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Research

Applying an Optimization Algorithm Based on the Cauchy Distribution for Active and Reactive Power Management with Batteries in Energy Distribution Systems

Aplicación de un algoritmo de optimización basado en la distribución de Cauchy para la gestión de potencia activa y reactiva con baterías en sistemas de distribución de energía

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Abstract

Context: This study developed an energy dispatch model (EDM) using the Cauchy-based distribution optimizer (CbDO) for coordinating battery energy storage units (BESUs) and photovoltaic (PV) sources in medium-voltage distribution networks, aiming to minimize energy losses and operating costs while observing to network constraints.
Method: The CbDO was implemented in MATLAB and benchmarked against the continuous genetic algorithm (CGA), the parallel particle swarm optimizer, the parallel vortex search algorithm, and a semidefinite programming (SDP) approach. The analyzed scenarios included unitary and variable power factor operation in order to test optimization performance.
Results: The CbDO outperformed traditional methods, achieving lower energy losses and CO_2 emissions, closely matching the SDP method's results in variable power factor scenarios. The most significant gains were observed when all DERs operated flexibly, validating our proposal's effectiveness in complex non-convex problems.
Conclusions: The CbDO is a viable and efficient solution for EDM, providing near-SDP performance with a simpler implementation. BESU integration and flexible power factor operation can notably enhance grid efficiency.

Keywords: energy dispatch model, Cauchy-based distribution optimizer, battery energy storage, distributed energy resources, power systems optimization

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Resumen

Contexto: Este estudio desarrolló un modelo de despacho de energía (EDM) utilizando el optimizador basado en la distribución de Cauchy (CbDO) para coordinar unidades de almacenamiento de energía en baterías (BESU) y fuentes fotovoltaicas (PV) en redes de distribución de media tensión, con el fin de minimizar las pérdidas de energía y los costos de operación a la vez que se cumplen las restricciones de la red.
Método: El CbDO fue implementado en MATLAB y comparado con el algoritmo genético continuo (CGA), el optimizador de enjambre de partículas en paralelo (PPSO), el algoritmo de búsqueda de vórtice en paralelo (PVSA) y un enfoque de programación semidefinida (SDP). Los escenarios analizados incluyeron la operación con factor de potencia unitario y variable, a fin de evaluar el desempeño de la optimización.
Resultados: El CbDO superó a los métodos tradicionales, logrando menores pérdidas de energía y emisiones de CO_2, siguiendo de cerca los resultados del método SDP en los escenarios con factor de potencia variable. Las mayores mejoras se observaron cuando todos los DERs operaban de manera flexible, validando la efectividad de nuestra propuesta en problemas no convexos complejos.
Conclusiones: El CbDO constituye una solución viable y eficiente para el EDM, ofreciendo un desempeño cercano al del método SDP con una implementación más sencilla. La integración de BESUs y la operación con factor de potencia flexible pueden mejorar significativamente la eficiencia de la red.

Palabras clave: modelo de despacho de energía, optimizador basado en la distribución de Cauchy, almacenamiento de energía en baterías, recursos energéticos distribuidos, optimización de sistemas de potencia

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Nomenclature

Acronyms

BESS Battery energy storage system

BESU Battery energy storage unit

CbDO Cauchy-based distribution optimizer

CGA Continuous genetic algorithm

DER Distributed energy resource

DG Distributed generator

EDM Energy dispatch model

EVCS Electric vehicle charging station

GTO Gorilla troop optimizer

PPSO Parallel particle swarm optimizer

PVSA Vortex search algorithm

RAM Random access memory

RES Renewable energy source

SDP Semidefinite programming

SoC State of charge

Parameters

Δ_h Time interval duration for h

ϵ Tolerance value for power flow convergence

γ_i Penalty coefficient for current constraint violations

$\mathbb{Y}_{k,m}$ Complex nodal admittance between nodes k and m

$C^{dg}_{\mathrm{CO}_2}$ CO_2 cost for the DG connected at node k

$C^{d}_{\mathrm{CO}_2,k}$ CO_2 cost for the conventional generator connected at node k

$I_{km,\text{máx}}$ Maximum allowable current in branch km

$P^b_{k,\text{máx}}$ Maximum power output of battery k

$P^b_{k,\text{mín}}$ Minimum power output of battery k

$V_{\text{máx}}$ Maximum voltage magnitude

$V_{\text{mín}}$ Minimum voltage magnitude

Subscripts

b Associated with battery systems

d Associated with demand/load nodes

dg Associated with DG

g Associated with conventional generation h Associated with time step h

km Associated with line between nodes k and m

Sets & Operators

$\mathbb{R}$ Real part operator

$\mathcal{H}$ Set of time steps

$\mathcal{L}$ Set of network branches

$\mathcal{N}$ Set of network nodes

$\text{diag}(\cdot)$ Diagonal matrix operator

$\text{mean}(\cdot), \text{std}(\cdot)$ Mean and standard deviation operators

Variables

$\alpha_k(x)$ Penalty function for the k-th constraint

β_j, λ_j Random numbers used in the global update rule

$\mathbb{V}_{d,h}^{t+1}$ Predicted voltage for the demand node d at time h and iteration $t+1$

$\mathbb{V}_{k,h}$ Voltage at node k and time h

$\mathbb{V}_{m,h}$ Voltage at node m and time h

μ_j, σ_j Mean and standard deviation for generating the initial population

$\text{SoC}_{k,h}^{b}$ SoC of the battery at node k and time h

F_f Fitness function used by the CbDO

h Number of time steps in the scheduling horizon

l Total number of branches in the network

n Total number of nodes in the distribution network

n_b Number of BESUs

n_g Number of conventional generators

n_{dg} Number of DGs

$S_{bk,h}$ Complex power from the battery at node k and time h

$S_{d,k,h}$ Complex load demand at node k and time h

$S_{gdk,h}$ Power from the DG at node k and time h

$S_{gk,h}$ Complex power from the conventional generator at node k and time h

x_{best}^{t} Best solution found at iteration t

x_j^p Solution vector for particle j in the CbDO

X_t Population matrix at iteration t in the CbDO

Y_t Candidate solution generated by the CbDO

1. Introduction

In response to the growing demand for sustainable energy and the urgent need to reduce the carbon footprint associated with traditional power systems, the integration of energy storage systems (ESS) and renewable energy sources (RES) into electrical distribution networks has become increasingly vital (1). Among the various ESS technologies, battery energy storage units (BESUs) are the most prevalent and widely deployed solutions (2). They play a pivotal role in achieving sustainability goals, as they store surplus energy for later use and provide active power support. Additionally, with the help of power electronics, BESUs can supply reactive power to the grid, which enhances their ability to stabilize voltage and improve power quality (3).

