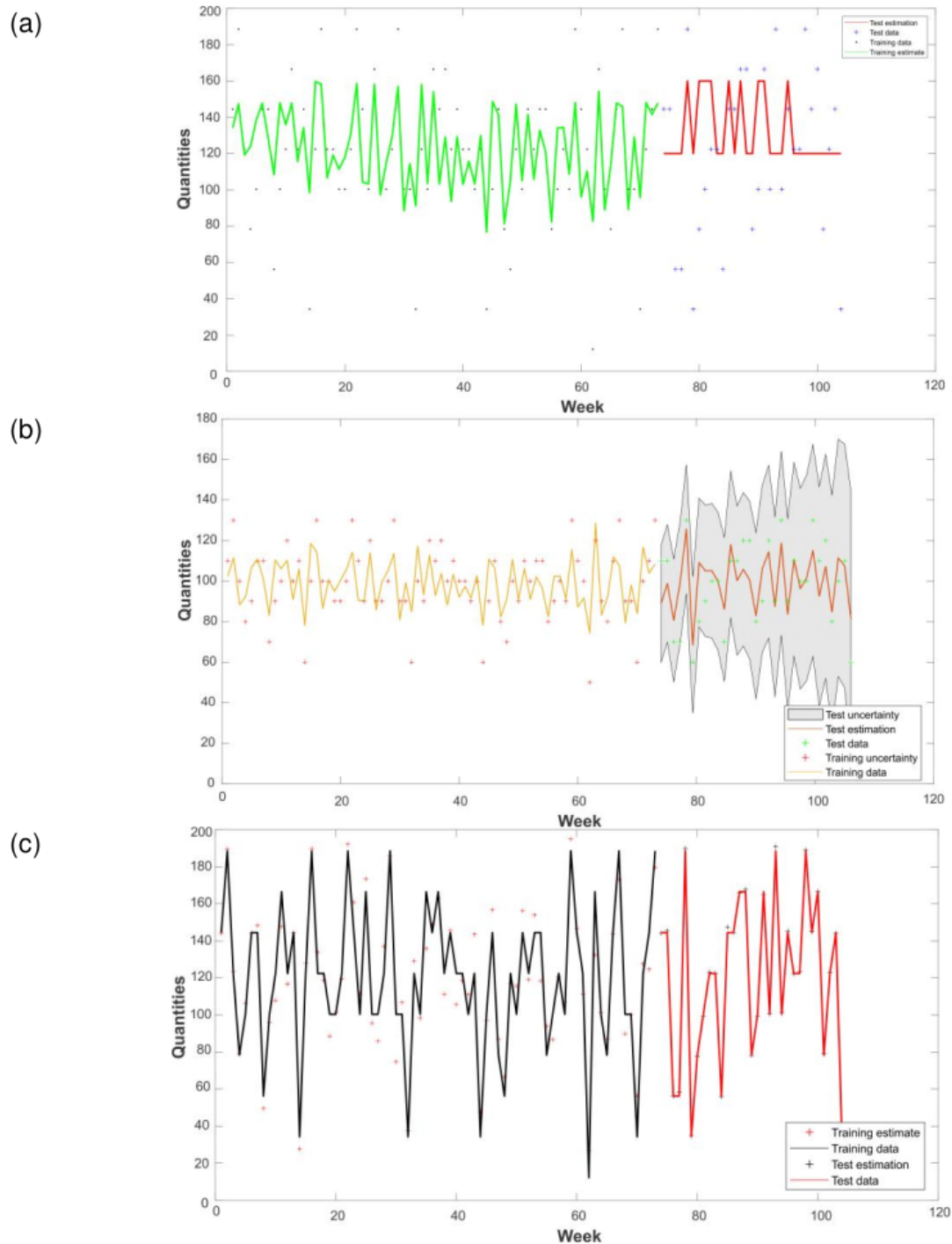
Figure 3. a) Demand estimation using the SVM method. b) Demand estimation using the GP method. c) Demand estimation using the ARX method

