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Planning of a supply chain for anti-personal landmine disposal by means of robots

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PLANEACIÓN DE LA CADENA DE ABASTECIMIENTO PARA LA DETECCIÓN Y ERRADICACIÓN DE MINAS ANTIPERSONALES POR MEDIO DE ROBOTS.

RESUMEN: El artículo presenta un Modelo de Programación Matemática Entera Mixta (MIP), que incorpora decisiones estratégicas y tácticas para la cadena de abastecimiento de detección y erradicación de minas antipersonales por medio de robots. El MIP está basado en un modelo de programación No-lineal Entero Mixto (MINLP) con elementos estocásticos, del cual es una aproximación. El MIP es obtenido por medio de dos procedimientos de solución que incluye la redefinición de variables, el tratamiento de restricciones estocásticas y no-lineales, y la incorporación de restricciones válidas. Finalmente, se presenta un análisis de sensibilidad de parámetros del MIP.

PALABRAS CLAVE: aplicaciones de programación matemática, programación entera, programación no-lineal, logística, cadena de abastecimiento, autómatas, minas anti-personales.

PLANIFICATION DE LA CHAÎNE D'APPROVISIONNEMENT POUR LA DÉTECTION ET L'ÉLIMINATION DE MINES ANTIPERSONNELLES AU MOYEN DE ROBOTS

RÉSUMÉ : L'article présente un Modèle de Programmation Mathématique Entière Mixte (MIP), incorporant des décisions stratégiques et tactiques pour la chaîne d'approvisionnement de détection et d'élimination de mines antipersonnelles au moyen de robots. Le MIP est basé sur un modèle de programmation Non Linéaire Entier Mixte (MINLP) avec des éléments stochastiques, dont il est une approximation, le MIP est obtenu au moyen de deux procédés de solutions incluant la redéfinition de variables, le traitement de restrictions stochastiques et non linéaires, et l'incorporation de restrictions valides. Finalement une analyse de sensibilité des paramètres du MIP est présentée.

MOTS-CLEFS: applications de programmation mathématique, programmation entière, programmation non linéaire, logistique, chaîne d'approvisionnement, automates, mines antipersonnelles.

PLANEJAMENTO DA CADEIA DE ABASTECIMENTO PARA A DETENÇÃO E ERRADICAÇÃO DE MINAS ANTIPESSOAIS POR MEIO DE ROBÔS

RESUMO: O artigo apresenta um Modelo de Programação Matemática Inteira Mista (MIP), que incorpora decisões estratégicas e táticas para a cadeia de abastecimento de detenção e erradicação de minas antipessoais por meio de robôs. O MIP está baseado em um modelo de programação não linear inteiro misto (MINLP) com elementos estocásticos, do qual é uma aproximação, o MIP é obtido por meio de dois procedimentos de solução que inclui a redefinição de variáveis, o tratamento de restrições estocásticas e não lineares, e a incorporação de restrições válidas. Finalmente, apresenta-se uma análise de sensibilidade de parâmetros do MIP.

PALAVRAS-CHAVE: aplicações de programação matemática, programação inteira, programação não linear, logística, cadeia de abastecimento, autômatos, minas antipessoais.

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ABSTRACT: The current paper presents a Mixed-Integer-Linear Programming Model (MIP) which incorporates strategic and tactical management decisions into the supply chain of an anti-personal landmine robotic detection and disposal system. Originally based on a mixed-integer-non-linear programming model (MINLP) with stochastic elements, of which it is an approximation, the MIP model is obtained by means of two solution procedures that include redefining variables, treating stochastic and non-linear constraints, and incorporating valid constraints. The model included considerations such as uncertain procurement, stochastic inventories in plants, production scales, supply-production-distribution capacities, particular distribution-production infrastructure, location-allocation considerations, stochastic demand, and BOM. Additionally, the models detail optimal helicopter operation by considering each period's trip frequency during the planning horizon. Finally, a sensibility analysis of the way in which parameters variations affect overall costs is presented. The suggested solution procedure is considered satisfactory in terms of time for the analyzed example.

KEYWORDS: mathematical programming applications, integer programming, non-linear programming, stochastic programming, logistics, supply chain, robots, anti-personal landmines

1. Introduction

The UN ban on the use of anti-personal landmines as war material (justified by their devastating impact on the world population, specially civilians who are not involved in the conflicts) materialised in the 1997 Ottawa Convention, which endeavoured to find an unanimous agreement on the establishment of observation centres for the surveillance over the de-mining of contaminated territories, the promotion of necessary measures for their disposal, the ban on their production and use as well as the destruction of all existing arsenals (Landmine Monitor Report 2009).

Mine-laying is a common practice in war conflicts due to the low cost and simplicity of construction of anti-personal mines (at least in low sophisticated devices).

In army operations, mines are used in a variety of ways: to block the advance of the enemy into specific areas, or to lead them into other ones where they can be more effectively attacked; to obstruct their movements during attacks; to prevent them from using resources in areas that will be abandoned (natural resources, facilities, equipment, communication routes, etc.); to reinforce natural or artificial obstacles; to prevent enemy retreat or to facilitate one's own, and to get in the way of the enemy's logistic support.