The combined management of active and reactive power through BESUs not only enhances the stability and quality of energy distribution; it also increases the resilience of distribution networks against fluctuations in renewable generation and variable demand (3). However, achieving an optimal control of these systems poses significant challenges, requiring advanced computational and technical strategies to effectively coordinate BESU operations under the dynamic conditions of distributed generation and load variability. The benefits of deploying battery energy storage systems (BESS) in distribution networks are illustrated in Fig. 1. By charging during low-demand periods and discharging during peak load intervals, BESS can effectively perform peak shaving, thereby reducing the need for additional generation capacity and alleviating stress on grid infrastructure. This operational flexibility not only supports more efficient asset utilization but also contributes to deferring costly grid reinforcements. Moreover, such control actions allow system operators to mitigate short-term power imbalances, reduce the curtailment of renewable sources, and maintain the voltage and frequency within acceptable limits.

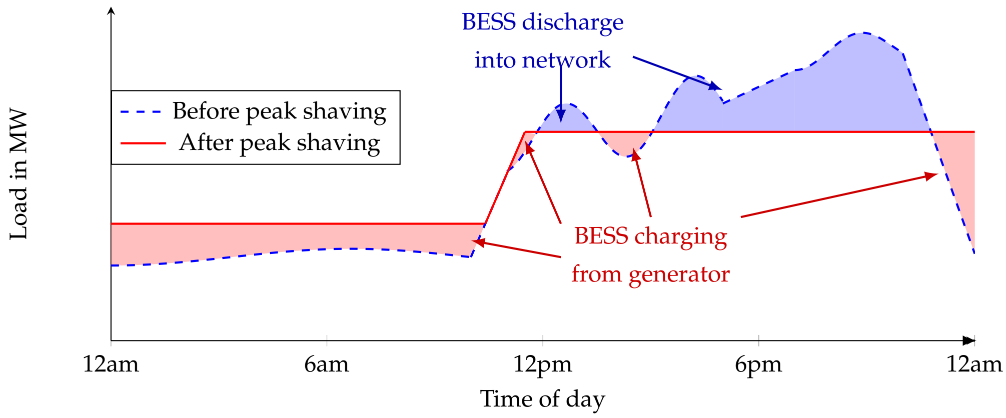

Figure 1. Effect of BESS peak shaving on daily load profiles

The literature presents a variety of approaches for managing RES and BESUs in distribution networks. Below are the key contributions in this field.

The authors of (4) proposed an optimal methodology for locating, selecting, and operating BESUs and renewable distributed generators (DGs) in medium- to low-voltage distribution systems. Using a mixed-integer non-linear programming (MINLP) model, the problem was divided into planning (device location and selection) and operation (optimal BESS scheme) stages. A simulated annealing algorithm with impedance-based sensitivity analysis was used for planning, while a novel decomposition method efficiently solved the operation problem, with near-globally optimal results. Validations on a 11-node feeder, a modified IEEE 135-node test system, and a real 230-node system confirmed the robustness and effectiveness of this methodology, which was demonstrated through comparative analysis across four scenarios.

The work by (5) proposed an optimization framework for distribution systems that combines photovoltaic (PV) and wind turbine (WT) units with BESUs at strategic locations. This study utilized the gorilla troop optimizer (GTO) to address the optimization problem as both a single- and a multi-objective task, targeting reduced power losses and total voltage deviation. In a two-stage approach, the GTO first determined the ideal siting and sizing of the PV and WT units. In the second stage, the GTO optimized the BESUs' operation while accounting for distinct electric vehicle charging station (EVCS) profiles on weekdays and weekends. The results indicated that integrating PV or WT units with BESUs into distribution networks yields substantial improvements.

The study by (6) described a practical power coordination approach for BESUs within a leader-follower framework. Here, the leader stage determines the state of charge (SoC) of the BESUs using the sine cosine algorithm (SCA), while the follower stage employs a power flow tool based on successive approximations to solve the power balance equations. This approach leverages an equivalent fitness function for the efficient exploration and exploitation of the solution space. Targeting the minimization of carbon dioxide emissions in a distribution grid with BESUs and renewable resources, this SCA-based model demonstrated a strong performance, outperforming alternative algorithms such as the continuous genetic algorithm (CGA), the particle swarm optimizer (PSO), and the vortex search algorithm (VSA) on a 33-bus test grid.

The authors of (7) presented an energy dispatch model (EDM) for microgrids, which was aimed at meeting the energy demand while minimizing both CO_2 emissions and operating costs. This EDM integrated model predictive control, a multi-objective optimization algorithm, and a decision tool. Model predictive control was used to adapt to dynamic operating conditions through a receding horizon approach, wherein optimization was recalculated at each time step based on updated data. The multi-objective optimization algorithm generated a Pareto front of trade-off solutions regarding emissions and cost, from which the decision tool selected the solution that best aligned with current operational priorities. Simulation results showed the robustness of this EDM under different forecasting accuracy conditions, highlighting the adaptability of model predictive control and the decision tool's effectiveness in handling the Pareto front.

The study by (8) examined the optimized dispatch of lithium-ion BESUs in commercial and industrial facilities as a strategy for reducing both CO_2 emissions and electricity costs. This multi-objective analysis evaluated 100 energy storage capacities, five discharge times, and two control strategies, with and without participation in event-based demand response programs. The results indicated that, unlike residential applications, standalone BESUs in large commercial settings could achieve substantial CO_2 emission reductions (>31 %) and cost savings (>10 %). Additionally, enrollment in demand response and dispatch under load-shifting control were found to be consistently optimal for minimizing both payback periods and emissions.

Various optimization techniques have been applied to enhance power coordination in BESUs and RES, particularly to curb CO_2 emissions. Among these approaches are second-order cone programming (9), ant colony optimization (10), convolutional neural networks (11), optimizers based on fuzzy logic (12), and artificial physics-based optimization (13). In addition, the comprehensive review by (14) examined EDM designs for integrating distributed energy resources into power networks. This study explored a range of solutions, including exact and probabilistic optimization methods, and highlighted the essential role of metaheuristic algorithms in the effective design and real-world application of EDM solutions.

This paper proposes an optimization approach based on Cauchy distribution to effectively manage active and reactive power, BESUs, and RES while aiming to minimize energy losses and operating costs in medium-voltage distribution networks. Unlike conventional algorithms, Cauchy-based optimization leverages the heavy-tailed nature of the Cauchy distribution to enhance the search process, enabling the exploration of more diverse regions of the search space and avoiding local optimum traps. This characteristic is particularly beneficial when dealing with complex, non-linear optimization problems, which are common in power systems.