other hand, Fig. **4**d shows that the SVM responds adequately to products with low variability, as is the case of the items in categories 1 and 7 (Table **II**).
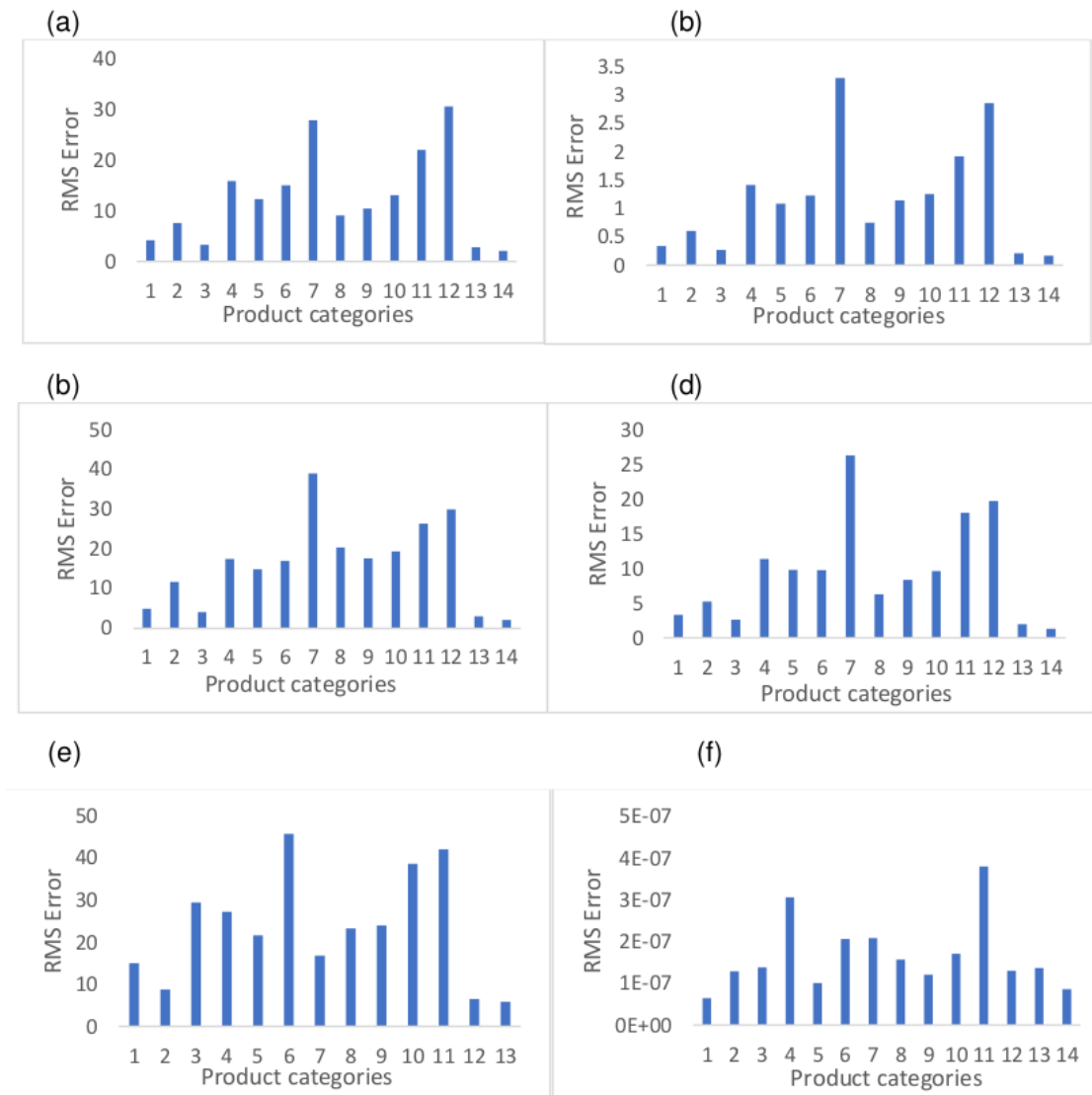
Figure 4. a) Training RMS error for the ARX method. b) Test RMS error for the ARX method. c) Training RMS error for the SVM method. d) Test RMS error for the SVM method. e) Training RMS error for the GP method. f) Test RMS error for the GP method