Mines have meant another issue in the horror of war because of the damage they cause to civilians. The main reasons are, on one hand, that landmine laying and clearance dynamics is hardly predictable and carelessly registered, and therefore impossible to analyse. On the other hand, partial mine-clearing of fields, also known in the military jargon as 'gap opening', is a general practice.

The consequences of mine explosions are burns, multiple wounds and infections caused by splinters. Wounds may have deadly consequences due to the impact of the explosion itself or the evolution of the injures received. Besides, there is constant fear amongst the affected population because of the risk of injure they are permanently exposed to.

The 2003 annual mine report informed that, by 2002, anti-personal mines had already injured 300.000 people around the world. The number of mutilated people in Angola is said to be around 20.000 according to the United Nations, and 70.000 according to "Doctors without Borders". For a 10 million people population, the rate could go from 1 out of 500 inhabitants, to 1 out of 145. In Somalia, the approximate rate is 1 out of 650, and in Cambodia, 1 out of 234. In Colombia, the number increased from 216 injured inhabitants in 2001 to 530 in 2002, between January and April of 2003, 151 and 1000 casualties and wounded people were reported in average and the following years. In addition, the 2009 annual mine report estimated the number of casualties in Colombia caused by mines, explosive remnants of war and victim-activated improvised explosive devices, to be 6.696 between 1999 and 2008.

The Red Cross International Committee, which has produced and delivered 7.402 prostheses and 311 orthoses for mine victims in 2011 (ICRC, 2011), estimated by 2003 that 800 people are killed by mines every month, 26 persons a day (ICBL, 2003), while figures from the U.S. Department of State talk about 26.000 casualties and wounded people every year (72 per day). According to estimations published in IDOC Internazionale (Dentico, 1995), for each victim surviving to a mine explosion, two people have died. In some countries, 75% of the survivors require amputations.

Figures are difficult to calculate since most highly-mined countries (with a recently ended conflict or still in conflict) lack the necessary infrastructure to transport and look after the victims on time.

According to "Physicians for Human Rights" (Human Rights Watch/Arms Project and Physicians for Human Rights, 1993), a full recovering medical treatment costs between US$3.000 and US$5.000, while the equipment needed by a child victim to walk again is priced above US$3.000.

The traditional landmine detection procedure on a land strip that has been classified as contaminated by mines is first carried out remotely by causing explosions, either by grenades dropped from aircraft or helicopters, by impacts of artillery shells, or with special vehicles that detonate landmines by direct contact. In a second step, specialized personnel are sent in order to detect remaining mines with hand held devices. Given the many types of mines and the mechanisms used to hide them, it becomes clear that this procedure is far from being efficient, cost saving, safe or environmentally sustainable.

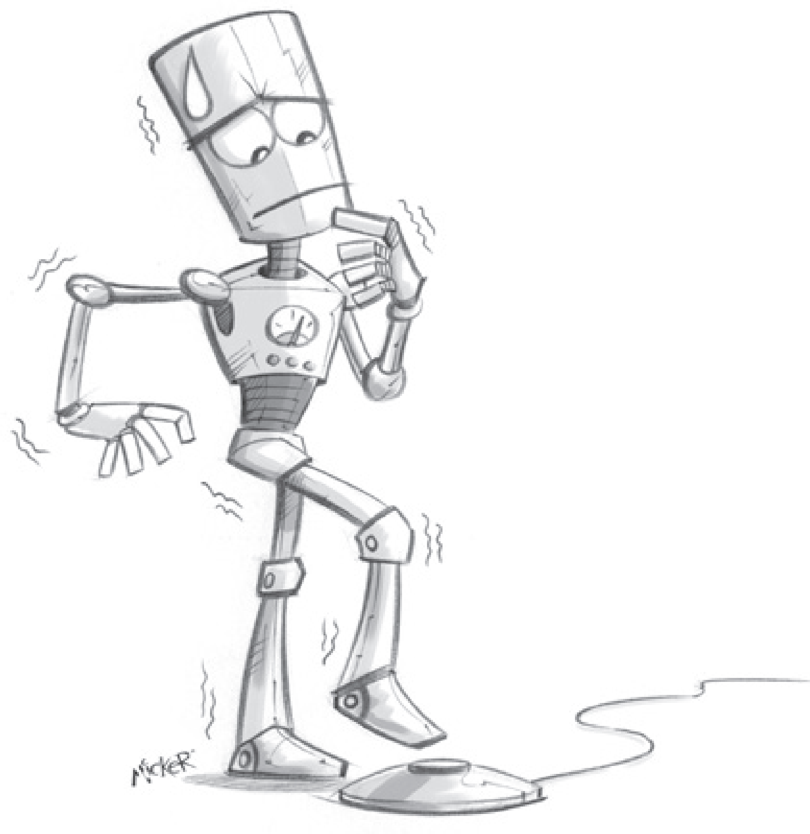
As a result of advances in robot technology, the latter has emerged as a viable innovative option for landmine detection and disposal. Although these developments do not allow large scale applications, it becomes clear that this procedure will enhance safety for specialised personnel, and is much more environmentally friendly when compared to other procedures currently in use.