The remainder of this document is organized as follows. Section 2 outlines the general optimization model for managing BESUs and RES in distribution networks. Section 3 describes the key components of the proposed leader-follower methodology, which employs the Cauchy-based distribution optimizer (CbDO) in the leader stage and the successive approximations method in the follower stage. Section 4 provides an overview of the modified 33-bus test system used in this study. Section 5 presents the main numerical results, and, finally, Section 6 provides some concluding remarks.

2. Exact problem formulation

Developing an effective EDM to coordinate BESUs and distributed energy resources (DERs) in medium-voltage distribution networks under a day-ahead planning scenario requires solving a non-linear programming (NLP) model (6). This type of model belongs to the family of non-convex optimization problems (15). Various performance indicators can be used to ensure the efficient coordination of these system components while considering technical, economic, and environmental metrics (16). This research focused on two specific objectives: a technical indicator related to daily energy losses and an environmental objective associated with carbon dioxide emissions (17). Each of these objective functions is outlined below.

2.1. Objective functions

The primary aim of an EDM for coordinating BESUs and DERs is to optimize key performance metrics for distribution companies, encompassing economic, technical, environmental, or social indicators or a combination thereof (18). This study focuses on minimizing two main objectives: the expected daily energy losses (D_{loss}) and the daily CO_2 emissions (D_{CO_2}), as defined by Eqs. (1) and (2) (17).

$$\text{mín } D_{\text{loss}} = \text{Re}\left\{\sum_{h\in\mathbf{H}}\sum_{k\in\mathbf{N}}\sum_{m\in\mathbf{N}} \mathbb{Y}_{k,m}\mathbb{V}^{\star}_{k,h}\mathbb{V}_{m,h}\Delta_h\right\}, \tag{1}$$

$$\text{mín } D_{\text{CO}_2} = \text{Re}\left\{\sum_{k\in\mathbf{N}}\sum_{h\in\mathbf{H}} \left(C^{g}_{\text{CO}_2,k}\mathbb{S}^{g}_{k,h} + C^{gd}_{\text{CO}_2,k}\mathbb{S}^{gd}_{k,h}\right)\Delta_h\right\}. \tag{2}$$

Here, $\text{Re}\{\cdot\}$ extracts the real part of the argument; $\mathbf{H}$ and $\mathbf{N}$ are the sets of analyzed time periods and network nodes, respectively; $\mathbb{V}_{k,h}$ and $\mathbb{V}_{m,h}$ denote the complex voltage at nodes k and m during time h; $\mathbb{Y}_{k,m}$ represents the complex nodal admittance between nodes k and m; Δ_h signifies the time discretization interval; and $\mathbb{S}^{g}_{k,h}$ and $\mathbb{S}^{gd}_{k,h}$ indicate the complex power output of the conventional and renewable sources connected to node k at time h, with $C^{g}_{\text{CO}_2,k}$ and $C^{gd}_{\text{CO}_2,k}$ being their corresponding CO_2 emission coefficients.

Notably, power generation using fossil fuels at medium-voltage levels is rare in urban distribution networks. Nonetheless, these systems contribute to equivalent CO_2 emissions by connecting to national power grids via transmission systems. Such grids often include mixed-generation matrices composed of renewable and fossil fuel sources (19). For instance, Colombia's electricity generation comprises approximately 67 % renewable sources (mainly hydropower), while the remaining 33 % relies on fossil fuels like coal, natural gas, and diesel (20). In this vein, the $C^{g}_{\text{CO}_2,k}$ coefficient models the equivalent emissions linked to distribution network operations (17).

2.2. Model constraints

The challenge of achieving an efficient operation of DERs in distribution networks involves adhering to a series of constraints derived from Kirchhoff's laws for each network node at every time interval, as well as to constraints related to the operational limits of the equipment used. Our EDM designed for the optimal coordination of DERs in medium-voltage distribution networks must consider the constraints outlined in Eqs. (3) to (18).

$$\mathbb{S}_{k,h}^{g,\star} + \mathbb{S}_{k,h}^{dg,\star} + \mathbb{S}_{k,h}^{b,\star} - \mathbb{S}_{k,h}^{d,\star} = \mathbb{V}_{k,h}^{\star} \sum_{m \in \mathbf{N}} \mathbb{Y}_{k,m} \mathbb{V}_{m,h}, \begin{Bmatrix} \forall\, h \in \mathbf{H} \\ \forall\, k \in \mathbf{N} \end{Bmatrix} \tag{3}$$

$$SoC_{k,h+1}^{b} = SoC_{k,h}^{b} - \varphi_k^b \mathrm{Re}\left\{\mathbb{S}_{k,h}^{b,\star}\right\} \Delta_h, \begin{Bmatrix} \forall\, h \in \mathbf{H} \\ \forall\, k \in \mathbf{N} \end{Bmatrix} \tag{4}$$

$$\mathbb{Z}_{km}\mathbb{I}_{km,h} = \mathbb{V}_{k,h} - \mathbb{V}_{m,h}, \; \{\forall\, h \in \mathbf{H}, \; \forall\, km \in \mathbf{L}\} \tag{5}$$

$$SoC_{k,h}^{b} = SoC_{k,i}^{b}, \; \{\forall\, h = h_{\text{mín}}, \; \forall\, k \in \mathbf{N}\} \tag{6}$$

$$SoC_{k,h}^{b} = SoC_{k,f}^{b}, \; \{\forall\, h = h_{\text{máx}}, \; \forall\, k \in \mathbf{N}\} \tag{7}$$

$$\left|\mathbb{S}_{k,h}^{g}\right| \leq S_{k,\text{nom}}^{g}, \; \{\forall\, h \in \mathbf{H}, \; \forall\, k \in \mathbf{N}\} \tag{8}$$

$$\mathrm{Re}\left\{\mathbb{S}_{k,h}^{g}\right\} \geq 0, \; \{\forall\, h \in \mathbf{H}, \; \forall\, k \in \mathbf{N}\} \tag{9}$$

$$\left|\mathbb{S}_{k,h}^{gd}\right| \leq S_{k,\text{nom}}^{gd}, \; \{\forall\, h \in \mathbf{H}, \; \forall\, k \in \mathbf{N}\} \tag{10}$$

$$\left|\mathbb{S}_{k,h}^{b}\right| \leq S_{k,\text{nom}}^{b}, \; \{\forall\, h \in \mathbf{H}, \; \forall\, k \in \mathbf{N}\} \tag{11}$$