Fig. **5** synthesizes the results for the *SMSE* metric. The GP fails to stabilize in the training phase regarding the categories {1,2,7, 12, 13}. On the other hand, the SVM and the ARx exhibit a similar qualitative behavior in all the product categories under study, both in the training stage and during the validation. The results suggest that the prediction computed by the three methods evaluated is stable. This corroborates what is evidenced in Figs. **5** and **6**, where the change in the datasets Z_{Train} and Z_{Test} fails to capture the dynamics of the data, nor does it significantly reduce the effective errors, mainly in the case of the SVM and the GP. The above insinuates that they fail to satisfactorily emulate the behavior of the demands. This may be due to the nature of the application, where the analysis time window (104 weeks) is restricted for these models.
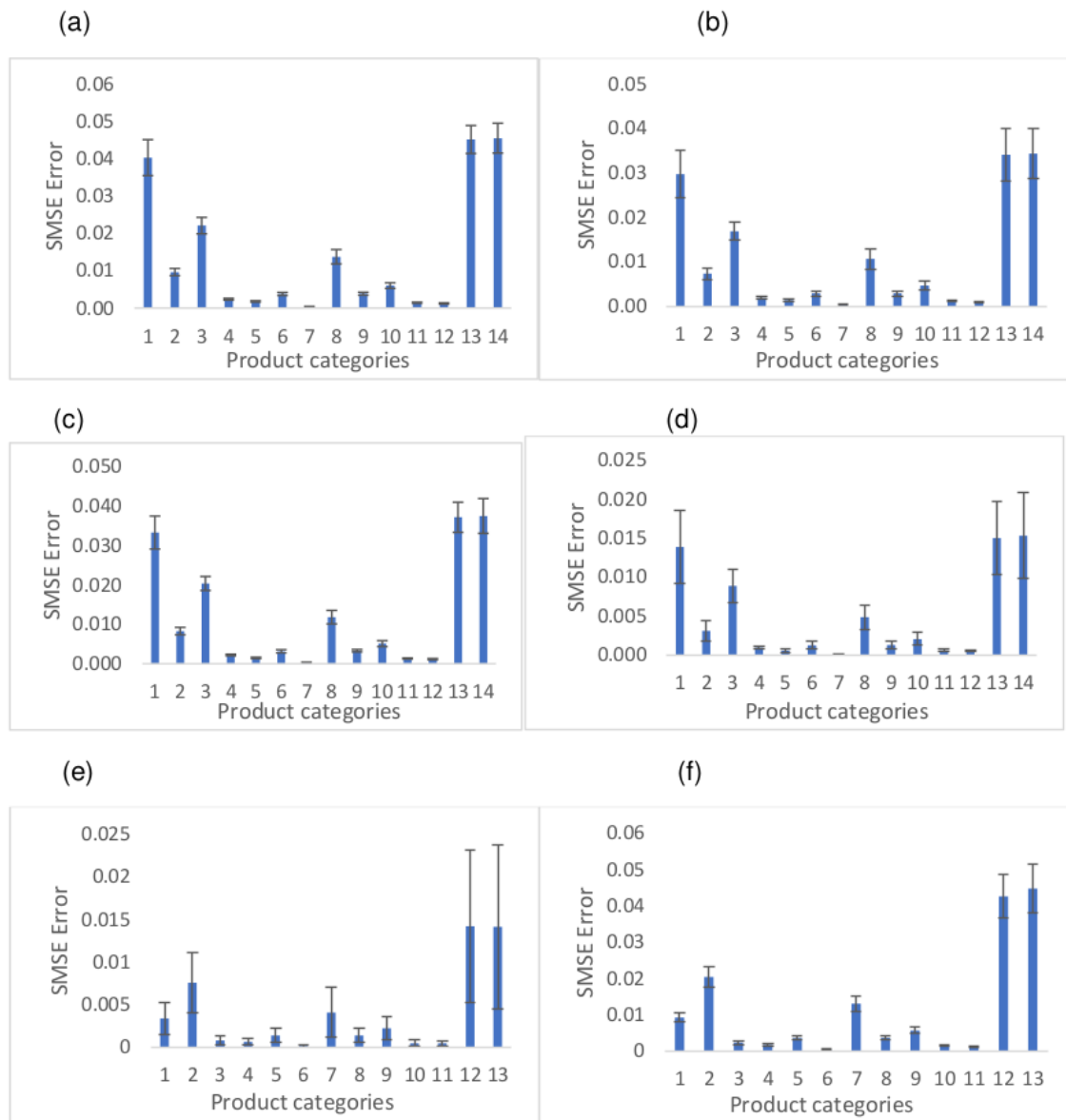
Figure 5. a) Training RMS error for the ARX method. b) Test RMS error for the ARX method. c) Training RMS error for the SVM method. d) Test RMS error for the SVM method. e) Training RMS error for the GP method. f) Test RMS error for the GP method

Table **VII** summarizes the results obtained by calculating the first expected value and the variance of the predictors for each of the studied products' demands. The ARx manages to approach the theoretical statistics, which is a significant result, suggesting that it manages to capture the variability and the mean value of the distributions associated with each of the demands. However, although the SVM and the GP approach the first theoretical moment of the data, they fail to capture the variability; these predictors have a variance bias in this application.