Considering the future viability of this option, a mathematical programming model to design supply chains for landmine disposal might probably become a sine qua non. The model would have to encompass technical robot requirements, as well as relevant production and logistic considerations. Such is the scope of the current paper.

2. Background

The following review focuses on some topics generally considered as relevant for strategic and tactical decision making in supply chains that have additionally been treated by means of mathematical programming models.

Among the first literature reviews on the topic, the one published by Aikens (1985) makes a description of the most relevant aspects of supply chain modelling for single echelon systems with deterministic demand. The fundamental aspects were associated to facilities location in plants and distribution centres (DCs), as well as raw material and final



product distribution. The problem size was then limited by the absence of a computationally adequate MIP optimiser.

The evolution of supply chain research has spread out in several directions since then. In the first place, as properly indicated by the above mentioned author, towards dynamic modelling considerations and handling of inventories associated with one agent (Cohen and Lee, 1988); in the second place, looking forward to some greater satisfaction of the final consumer (Arntzen *et al*., 1995); in the third place, towards the development of distribution systems (Geoffrion and Powers, 1995); and last, seeking for a better co-ordination of logistic operations in different stages of the supply chain (procurement, production and distribution) (Thomas and Griffin, 1996). The aspects considered by these authors included: lead times, procurement and distribution channel capacity, scale economies and bill of materials, among others.

Towards the end of the twentieth century, due to increased pressures caused by the internationalisation of economy, and taking in new developments in computational processing, the characterization of the global supply chain concept was consolidated by Vidal and Goetschalckx (1997). In order to solve these new aspects, optimisation and heuristic procedures supported by commercial software were developed by Goetschalckx *et al*. (2002), resulting in satisfactory practical results (Vidal and Goetschalckx, 1997). Some study topics on supply chains have included uncertainty in the procurement (Gupta and Maranas, 2003; Chen and Lee, 2004). Dong *et al*. (2004) developed a model oriented towards the equilibrium of the supply chain when there is stochastic demand from retailers to manufacturing plants. The model considered a number of manufacturing plants that produce homogeneous articles, which are later bought by the retailers. The objective consisted on maximizing the total profit. Eskigun *et al*. (2005) proposed the design of a Supply Chains distribution network, considering delivery times, and locations and vehicles capacities. The study by Agarwal *et al*. (2007) uses an interpretative structural model that allows analyzing the way in which the relationships between the variables that make up the supply chain, affect its agility and flexibility. An architecture that allows the assurance, simplification and synchronization of simulation models within a supply chain, is proposed in Iannone *et al*. (2007). Actual and expected delivery times are considered within the problems objective. Finally, Sawik (2009) achieved the integration of material scheduling, material supply and product assembly using a mixed integer programming approach product.

Towards the end of the twentieth century the increased pressure by the economic internationalization of supply chain, new global aspects had to be considered these included: transfer prices (Vidal and Goetschalckx, 2001) and particular topics on design of operations (Mula *et al*., 2011). Recently García *et al*. (2011) presented a work on domestic supply chain environments that included new aspects like reliability, scale economies, organizational considerations, uncertain demand, among other relevant aspects.

Such is the topic of the current document, which boasts an incorporation of strategic and tactical aspects in both static and dynamic contexts, including particular infrastructure and logistics considerations about distribution and production, together with some other important items like reliability of procurement channels, factory location, and stochastic demand. Summarising, the paper presents a model, a solution procedure and a sensibility analysis, applied to a particular example. In order to explain the model, each of the aspects it deals with is presented in both conceptual and mathematical contexts.

3. The Model

Although more complex, supply chain models integrating strategic and tactical decisions are known to allow closer approximations to reality than those only linked to either type of decision (Goetschalckx *et al*., 2002). In fact, the medium scale optimisation instances found in the realm of this particular case make it advisable to apply the former model type. In order to introduce the MINLP and MIP models, a consistent notation is presented as the different aspects of the supply chain are introduced.

Aspects taken into account

The different aspects considered in the model are treated in detail in the following sections.

3.1. Procurement reliability

A very important consideration in supply chain strategic management has to do with channel selection, not only because it involves raw material and supply costs, but procurement lead times and raw material quality too, which all come up as reliability requirements in the production stage. Special attention was given in this paper to the proposals of Vidal and Goetschalckx (2001), which used binary variables to model procurement channel reliability constraints, and then applied them as relevant choosing criteria.

Sets

AIP(i,p) : raw material used up by supplier *i* in product *p*

IJ(j) : plant *j* suppliers

J : plant locations

P : robot types

PA(j) : robot types made in factory *j*

Parameters

$PROB_{ij}^{a}$= reliability of the supply channel that links supplier *i* with plant *j* through raw material *a* (percentage).