$$0 \leq \mathrm{Re}\left\{\mathbb{S}_{k,h}^{dg}\right\} \leq S_{k,\text{nom}}^{dg} G_h^{dg}, \; \{\forall\, h \in \mathbf{H}, \; \forall\, k \in \mathbf{N}\} \tag{12}$$

$$P_{k,\text{mín}}^{b} \leq \mathrm{Re}\left\{\mathbb{S}_{k,h}^{b}\right\} \leq P_{k,\text{máx}}^{b}, \; \{\forall\, h \in \mathbf{H}, \; \forall\, k \in \mathbf{N}\} \tag{13}$$

$$V_{\text{mín}} \leq |\mathbb{V}_{k,h}| \leq V_{\text{máx}}, \; \{\forall\, h \in \mathbf{H}, \; \forall\, k \in \mathbf{N}\} \tag{14}$$

$$|\mathbb{I}_{km,h}| \leq I_{km,\text{máx}}, \; \{\forall\, h \in \mathbf{H} \; \forall\, km \in \mathbf{L}\}. \tag{15}$$

Here, $\mathbb{S}_{k,h}^{d,\star}$ denotes the complex power consumption at node k during time period h. The SoC of the type-b battery connected to bus k at times h and $h+1$ is represented by $SoC_{k,h}^{b}$ and $SoC_{k,h+1}^{b}$, respectively. The parameter φ_k^b is the charge/discharge efficiency coefficient of the battery. The variables $SoC_{k,i}^{b}$ and $SoC_{k,f}^{b}$ denote the initial and final SoC values expected for the type-b battery at bus k. The impedance of the branch connecting nodes k and m is expressed by $\mathbb{Z}_{km}$, with $\mathbb{I}_{km,h}$ representing the current that flows through this branch at time h. The symbols $S_{k,\text{nom}}^{g}$ and $S_{k,\text{nom}}^{dg}$ specify the maximum apparent power capabilities of the conventional and distributed generators at bus k. The generation availability profile for a distributed source at bus k is denoted by $G_{k,h}^{dg}$. The maximum power transfer capacity of the power electronic converter associated with the type-b battery at bus k is represented by $S_{k,\text{nom}}^{b}$. The parameters $P_{k,\text{mín}}^{b}$ and $P_{k,\text{máx}}^{b}$ set the limits for active power injection or absorption for the battery at bus k. The variables $V_{\text{mín}}$ and $V_{\text{máx}}$ indicate the minimum and maximum voltage allowable in all network nodes at any given time, while $\mathbb{I}_{km}^{\text{máx}}$ defines the maximum current limit for the line connecting nodes k and m. Finally, the set $\mathbf{L}$ contains all network branches, and $h_{\text{mín}}$ and $h_{\text{máx}}$ represent the start and end of the period set $\mathbf{H}$.

The constraints outlined in (3)-(18) can be interpreted as follows.

Eq. (3) outlines the power balance at each network node for each time interval, ensuring compliance with Kirchhoff's current law (21). Eq. (4) details the energy storage dynamics of the batteries, modeled through the SoC for each time period (4). Eq. (5) applies Ohm's law to each branch in the network, describing the relationship between voltage and current across branch impedances, and Eqs. (6) and (7) specify the initial and final SoC for the batteries at the start and end of the scheduling period, thereby establishing consistent energy management across the timeline (6).

Constraints (8) and (9) impose upper bounds on the power flowing from the substation and ensure that it only provides active power. Constraints (10) and (11) set the power transfer limits for the DGs and BESUs. Eq. (12) ensures that the DGs inject power only when renewable resources are available, thus preventing power absorption. Eq. (13) defines the permissible range for active power charging/discharging by the type-b battery at node k during period h. Finally, Box-Type Constraint (14) ensures that the voltage at all network nodes adheres to regulatory standards for medium-voltage systems (22), and Constraint (18) guarantees that the current flowing through any network branch remains below the thermal capacity of the conductor, thus maintaining a safe operation.

To address the challenges posed by the non-convex constraints inherent in the Optimization Model (1)-(18), this research employs a novel metaheuristic optimizer: the CbDO. The Set of Constraints (3)-(18) encapsulates the technical requirements for operating a power distribution network with DERs. While many of these constraints are convex and manageable within classical optimization frameworks, critical aspects such as the Power Balance Constraint (3) and the Voltage Regulation Bounds (14) introduce significant non-convexities. These complexities arise from nonlinear relationships and inequalities, particularly due to the complex product of voltage terms and the structure of box-type constraints involving inequality conditions.

In light of these characteristics, conventional solvers often struggle to find feasible or globally optimal solutions, especially when handling non-convex, multi-modal search spaces. The aforementioned optimizer, as a metaheuristic technique inspired by the Cauchy distribution, excels in exploring such spaces efficiently (23). Its design allows for a better handling of diverse solution landscapes, leveraging the long-tailed nature of the Cauchy distribution, which promotes extensive exploration and reduces the risk of premature convergence. This is particularly valuable for solving non-convex optimization problems, where local optima may otherwise hinder traditional algorithms.

Thus, employing the CbDO for optimizing complex power system models, such as the one represented in (1)-(18), provides a robust alternative that aligns with the nature of the problem. This approach facilitates the search for high-quality solutions that adhere to technical constraints while effectively navigating intricate non-convexities. The set of variables related to the Optimization Model (1)-(18) is presented in Table **I**.

3. Solution methodology

Our methodology for solving the Optimization Model (1)-(18) involves applying the CbDO to the active and reactive power dispatch problem while considering BESUs. The CbDO, known for its strong exploration capabilities, serves as the leader stage, while a power flow analysis tool acts as the follower stage (6, 24). The leader defines the SoC of the BESUs, represented by $SoC^{b}_{k,h}$. These values are then

Table I. Number of variables in the Optimization Model (1)-(18).

Type	**Number**
Voltage at each node and time step	$n \times h$
Current in the branches and at each time step	$l \times h$
Apparent power from conventional generators at each time step	$n_g \times h$
Apparent power from DGs at each time step	$n_{dg} \times h$
Apparent power from batteries at each time step	$n_b \times h$
SoC of the batteries at each time step	$n_b \times h$
Total variables	$(n + l + n_g + n_{dg} + 2n_b) \times h$

used in the follower stage, where a power flow solver ensures compliance with the power balance constraints by determining voltage profiles and power generation values (25). The main components of the proposed methodology are outlined below.