Table VII. Results regarding the expected value and variance for each of the predictors

|	Theoretical		GP		SVM		ARX	
	E[Y]	$\sqrt{VAR}$	E[Y]	$\sqrt{VAR}$[Y]	E[Y]	$\sqrt{VAR}$	E[Y]	$\sqrt{VAR}$[Y]
Product 1	12	5	12.18	11.84	14.35	0.86	**11.25**	**4.07**
Product 2	25	8	37.44	28.42	32.84	1.24	**22.71**	**7.76**
Product 3	18	4	18.70	17.84	20.64	0.64	**16.91**	**3.55**
Product 4	50	17.3	54.35	49.35	48.39	2.96	**48.03**	**16.60**
Product 5	60	15	52.48	59.00	57.73	2.56	**60.28**	**12.02**
Product 6	40	15	34.35	39.45	36.42	2.49	**38.42**	**16.71**
Product 7	120	40	115.95	110.35	117.24	7.10	**120.80**	**25.32**
Product 8	20	10	18.19	21.03	19.08	1.50	**23.85**	**8.22**
Product 9	40	13	43.18	36.90	38.64	2.26	**39.03**	**10.00**
Product 10	30	15	38.93	30.65	36.75	2.45	**28.64**	**12.61**
Product 11	62.5	27.4	65.22	69.39	64.72	4.10	**66.71**	**23.89**
Product 12	70	28.9	52.74	60.87	45.69	4.70	**52.72**	**31.38**
Product 13	12	3	14.96	12.65	11.99	0.43	**11.95**	**2.92**
Product 14	12	2	11.81	12.06	12.35	0.30	**11.56**	**2.22**

Given that the ARx exhibits a good performance in estimating the demands in each of the categories, and considering that the SVM and GP do not reach competitive performances (Table **VII**, Figs. **3**, **4**, **5**), the outputs Z inferred by the ARx are selected to build the integer programming model described in Eq. (11) in such a way that it is possible to refine the quantities Z by defining a weekly investment index to determine Z^*. Fig. **6** shows the difference between the profits when considering a sufficient investment to acquire the amounts Z versus the profits gained when examining a smaller investment to obtain Z^* in a cumulative period of 22 weeks. Notice how, in the linear programming method, the profits approach the maximum values as the investment grows. This is an expected result since the formulation aims to provide further weight to the products with the highest profit; the method suggests the amounts that maximize profit to the shopkeeper. It should be noted that, from an investment of 1.200.000 COP, it is possible to obtain the same profit as the one obtained with the Z amounts, *i.e.*, the values suggested by the ARx.

Fig. **7**a shows the expected profit for 22 weeks according to the amounts Z. It also shows the profit for the amounts Z^* corresponding to an investment of 1.220.000 COP. This shows that the linear programming model does not affect the trend of expected profits. Note that this result is still valid for smaller investments (Fig. **7**b), and it is important because the method guarantees proportionality in profits, even if the investment is low.