PRO_{j}^{p}= reliability goal in the production of robot type *p* at plant *j* (percentage)

Variables

$$v_{ij}^{a} = \begin{cases} 1 \text{ if supplier } i \text{ provides plant } j \text{ with raw material } a \\ 0 \text{ otherwise} \end{cases}$$

Expression for supply channel choice reliability:

$$\prod_{i\in IJ(j)}\prod_{a\in AIP(i,p)}\left(PROB_{ij}^{a}\right)^{V_{ij}^{a}} \geq PRO_{j}^{p} \quad j\in J, p\in PA(j) \qquad (3.1)$$

Reliability is modelled through the supply compliance percentage required by the factories for each raw material, which can be obtained by means of a feasible combination of their suppliers' reliability percentage measurements.

3.2. Bill of Materials (BOM)

The bill of materials constraint has a twofold function in the model, both linking the procurement and distribution stages (between plants and demand zones (DZs)), and quantifying the raw material amounts required to satisfy the demand. Given the importance of these two aspects, this constraint becomes particularly necessary when choosing suppliers and procurement channels in the planning horizon.

Sets

A : raw material

PA(a) : robot types using raw material type *a*

RIJ : supply network made up of logistic links between suppliers and plants

RJK : supply network composed of logistic links between plants and DCs

Parameters

q^{ap} = amount of type *a* raw material used for the construction of one type *p* robot (resource units / robot)

Variables

x_{jk}^{p} = average periodic amount of type p robots delivered from plant j to DZ k in the planning horizon (units / planning horizon)

s_{ij}^{a}= average periodic amount of raw material type a provided by supplier i to production plant j in the planning horizon (units / planning horizon)

Expression for BOM: The proposal includes a constraint for each factory - raw material combination required.

$$\sum_{(i,j)\in RIJ} s_{ij}^{a} = \sum_{p\in PA(a)} \sum_{(j,k)\in RJK} q^{ap} x_{jk}^{p} \qquad j\in J, a\in \qquad (3.2)$$

3.3. Allocation of DCs regarding one singular supply source

The geographical location of production plants and distribution centres depends on the particular operational conditions of the army. A national army generally consists of divisions that are themselves composed of brigades, which are in turn subdivided into battalions that can have assigned special tasks like engineering, cavalry, artillery, infantry, special forces, etc. Yet, the division commanding tasks are assigned to higher level battalions, namely the division ones. To avoid overlapping of competencies, each brigade has its assigned territory. In this manner, the security and efficiency of army operations and the monopoly of war material are safeguarded.

Consequently, any decision making endeavours to outline the location of production plants within division battalions, and their associated DCs in brigade battalions. Additionally, for army operation security reasons, each DC is responsible for the de-mining of just one DZ. In modelling this aspect, Geoffrion and Grave's [16] proposal has been used as a referential milestone, as far as it links the aggregate demand of each DZ to a single procurement source. This constraint is particularly relevant due to the aforementioned army hierarchy. The correspondent expressions are shown below:

Sets

JK(k) : DZs supplied by facility k

K : demand zones

Variables

$$B_{jk} = \begin{Bmatrix} 1 & \text{if plant } j \text{ supplies DZ } k \\ 0 & \text{otherwise} \end{Bmatrix}$$

Expression for single supply source selection

$$\sum_{j\in JK(k)} B_{jk} = 1 \qquad k\in K \qquad (3.3)$$

3.4. Integration of production, distribution and allocation stages

The construction of a robot is typically modular, and it is foreseeable that the main production activities (assembly or manufacture) are carried out in factories, whereas the repairing activities are restricted to distribution centres (DCs). However, the constraint presented here is not only to aim these simple tasks, but also the production of new (or innovative) robot components. Additionally, in the tactical aspect, the model defines robot production and distribution for each planning period.

In proposing these constraints we seek to establish a cross link between the strategic and tactical decisions of the chain. In regard to the former ones, the average aggregate distribution from plants and DCs is linked to the production periods in the factories, in order to correctly choose their relative location and assignment. Division and brigade battalions are usually lodged in strongly guarded locations and have various means of transportation among which their habitually well maintained roads are the most common.

Sets

PJK(j,k) : robot types sent from factory j to DZ k

PK(k) : robot types sent to DZ k

T : periods

Parameters

N = number of periods.

Variables

d_{k}^{p} = average periodic amount of type p robots used at DZ k in the planning horizon (units/period)

y_{kt}^{p} = amount of type p robots delivered at DZ k during period t (units / period)

Expression for unique permissible production and distribution source choice for a DC:

$$x_{jk}^{p} = B_{jk}\, d_{k}^{p} \qquad (j,k)\in RJK,\ p\in PJK(j,k) \qquad (3.4)$$

Expression for link between both average periodical production and periodical production, delivered at a DZ:

$$\sum_{t\in T} y_{kt}^{p} = N\, d_{k}^{p} \qquad k\in K, p\in PK(k) \qquad (3.5)$$

Note that both sides of the balance equation show the aggregated amount of robots at DZs.

3.5. Selection of procurement channels and location of factories

Procurement and location are core decisions in logistics, due to their associated strategic and tactical potential costs along the entire planning horizon. In addition, each supplier bases his calculations on a minimum offer, which depends on his procurement policies and on a maximum supply bound which in turn relates to his production capacity. Consequently, the suggested model includes throughput capacity constraints for each supply channel and production plant.