3.1. Fitness function

In metaheuristic optimization, fitness functions are essential for managing constraints through penalty methods (6, 24). For the problem under study, the fitness function F_f is defined as shown in (16):

$$F_f = D_{\mathrm{CO}_2} + \sum_{k \in \mathbb{C}} \alpha_k(x), \tag{16}$$

where $\alpha_k(x)$ denotes the penalty applied for violating the k^{th} constraint, and $\mathbb{C}$ is the set of constraints considered in the fitness function. The constraints included in calculating the penalty ensure compliance with operating and technical limits, except for the constraints already handled by the follower stage, as is the case with (3).

The penalty function for the current limit of the distribution branches, as expressed in (18), is given below (6):

$$\alpha_I(x) = \gamma_i \max_{h \in \mathbf{H}, km \in \mathbf{L}} \{|\mathbb{I}_{km,h}| - I_{km,\text{máx}}, 0\}, \tag{17}$$

where γ_i is a positive penalty coefficient.

This penalty function only contributes a non-zero value if any branch current exceeds its thermal limit, thereby ensuring that the solutions remain feasible under real-world operating conditions (26).

3.2. Follower stage: power flow solution

With the aim of evaluating the solutions proposed by the CbDO, a power flow solver based on the successive approximations method is utilized to solve the nonlinear power balance constraint (3) (6). The iterative power flow update formula is presented in (18).

$$\mathbb{V}_{d,h}^{t+1} = -\mathbb{Y}_{d,d}^{-1} \left[\operatorname{diag}^{-1}\left(\mathbb{V}_{d,h}^{t,\star}\right) \mathbb{S}_{d,h}^{\star} - \mathbb{Y}_{d,g}\mathbb{V}_{g,h} \right], \tag{18}$$

where $\mathbb{V}_{d,h}^{t+1}$ is the updated vector of demand-node voltages for time h, and $\mathbb{Y}_{d,d}^{-1}$ is the inverse of the admittance matrix relating the demand nodes. The term $\mathbb{S}_{d,h}^{\star}$ represents the net power at these nodes, including the contributions of BESUs and RES.

The iterative process converges when the stopping criterion in (19) is met.

$$\max_{t} \left| \mathbb{V}_{d,h}^{t+1} - \mathbb{V}_{d,h}^{t} \right| \leq \varepsilon, \ \forall h \in \mathbf{H}, \tag{19}$$

where ε is a user-defined tolerance, typically set at 1×10^{-10}.

Once convergence is achieved, the power injected by the conventional source is calculated as per (20).

$$\mathbb{S}_{g,h} = \mathbb{Y}_{d,g}\mathbb{V}_{g,h} + \mathbb{Y}_{g,d}\mathbb{V}_{d,h}, \ \forall h \in \mathbf{H}. \tag{20}$$

This final calculation ensures that the power dispatch observes the network constraints, allowing for an accurate evaluation of the Objective Function (1).

3.3. Leader stage: CbDO

The CbDO is a new math-inspired metaheuristic algorithm designed to explore and exploit the solution space through the properties of the Cauchy distribution (23). Below, we outline its application for the active and reactive power dispatch problem involving BESUs. The SoC of the BESUs was chosen as the set of decision variables for the algorithm. The encoding for these variables is illustrated in Eq. (21), showing the SoC values for two BESUs, one situated at bus k and the other at bus m (6).

$$x_j^p = \underbrace{SoC_{k,h_{\text{mín}}}^b \quad SoC_{k,h}^b \quad \cdots \quad SoC_{k,h_{\text{máx}}}^b}_{\text{BESU } k} \quad \cdots \quad \underbrace{SoC_{m,h_{\text{mín}}}^b \quad SoC_{m,h}^b \quad \cdots \quad SoC_{m,h_{\text{máx}}}^b}_{\text{BESU } m}. \tag{21}$$

Once the SoC variables for the BESUs have been established, the CbDO determines the corresponding active and reactive power injection into the network. This step ensures compliance with all the operating constraints specified in the Optimization Model (1)-(18) for the proper functioning of the BESUs.

3.3.1. Main parameters of the CbDO

The CbDO leverages the Cauchy distribution, which is characterized by the probability density function defined in 22 (27).

$$f(x; x_0, \gamma) = \frac{1}{\pi\gamma \left[1 + \left(\frac{x - x_0}{\gamma}\right)^2\right]}, \tag{22}$$

where x is the random variable, x_0 is the location parameter, and γ is the scale parameter.

The corresponding cumulative distribution function (CDF) is obtained as follows (22):

$$F(x; x_0, \gamma) = \frac{1}{\pi} \arctan\left(\frac{x - x_0}{\gamma}\right) + \frac{1}{2}. \tag{23}$$

The CDF and its properties form the basis for defining the CbDO's exploration and exploitation mechanisms (28).

3.3.2. Constructing the initial population

To initialize the population, the CbDO defines the center of the solution space, μ, and its standard deviation, σ:

$$\mu = \text{mean}\left\{\begin{bmatrix}\mathbf{x}_{\text{mín}} \\ \mathbf{x}_{\text{máx}}\end{bmatrix}\right\}, \tag{24}$$

$$\sigma = \text{std}\left\{\begin{bmatrix}\mathbf{x}_{\text{mín}} \\ \mathbf{x}_{\text{máx}}\end{bmatrix}\right\}, \tag{25}$$

where $\mathbf{x}_{\text{mín}}$ and $\mathbf{x}_{\text{máx}}$ represent the lower and upper bounds of the variables.

The initial population X^t at iteration $t = 0$ is constructed using

$$X_{ij}^t = \mu_j + \alpha_j \sigma_j, \quad \left\{\begin{matrix} i = 1, 2, ..., n_s \\ j = 1, 2, ..., n_v \end{matrix}\right\}, \tag{26}$$

where α_j is a uniformly distributed random number between 0 and 1.

Remark 1 *Each position in the initial population is checked in order to ensure that it lies within the bounds* $\boldsymbol{x}_{\text{mín},j} \leq X_{ij}^t \leq \boldsymbol{x}_{\text{máx},j}$. *If it does not, a correction is applied:*

$$X_{ij}^t = \boldsymbol{x}_{\text{mín},j} + \alpha_j(\boldsymbol{x}_{\text{máx},j} - \boldsymbol{x}_{\text{mín},j}). \tag{27}$$

3.3.3. Exploration and exploitation mechanisms

The CbDO's local exploration uses the quantile function derived from (23), which is defined as follows:

$$Q(p) = x_0 + \gamma \tan\left(\pi\left(p - \frac{1}{2}\right)\right), \tag{28}$$

where p is a random number in the range [0, 1]. This is used in the local update rule:

$$Y_{ij}^t = X_{ij}^t + \alpha_j \gamma^t \tan\left(\pi\left(p_j - \frac{1}{2}\right)\right). \tag{29}$$

For its global exploration, the algorithm calculates the mean $\mathbf{m}^t$ and standard deviation $\mathbf{s}^t$ of the current population:

$$\mathbf{m}^t = \text{mean}(X^t), \tag{30}$$

$$\mathbf{s}^t = \text{std}(X^t). \tag{31}$$

The global update rule is as follows:

$$Y_{ij}^t = X_{ij}^t + \gamma^t \mathbf{s}_j^t \left[\beta_j \left|\mathbf{x}_{\text{best},j}^t - X_{ij}^t\right| + \lambda_j \left|\mathbf{m}_j^t - X_{ij}^t\right|\right], \tag{32}$$

where β_j and λ_j are uniformly distributed random numbers between -1 and 1.