Fig. **8** shows how the solution to the problem given by Eq. (11) refines the number of products and prioritizes those that generate greater profits. For example, in Fig. **8**a, the model gives priority to
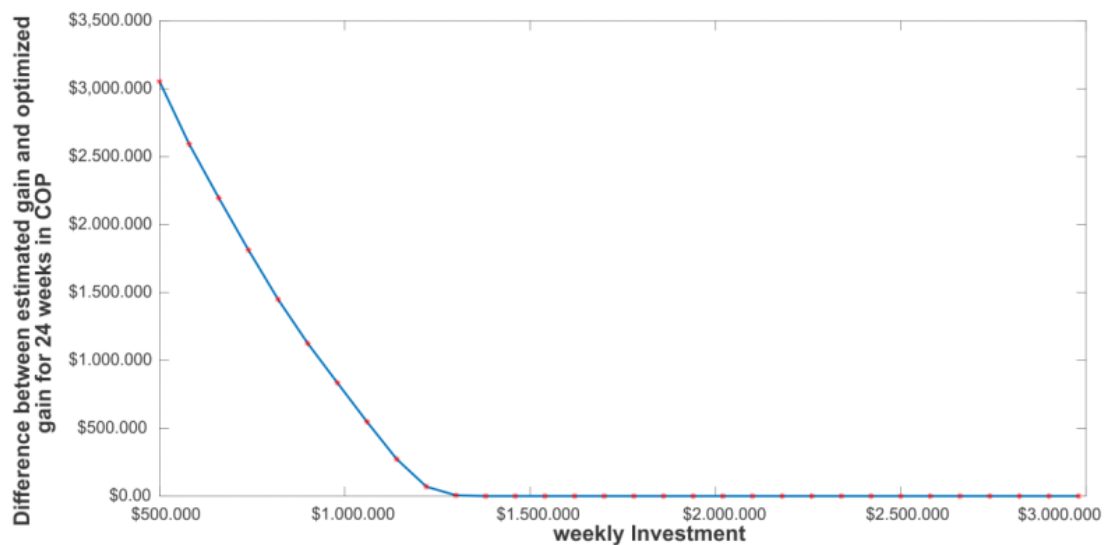
Figure 6. Difference between the profit projected by the estimation of the product and the profit obtained by the integer programming model for 22 weeks of analysis
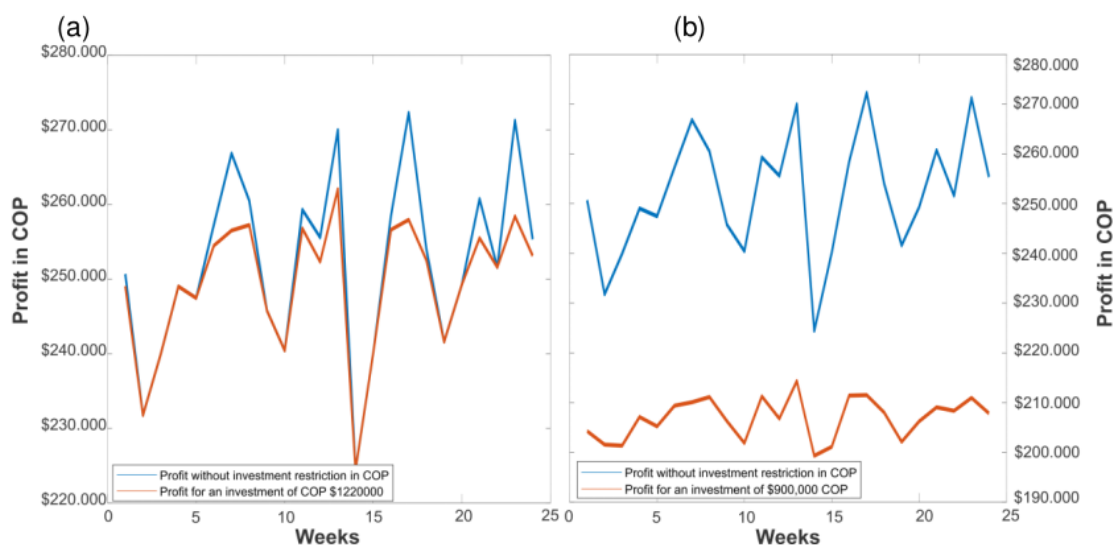
Figure 7. a) Expected profit with Z vs. profit obtained with Z^* for 22 weeks, corresponding to an investment of 1.220.000 COP. b) Expected profit with Z vs. profit obtained with Z^* for 22 weeks given an investment of 920.000 COP