Sets

AIJ(i,j) : raw material types provided by supplier *i* at plant *j*

JI(i) : plants provided by supplier *i*

KJ(j) : DZs supplied by factory *j*

Parameters

CAP_j^p = periodic production capacity of plant *j* for manufacturing type *p* robot, in the planning horizon (units / period)

O^p = capacity fraction used in the production of type *p* robots (%)

$SMAX_i^a$ = maximum periodic amount of type *a* raw material provided by supplier *i* in the planning horizon (units of raw material / period)

$SMIN_i^a$ = minimum periodic amount of type *a* raw material provided by supplier *i* in the planning horizon (units of raw material / period)

Variables

$$a_j = \begin{cases} 1 & \text{if the plant is located in } j \\ 0 & \text{otherwise} \end{cases}$$

Logical expression linking suppliers to plants, which becomes necessary because a procurement channel can only be selected if its supplied plant location is selected too.

$$v_{ij}^a \le a_j \qquad j \in J, i \in IJ(j), a \in AIJ(i,j) \tag{3.6}$$

Expressions for procurement capacity:

$$SMIN_i^a v_{ij}^a \le s_{ij}^a \le SMAX_i^a v_{ij}^a \quad i \in I, j \in JI(i), a \in AIJ(i,j) \tag{3.7}$$

Raw material flow from suppliers to factories depends on procurement channel reliability. The left (right) side of the above constraint allows to model minimum (maximum) procurement conditions, traditionally imposed by some suppliers, deriving from their production and logistic inflow capacity.

Expressions for production capacity:

$$\sum_{k \in KJ(j)} O^p y_{kt}^p \le CAP_j^p \, a_j \qquad j \in J, t \in T, p \in PJ(j) \tag{3.8}$$

Robot production is bounded by factory capacity, which can be feasible, and consequently positive, only if factory location is also feasible.

3.6. Scale economies

Operation levels define production and distribution (from plants to DZs) scale economies in each time period, a relation that can be modelled through mathematical programming, by defining the production and distribution range sizes for which a related differential cost has been established. The average unitary cost decreases gradually along ranges, up to the point where the capacity is saturated. From then on, an increase can be observed as the demand grows beyond the available capacity and has to be satisfied either by outsourcing or reinvestment. Scale economies have been considered in this article because of their relevance to the design of the supply chain.

Sets

E(p) : type *p* robot production scales

Parameters

$GMAX_k^{pe}$ = maximum production bound for type *p* robots delivered at DZ *k* in operation scale *e* (robot units / period)

$GMIN_k^{pe}$ = minimum production bound for type *p* robots delivered at DZ *k* in operation scale *e* (robot units / time period)

Variables

y_{kt}^{pe} = type *p* robots delivered at DZ *k* in operation scale *e* during period *t* (units / period)

$$w_t^{pe} = \begin{cases} 1 & \text{if product } p \text{ is produced in operation scale } e \text{ during period } t \\ 0 & \text{otherwise} \end{cases}$$

Expression for operation scale: the number of robots distributed during a given time period must have been produced in any of such period's production scales.

$$y_{kt}^p = \sum_{e \in E(p)} y_{kt}^{pe} \quad k \in K, t \in T, p \in PK(k) \tag{3.9}$$

Logical expression for operation scales: one only robot production scale at the most, can be activated in a given time period.

$$\sum_{e \in E(p)} w_t^{pe} \le 1 \qquad t \in T, p \in PK(k) \tag{3.10}$$

Expression for operation scale bounds: in order to match periodical robot production to its corresponding scale, the following constraint is used:

$$(GMIN_k^{pe})w_t^{pe} \leq y_{kt}^{pe} \leq (GMAX_k^{pe})w_t^{pe} t \in T, p \in PK(k), e \in E(p) \quad (3.11)$$

Such constraint allows defining the operation scale at which the production of each robot type is carried out, framing it within its two corresponding bounds. Additionally, and together with constraints (3.9) and (3.10), it assures that the number of robots delivered from each plant to its associated DZs comes from only one production scale.

3.7. Distribution infrastructure

Distribution activities inside the demand zones (from brigades to mine contaminated areas) are usually hampered by lack or bad condition of access roads and by proneness to assaults by enemy forces. Consequently, the access is usually carried out using helicopters. Those aspects of the supply chain that are related to delivery into DZs are very important due to expensiveness of helicopter fleet operation and buying cost. The helicopter fleet size can be obtained by means of the following expression:

Variables

h_{kt} = number of helicopters used at DZ k during period t (units / time period)

$hmax_k$ = minimum number of helicopters used at DZ k in the planning horizon (units)

Expression for helicopter fleet minimum size

$$h\max_k \geq h_{kt} \qquad k \in K, \quad t \in T \quad (3.12)$$

The helicopter fleet minimum size at each DZ corresponds to the maximum number of helicopters used during a given time period within the studied planning horizon.