Remark 2 *Note that $\mathbf{x}_{best}^t$ represents the best solution in the population. In minimization problems, this is the solution with the lowest objective function value, whereas, in maximization problems, it corresponds to the solution with the highest value.*

3.3.4. Population replacement and stopping criteria

New solutions generated by the local and global rules are checked and corrected so that they remain within the specified bounds. The replacement rule is as follows:

$$\mathbf{x}_i^{t+1} = \begin{cases} \mathbf{y}_i^t & \text{if } F_f(\mathbf{y}_i^t) < F_f(\mathbf{x}_i^t), \\ \mathbf{x}_i^t & \text{otherwise.} \end{cases} \tag{33}$$

The algorithm stops when the maximum number of iterations $t_{\text{máx}}$ is reached. Fig. 2 presents a flowchart of the proposed leader-follower strategy.

4. Test feeder characterization

To simulate a representative urban distribution network in Colombia, we adapted a 33-bus grid by incorporating the typical demand and solar generation characteristics of Medellín (Fig. 3). This system is supplied with 12.66 kV at the substation terminals (phase-to-ground voltage). Table II presents the detailed electrical configuration, peak demand values, and conductor thermal constraints of the 33-bus grid. This information was taken from (29).

In this adaptation, renewable generation through PV systems and energy storage provided by lithium-ion BESUs were included in order to study their daily operation within the network. Additional specifics are available in (29). Three PV units were strategically positioned at nodes 13, 25, and 30, with nominal capacities of 1125, 1320, and 999 kW, respectively. Moreover, BESUs (types C, A, and B) were placed at nodes 6, 14, and 31. The voltage regulation parameters for the system allowed for deviations of ± 10 %. To ensure the proper operation of the BESUs, the allowable range for the SoC was set between 10 and 90 %, with initial and final SoC levels of 50 %.