products 1, 2, 3, 4, 13, and 14 when comparing them against the utility portions defined in Table **7**. They coincide with the items that report the highest percentage of utility per sale. Similarly, it should be noted (Fig. **8**b) that, as the investment value increases, the amounts Z^* tend to equal the values of Z, which corrects the results reported in Figs. **6** and **7**.
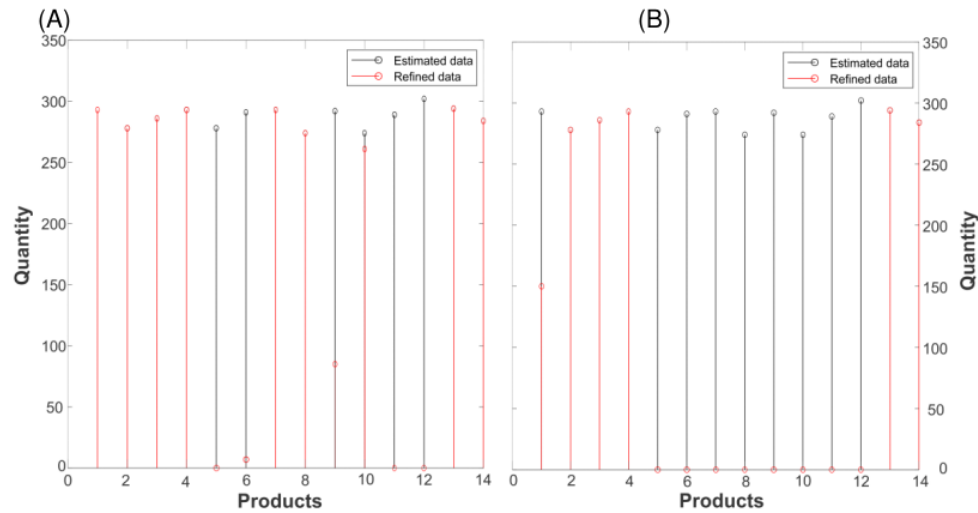
Figure 8. a) Estimated amounts for the 14 products in 22 weeks vs. refined amounts for a weekly investment of 500.000 COP. b) Estimated amounts for the 14 products in 22 weeks vs. refined amounts for a weekly investment of 1.000.000 COP

5.   Conclusions

In this research, a methodology is designed and implemented with the aim of estimating the demand for products sold in neighborhood stores. This is achieved based on the history of the weeks of sales for previous months and weeks. This method estimates demand and suggests which products to buy in order to maximize profit based on an available investment cost. Based on the results, it is observed that the model proposed in Eq. (11) is functional to the extent that the investment available per week is not sufficient to acquire the amounts Z. This is inferred by analyzing that reported in Figs. **8**a and **8**b, where the behavior of the amounts Z and Z^* is described as an investment value. The aforementioned is an expected result because, at a higher level of investment, the amounts Z^* are closer to Z. Therefore, it is concluded that the refinement process of the proportions of Z becomes unnecessary at high levels of investment.

On the other hand, the results show that, for this application, the ARx is the model that is closest to the data dynamics (Table **VII**), managing to reproduce the first moment and the variance of the probability distributions assumed in Table **III**. This is verified by analyzing the values of the effective errors reported in Fig. **4**b, where the ARx proves to be more competitive than the SVM and the GP.

The differences per product regarding the behavior of the effective error for the studied learning models can be explained by analyzing some categories' variances. For example, from Table **IV**, items 11 and 12 have a high variability concerning the others, *i.e.*, they exhibit a greater range of fluctuation in demand. Note that, in these particular cases, the effective error is significantly higher when compared to products with low variability, as for those belonging to category 1. The above suggests that the prediction models' yields are sensitive to solid changes in demand.

As further work, the proposed method must incorporate other relevant variables, allowing for qualitative and/or quantitative variables to be weighted in order to refine the Z quantities. This is essential, given that, in a competitive environment such as the neighborhood grocery store market, the supply of different products should not be strictly conditioned as being a problem of maximizing the shopkeeper's profits according to the items that yield the highest profit. However, on the contrary, it should consider the client's needs, so that these are incorporated into the strategy for calculating Z^*. On the other hand, it is also important to validate this methodology with different neighborhood stores, as well as to measure the possible impact of the methodology, for it is imperative to build software that allows the shopkeeper to manage it with ease. This information can be valuable to grow the model and thus correlate other variables currently unknown to the method.

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7. Author Contributions

The study was designed, implemented, and validated by Carlos Alberto Henao Baena. Andrés Felipe Calvo Salcedo supported the experiments. The collaborative efforts of Bibiana Zuluaga Zuluaga and Julián Galeano Castro involved identifying the problem and constructing the database. Edward Jhohan Marín García reviewed and supported the editing process of the paper. It is important to note that all authors have rigorously reviewed the content and have provided their collective consent and approval for the final manuscript.

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