3.8. Some other considerations about supply chain capacity

As it has been shown, in studying the previous aspects, certain relevant capacity constraints were added to the supply chain model. Nevertheless, it is necessary to include some additional capacity considerations. Two relevant constraints are respectively associated to throughput production capacity due to raw material procurement bounds, and to the necessary infrastructure to carry out the distribution process from plants to DCs and within the DZs.

Sets

$PIA(i,a)$: robots types that use raw material type a provided by supplier i

Parameters

HE_k = number of available helicopters in DZ k

$XMAX_{jk}^p$ = distribution capacity for type p robot from plant j to DZ k in the planning horizon (units / period)

$ZMAX_k$ = maximum number of feasible helicopter trips in DZ k

β^p = helicopter pay load capacity for transporting type p robots (robot units / helicopter)

Variables

z_{kt} = number of helicopter trips at DZ k during period t (trips / period)

Expression for distribution capacity bounds, due to supply channel capacity:

$$SMIN_i^a \leq \sum_{k \in KJ(j)} \sum_{p \in PIA(i,a)} q^{ap} y_{kt}^p \leq SMAX_i^a \quad i \in I, j \in JI(i), t \in T, a \in AIJ(i,j) \quad (3.13)$$

The raw material used for a certain product must be adjusted within the raw material procurement bounds established by each supplier.

Expression for distribution capacity from plants to DCs:

$$y_{kt}^p \leq XMAX_{jk}^p \qquad j \in J, k \in K, \ t \in T, \ p \in PJK(j,k) \quad (3.14)$$

The above constraint takes into account the distribution capacity between plants and DCs.

Expression for distribution capacity within DZs:

$$y_{kt}^p \leq \beta^p \ z_{kt} \ h_{kt} \qquad k \in K, t \in T, \ p \in P \quad (3.15)$$

The above constraint contemplates both payload capacity and operational frequency of helicopter trips within DZs.

Expression for maximum number (maximum bound) of helicopters within DZs:

$$h_{kt} \leq HE_k \qquad k \in K, \quad t \in T \quad (3.16)$$

Expression for maximum number (maximum bound) of helicopter trips per time period within DZs:

$$z_{kt} \leq ZMAX_k \qquad k \in K, \quad t \in T \quad (3.17)$$

3.9. Deamand

Data collected by the local observatory will allow the classification of an area as contaminated by mines. The consequent demand for robots is not determined by the number of mines to be detected, but the extension of the contaminated area, due to the fact that if a specific land strip is contaminated, then it has to be entirely scanned by robots, in disregard of how many mines it might have.

As for the robots, it has to be taken into account that a percentage of them will periodically be rendered useless. Additionally, when considering a DZ, the model assumes a stochastic contaminated area which is scanned by robots, where each robot type has an average scanning speed defined for each period. In order to suitably model the stochastic area, chance constraints are included (Charnes and Cooper, 1959); but this requires a certain level of compliance probability for each given constraint. The associated constraint (eq 3.16) is shown below:

Parameters

α = significance level

θ^p = type *p* robot expected scanning speed in a given planning period (scanned area units ⁄ robot)

φ^p = expected contingent fraction of type *p* robots in a given planning period (percentage ⁄ period)

π^p = type *p* robot estimated operational life span (time units)

$\Theta_k(\xi)$ = mine contaminated stochastic area in DZ *k* in the planning horizon (area units)

Chance constraint for DZ stochastic area robotic de-mining in the planning horizon

$$P\left\{\sum_{p\in P}\sum_{e\in E(p)}\left[\sum_{t=1}^{N-\pi^p}\theta^p\phi^p\pi^p y_{kt}^p w_{kt}^{pe}+\sum_{l=0}^{\pi^p-1}\theta^p\phi^p\left(\pi^p-l\right)y_{k,N-\pi^p+1+l}^p\ w_{k,N-\pi^p+1+l}^{pe}\right]\geq\Theta_k(\xi)\right\}\geq 1-\alpha\quad k\in K \tag{3.18}$$

The above constraints express the minimum permissible target probability required to scan the mine contaminated area in the planning horizon.

3.10. Variable bounds

$$\begin{aligned}
&s_{ij}^a \geq 0 && i\in I,\ j\in J,\ a\in A\\
&x_{jk}^p \geq 0 && j\in J,\ k\in K,\ p\in PK(k)\\
&d_k^p \geq 0 && k\in K,\quad p\in PK(k)\\
&y_{kt}^p \geq 0 && k\in K,\ t\in T,\ p\in PK(k)\ \text{and integer}\\
&y_{kt}^{pe} \geq 0 && k\in K,\ t\in T,\ p\in PK(k),\ e\in E(p)\ \text{and integer}\\
&z_{kt} \geq 0 && k\in K,\quad t\in T\ \text{and integer}\\
&h_{kt} \geq 0 && k\in K,\quad t\in T\ \text{and integer}\\
&h\max_k \geq 0 && k\in K,\quad t\in T\ \text{and integer}
\end{aligned} \tag{3.19}$$

3.11. Objective function

The planning model has to take into account the fixed costs of a series of supply chain activities, like construction (installation) and operation of factories and DCs, or logistic flows of raw materials and manufactured products between suppliers, plants, and DZs. In sum, the fixed costs of the procurement, production and distribution stages. Additionally, the model not only considers certain fixed costs (e.g., infrastructure) that can be estimated by assigning them a constant value along the entire planning horizon, but also those that go through periodical changes (like production and distribution), and are equally included in the planning horizon. Finally, the model follows Silver and Perterson's proposal (1985) of inventory quantification, which includes safety stock factors, lead times, and inventory cycle factors. The proposal assumes that stochastic demand and deterministic lead times are independent for each raw material.