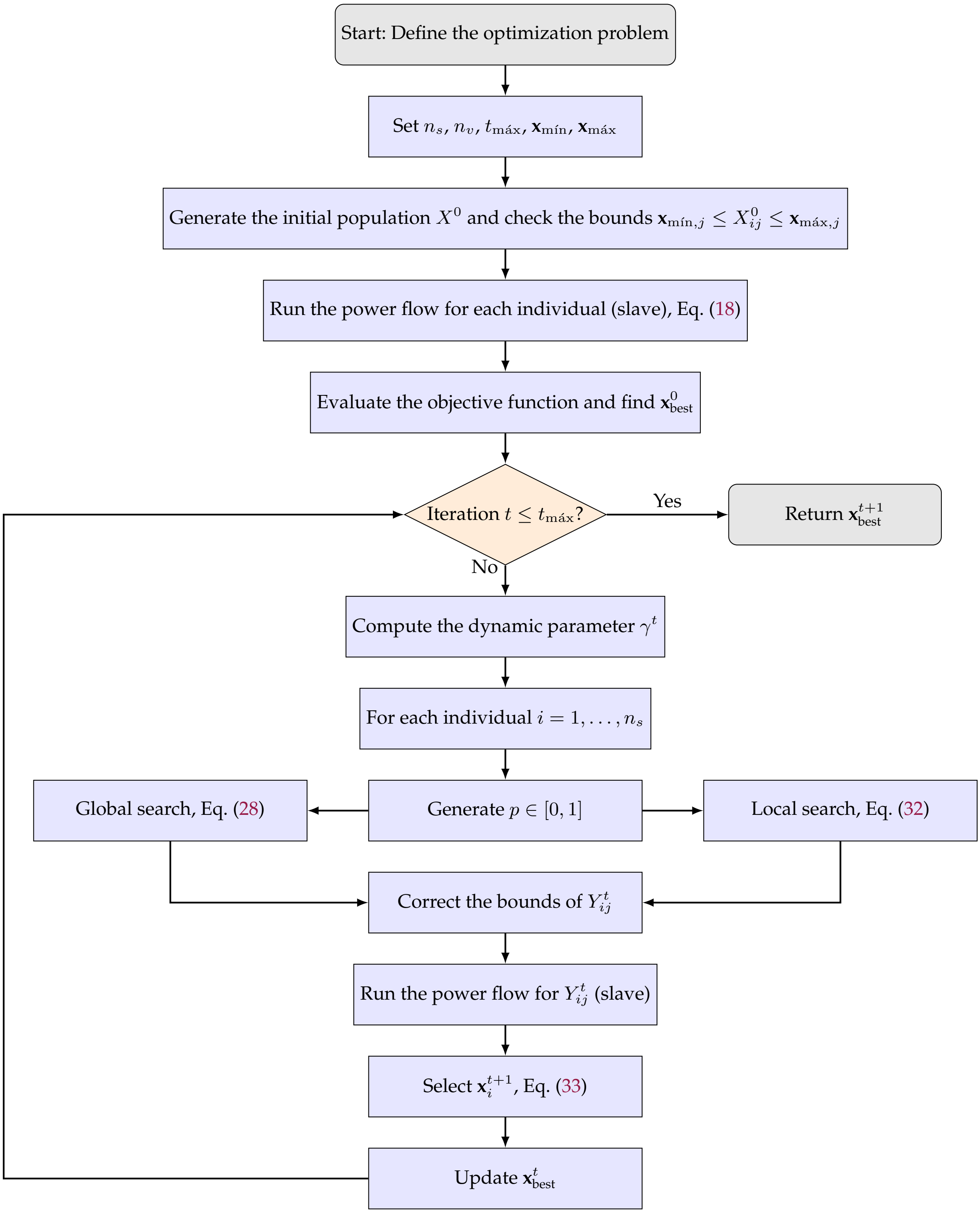

Figure 2. Detailed flowchart of the proposed CbDO-based leader-follower strategy

Table II. Main parameters for the 33-node test system

Node i-j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node i-j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1-2	0.0922	0.0477	100	60	17-8	0.7320	0.5740	90	40
2-3	0.4930	0.2511	90	40	2-19	0.1640	0.1565	90	40
3-4	0.3660	0.1864	120	80	19-20	1.5042	1.3554	90	40
4-5	0.3811	0.1941	60	30	20-21	0.4095	0.4784	90	40
5-6	0.8190	0.7070	60	20	21-22	0.7089	0.9373	90	40
6-7	0.1872	0.6188	200	100	3-23	0.4512	0.3083	90	50
7-8	1.7114	1.2351	200	100	23-24	0.8980	0.7091	420	200
8-9	1.0300	0.7400	60	20	24-25	0.8960	0.7011	420	200
9-10	1.0400	0.7400	60	20	6-26	0.2030	0.1034	60	25
10-11	0.1966	0.0650	45	30	26-27	0.2842	0.1447	60	25
11-12	0.3744	0.1238	60	35	27-28	1.0590	0.9337	60	20
12-3	1.4680	1.1550	60	35	28-29	0.8042	0.7006	120	70
13-14	0.5416	0.7129	120	80	29-30	0.5075	0.2585	200	600
14-15	0.5910	0.5260	60	10	30-31	0.9744	0.9630	150	70
15-16	0.7463	0.5450	60	20	31-32	0.3105	0.3619	210	100
16-17	1.2860	1.7210	60	20	32-33	0.3410	0.5302	60	40

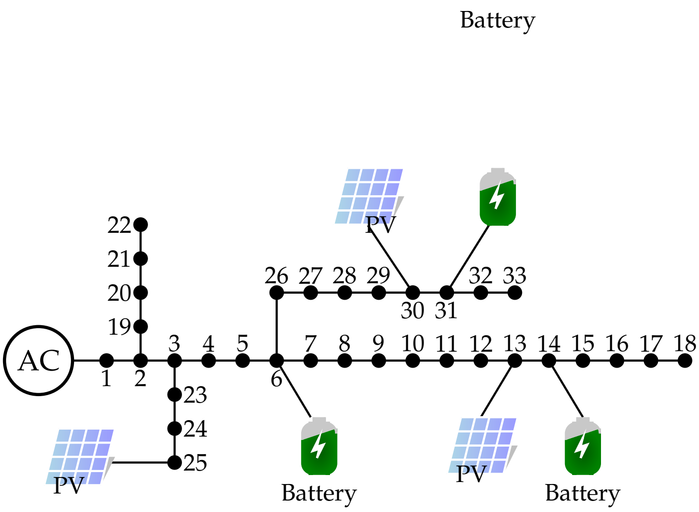

Figure 3. Adaptation of the 33-bus grid for emulating an urban network

Further insights into the classification and characteristics of the different BESU types can be obtained by consulting (30). For a comprehensive view of the hourly daily generation and demand profiles utilized in this study, please refer to (29). Additionally, the CO_2 emissions factor for the PV generators was assumed to be zero (denoted as $C^{dg}_{CO_2} = 0$), while the emissions factor associated with conventional power generation from the substation ($C^{g}_{CO_2}$) was set as 0.16438 g/Wh, according to data from (29).

5. Numerical validations, analysis, and discussion

For the computational implementation of our proposal, we employed the MATLAB programming environment (version 2022b). The simulations were performed on a PC equipped with an AMD Ryzen 7 3700 processor at 2.3 GHz and 16.0 GB of RAM running a 64-bit version of Microsoft Windows 10 Single Language. The optimization of the semidefinite programming (SDP) model was also conducted in the MATLAB software, using the CbDO to solve the studied problem.

5.1. Computational validation against literature reports

A comparative analysis against a recent metaheuristic approach demonstrated that the proposed CbDO method can solve the problem regarding the effective coordination of BESUs and solar generation systems. For the sake of comparison, we conducted two analyses of the proposed optimization approach:

i. A distribution network analysis considering a unitary power factor for the BESUs and PV generators, in order to compare our approach against the continuous genetic algorithm (CGA) and the parallel versions of the particle swarm optimizer (PPSO) and the vortex search algorithm (PVSA). Here, $C^{g}_{\mathrm{CO}_2,k}$ was defined as $0{,}16438$ kg/kWh, and $C^{dg}_{\mathrm{CO}_2,k}$ was assumed to be zero since solar generation, in its operation stage, is regarded as a green energy technology.

ii An evaluation of the effectiveness of the proposed SDP relaxation in managing DERs in medium-voltage distribution networks with a variable power factor, considering three simulation scenarios. The first scenario ($\mathbf{S}_1$) involved the daily dispatch of PV sources operating with a unitary power factor and BESUs with a variable power factor. In the second scenario ($\mathbf{S}_2$), the PV sources were dispatched on a daily basis with a variable power factor, while the BESUs maintained a unitary power factor. Finally, in the third scenario ($\mathbf{S}_3$), all DERs operated with a variable power factor. These scenarios were designed to provide a thorough assessment of the approach under different operating conditions.

5.2. Unitary power factor analysis

Applying the CbDO to solve the EDM developed for dispatching BESUs and PVs in distribution networks while considering a unitary power factor yielded the results presented in Table III.

Table III. Comparison of results regarding both objective functions analyzed

Method	D_{CO_2} (kg/day)	D_{loss} (kWh/day)
Benchmark case	9887.4082	2484.5747
CGA	9878.0207	2431.4745
PPSO	9869.6348	2380.8336
PVSA	9869.5623	2377.8028
CbDO	9869.2134	2374.6028

Based on these numerical results, the following can be stated:

i. The CbDO performs better than the other metaheuristic methods (CGA, PPSO, and PVSA) in relation to both objective functions. Specifically, regarding the expected daily energy losses, CbDO reports a reduction of 3.2000 kWh/day compared to PVSA. As for CO_2 emissions, the CbDO shows a decrease of 0.3489 kg/day with respect to PVSA. Although these improvements may appear modest, they underscore CbDO's ability to effectively explore the solution space and avoid local optima, wherein other metaheuristics might exhibit some limitations.

ii. In comparison with the benchmark case (*i.e.*, grid operation with PVs but without BESUs), the CbDO shows significant reductions in both metrics. The daily energy losses are lowered by approximately 4.4261 %, while the CO_2 emissions are reduced by about 0.1840 %. This confirms the potential benefits of integrating BESUs into distribution networks for optimal energy management, as it yields notable improvements in terms of technical efficiency and environmental sustainability.

5.3. Variable power factor analysis

To validate the effectiveness of the proposed optimization approach regarding a variable power factor operation of DERs, the objective function related to energy losses was analyzed in comparison with the results reported in (31), which employed an SDP approach.

In this analysis, the benchmark case corresponds to the numerical results reported in Table IV, which pertain to the unitary power factor operation of the PVs and BESS. Here, the CbDO results are labeled as $\mathbf{S}_0$.

Table IV. Comparative analysis considering a variable power factor operation for minimizing energy losses

Method	SDP	CbDO
$\mathbf{S}_0$	2373.4053	2374.6028
$\mathbf{S}_1$	1470.3694	1472.1725
$\mathbf{S}_2$	1297.2639	1298.9856
$\mathbf{S}_3$	1266.7221	1267.8945

This comparison aids in evaluating the performance of the proposed CbDO and the SDP approach under various operating scenarios regarding DERs with variable power factor capabilities. Here, the energy losses function serves as the primary metric for assessing the effectiveness of the aforementioned optimization methods. For context, $\mathbf{S}_0$ represents the benchmark case where the PVs and BESUs operate with a unitary power factor.

- **Benchmark comparison ($\mathbf{S}_0$).** The baseline $\mathbf{S}_0$ scenario, wherein the PVs and BESUs are dispatched with a unitary power factor, provides the initial energy losses. According to the results, the SDP approach outperforms the CbDO in this case, with a value of 2373.4053 kWh/day *vs.*

our proposal's 2374.6028 kWh/day. The marginal difference between these results suggests that, while both optimization methods are effective, the SDP approach has a slight edge in this specific configuration.

- **Variable power factor for the BESUs ($\mathbf{S}_1$)**. In scenario $\mathbf{S}_1$, the BESUs are operated with a variable power factor, while the PVs continue to function at a unitary power factor. Here, both the SDP approach and CbDO demonstrate significant improvements in reducing energy losses compared to the benchmark case. The SDP approach achieves energy losses of 1470.3694 kWh/day, while CbDO yields 1472.1725 kWh/day. This substantial reduction *vs.* the $\mathbf{S}_0$ scenario suggests that allowing BESUs to operate with a variable power factor greatly enhances network efficiency. The close difference between the analyzed methods confirms our proposal's comparable performance.
- **Variable power factor for the PVs ($\mathbf{S}_2$)**. Scenario $\mathbf{S}_2$ explores the impact of operating PVs with a variable power factor while the BESUs maintain a unitary power factor. In this scenario, the SDP approach yields energy losses of 1297.2639 kWh/day, while the CbDO reports a slightly higher value (1298.9856 kWh/day). These results highlight the fact that incorporating a variable power factor operation for PVs contributes to further reducing energy losses, suggesting that flexibility in DER operation is crucial for improving distribution system performance.
- **Variable power factor operation ($\mathbf{S}_3$)**. Scenario $\mathbf{S}_3$, where all DERs are dispatched with a variable power factor, reports the most significant improvement in energy losses minimization. The SDP approach achieves the lowest energy losses (1266.7221 kWh/day), followed closely by CbDO (1267.8945 kWh/day). This scenario confirms that a variable power factor operation for all DERs provides the best overall performance. The minimal difference between the results for the SDP and CbDO methods further underscores the efficacy of our proposal as a reliable optimization tool that closely approaches the more complex SDP method.

In summary, the analysis in Table **IV** reveals that the proposed CbDO provides competitive results across all scenarios, closely matching the performance of the SDP solution. The most significant energy losses reductions occur when operating both the PVs and the BESUs with a variable power factor($\mathbf{S}_3$). While the SDP approach slightly outperforms CbDO in each scenario, the differences are small enough to validate our proposal as a practical and efficient alternative for energy losses minimization in medium-voltage distribution networks with DERs. This validation emphasizes the importance of employing a flexible power factor in DER operation to optimize network efficiency.

6. Conclusions and future works

The findings and numerical validations outlined in Section 5 provide important insights into the effectiveness of the proposed CbDO in solving the energy dispatch problem for medium-voltage distribution networks with DERs. In this vein, the following conclusions can be drawn:

- **Performance comparison against metaheuristic methods.** The CbDO consistently outperforms traditional metaheuristic algorithms such as the CGA, PPSO, and PVSA; Table **III** demonstrates that our proposal achieves the lowest expected daily energy losses and CO_2 emissions. Although modest, the reductions indicate that CbDO's design effectively navigates the solution space, mitigates the risk of local optimum traps, and delivers superior optimization outcomes.

- **Benefits of BESU integration.** When comparing the benchmark case against scenarios involving BESU integration, CbDO's results reveal substantial improvements in terms of both energy losses and CO_2 emissions. The inclusion of BESUs facilitates enhanced energy management and system efficiency, with approximately 4.4261 % less energy losses and a 0.1840 % reduction in emissions. This underscores the importance of optimal BESU coordination to achieve technical and environmental gains.
- **Impact of variable power factor operation.** The comparative analysis shown in Table IV highlights the value of implementing variable power factor operation for all DERs. While the SDP approach delivers slightly better results, CbDO's performance is closely aligned, especially in $\mathbf{S}_3$, where all DERs operate with a variable power factor. This scenario yields the most significant decrease in energy losses, demonstrating that flexibility regarding the power factor maximizes network performance. The small margin between CbDO and the SDP approach confirms that our proposal is a viable, efficient, and practical solution for complex, non-linear optimization problems in power systems.

Three future research directions could further advance the findings of this study. Firstly, the proposed CbDO could be enhanced to incorporate adaptive mechanisms that dynamically adjust the balance between exploration and exploitation during optimization, potentially improving convergence rates and solution accuracy. Secondly, applying the CbDO to more extensive and more complex distribution networks, including real-time simulation and control integration, could validate its scalability and practical applicability in diverse grid environments. Lastly, exploring the hybridization of our proposal with other optimization methods, *e.g.*, combining it with machine learning techniques for improved predictive capabilities, could enhance its robustness and adaptability to changing grid conditions and uncertain renewable energy generation profiles.

7. CRediT author statement

Oscar Danilo Montoya and Walter Gil-González conceived the idea and conducted the background research. Maria Camila Vega-Peña and Oscar Danilo Montoya supported the experiments. The collaborative efforts of Maria Camila Vega-Peña, Oscar Danilo Montoya, and Walter Gil-González involved problem identification and data curation. Oscar Danilo Montoya and Walter Gil-González reviewed and supported the editing of the paper. Maria Camila Vega-Peña, Oscar Danilo Montoya, and Walter Gil-González supervised the research and provided critical feedback. It is important to note that all authors have rigorously reviewed the content and have provided their collective consent and approval for the final manuscript.

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