Parameters

F_{ij}^a = type *a* raw material inter arrival time from supplier *i* to plant *j* (time units ⁄ planning horizon)

F_{jk}^p = type *p* robot inter arrival time between plant *j* and DZ *k* (time units ⁄ planning horizon)

FCI = inventory cycle factor (percentage)

FIS_j^a = safety stock factor for type *a* raw material at plant *j* (time units ⁄ planning horizon)

FIS_k^p = type *p* robot safety stock factor at DZ *k* (time units ⁄ planning horizon)

H = holding cost ($ ⁄ $ planning period)

L_{ij}^a = expected lead time for delivering raw material from supplier *i* to plant *j*

L_{jk}^p = expected lead time for delivering type *p* robots from plant *j* to DZ *k* (time ⁄ raw material)

Costs:

C_i^a = procurement cost of raw material *a* provided by supplier *i* (including transportation and duties) ($ ⁄ robot)

C_k^p = distribution cost of type *p* robots employed at DZ *k* (including transportation and duties) ($ ⁄ robot)

C_{jk}^p = cost of type *p* robot distribution from plant *j* to DZ *k* ($ ⁄ period)

C_{kt}^{pe} = production cost of type *p* robots used in DZ *k* in operation scale *e* during period *t* ($ ⁄ robot)

CC_{kt} = fuel costs at DZ k in period t (\$ / helicopter trip)

CF_j = fixed cost of factory j (\$/ planning horizon)

CF_k= operation fixed cost for DZ k (\$ planning horizon)

CH = helicopter cost (\$ / helicopter)

CH_{kt} = helicopter operation fixed cost at DZ k during period t (\$/period)

CI_j^a = raw material a inventory cost at production site j (\$ / planning horizon)

CI_k^p = type p robot inventory cost at DZ k (\$ / period)

CM^p = type p robot maintenance cost (\$ / robot)

$$\begin{aligned}
MIN \quad & \sum_{k\in K}(CH)HMAX_k + \sum_{k\in K}\sum_{t\in T}(CH_{kt})h_{kt} + \sum_{i\in I} CF_j a_j + \sum_{k\in K}\sum_{t\in T}\sum_{p\in PK(k)}\sum_{e\in E(p)} C_{kt}^{pe} y_{kt}^p w_t^{pe} \\
&+ \sum_{j\in JI(i)}\sum_{a\in AIJ(i,j)} N\,C_i^a s_{ij}^a + \sum_{j\in JI(i)}\sum_{a\in AIJ(i,j)} N\left(CI_j^a H\right)\left[L_{ij}^a + (FCI)F_{ij}^a + FIS_j^a\sqrt{L_{ij}^a}\right]s_{ij}^a \\
&+ \sum_{k\in K}\sum_{p\in P} N\,C_k^p d_k^p + \sum_{k\in KJ(j)}\sum_{p\in PJK(j,k)} N\left(CI_k^p H\right)\left[L_{jk}^p + (FCI)F_{jk}^p + FIS_k^a\sqrt{L_{jp}^p}\right]x_{jk}^p + \\
&\sum_{k\in K}\sum_{t\in T}\sum_{p\in P}\sum_{e\in E(p)} CM^p(1-\phi^p)y_{kt}^p w_t^{pe} + \sum_{k\in K}\sum_{t\in T}(CC_{kt})z_{kt} + \sum_{k\in K} CF_k
\end{aligned} \qquad (3.20)$$

4. Solution Procedure 1

The design of the supply chain model was based on conceiving the strategic aspects in a single time period (the planning horizon) and the tactical ones in all periods of the temporary horizon, therefore having respect for the specific nature of both types of decision. The following steps are suggested in order to obtain an MIP model, which is used as an approximation of an MINLP one:

4.1. Step 1: approximation to non-linearity

With respect to non-linear constraints, those related to distribution capacity by means of helicopters at DZs (3.15), procurement reliability (3.1) and stochastic demand area (3.18) are treated here. First of all, the variables associated to helicopter distribution capacity constraints had to be redefined. In the case of the reliability constraint (3.1), logarithmic and absolute value function properties are used. With regards to the stochastic area expression (3.18), a Gaussian area is assumed and then a deterministic approximation of such constraint is obtained. In conclusion, as a result of step 1, a less complex problem is obtained, as shown next:

- As for helicopter distribution constraints, given that:

$$\eta_{kt} = z_{kt}\, h_{kt} \qquad k\in K, \quad t\in T \text{ and integer} \qquad (4.1)$$

and replacing that expression in eq. (3.15), we have:

$$y_{kt}^p \le \beta^p\, \eta_{kt} \qquad k\in K, \quad t\in T \qquad (4.2)$$

Expression for maximum number of periodic distribution trips per fleet at DZ:

$$\eta_{kt} \le MIN\left\{z_{kt}\, HE_k, ZMAX_k\, h_{kt}\right\} \quad k\in K, \quad t\in T \qquad (4.3)$$

- The procurement reliability constraint (3.1) is in turn replaced by the following equivalent:

$$\sum_{i\in IJ(j)}\sum_{a\in AIJ(i,j)} v_{ij}^a \left|\ln(PROB_{ij}^a)\right| \le \left|\ln(PRO_j^p)\right| \quad j\in J, p\in PJ(j) \qquad (4.4)$$

In order to deal with the non-linearity associated to the product of the continuum and binary variables, the approximation suggested by Hanson and Martin (1990) is applied, including variable redefinition and incorporating additional constraints. The procedure is carried out as follows:

Assuming that product Ω x Ξ appears in the model, where Ξ is a binary variable {0.1}, while Ω is a continuous non-negative variable, then the following procedure can be carried out:

$$\begin{aligned}
&\Delta \le \Omega \\
&\Delta \le \Xi\, M_{\Xi} \\
&\Delta \ge \Omega - (1-\Xi)M_{\Omega}
\end{aligned} \qquad (4.5)$$

Where:

M_{Ω}: maximum bound of Ω, corresponding to a positive integer parameter

M_{Ξ}: maximum bound of product Ω x Ξ, corresponding to a positive integer parameter

The transformation can be used to approximate the non-linearity of equations (3.20) and (3.4). As a consequence, the following variables are defined and substituted in the aforementioned constraints:

$$\psi_{kt}^{pe} = y_{kt}^p w_t^{pe} \quad k\in K, t\in T, p\in PK(k), e\in E(p) \qquad (4.6)$$

$$x_{jk}^p = B_{jk} d_k^p \quad j\in J, k\in K, p\in PJK(j,k) \qquad (4.7)$$

Finally, the following constraints are incorporated to the suggested model

$$\psi_{kt}^{pe} \le y_{kt}^p \quad k\in K, t\in T, p\in PK(k), e\in E(p) \qquad (4.8)$$

$$\psi_{kt}^{pe} \le GMAX_k^{pe}\, w_t^{pe} \quad k\in K, t\in T, p\in PK(k), e\in E(p) \qquad (4.9)$$

$$\psi_{kt}^{pe} \ge y_{kt}^p - (1-w_t^{pe})M \quad k\in K, t\in T, p\in PK(k), e\in E(p) \qquad (4.10)$$

$$x_{jk}^p \le d_k^p \quad j\in J, k\in K, p\in PJK(j,k) \qquad (4.11)$$

$$x_{jk}^p \le \left(z_{1-\alpha}\sigma(\Theta_k(\xi)) + E(\Theta_k(\xi))\right)B_{jk} \quad j\in J, k\in K, p\in PJK(j,k) \qquad (4.12)$$

$$x_{jk}^p \ge d_k^p - (1-B_{jk})M \quad j\in J\ k\in K, p\in PJK(j,k) \qquad (4.13)$$

- The stochastic demand constraint (3.18) is replaced by the expression below (the corresponding procedure is presented in the Appendix 1). In consequence, the stochastic area in the mentioned equation is simplified into a Gaussian distribution, which is a commonly encountered condition in practical cases

$$\sum_{p\in P}\sum_{e\in E(p)}\left[\sum_{t=1}^{N-\pi^p}\theta^p\phi^p\pi^p\psi_{kt}^{pe}+\sum_{l=0}^{\pi^p-1}\theta^p\phi^p\left(\pi^p-l\right)\psi_{k,N-\pi^p+1+l}^{pe}\right]\geq z_{1-\alpha}\sigma\left(\Theta_k(\xi)\right)+E\left(\Theta_k(\xi)\right)\quad k\in K \tag{4.14}$$

4.2. Step 2: Acquisition of valid constraints

The minimum and maximum objective bounds of the supply chain are obtained by means of a procedure that stipulates a single production scale range, containing two linear problems to be solved. In the first (second) one, the minimum (maximum) bound, is obtained by determining the maximum (minimum) scale cost for each factory-robot-period combination.

For both models, the associated procedure goes as follows: in the first place, each DZ's product flow bounds are established:

$$GMAX_k^p = Max_{e\in E(p)}[GMAX_k^{pe}]\quad k\in K,\ p\in PK(k) \tag{4.15}$$

$$GMIN_k^p = Min_{e\in E(p)}[GMIN_k^{pe}]\quad k\in K,\ p\in PK(k) \tag{4.16}$$

Flow costs are determined as shown below:

- Model 1:

$$C_{kt}^p = Max_{e\in E(p)}[C_{kt}^{pe}]\quad k\in K, t\in T,\ p\in PK(k) \tag{4.17}$$

- Model 2:

$$C_{kt}^p = Min_{e\in E(p)}[C_{kt}^{pe}]\quad k\in K, t\in T,\ p\in PK(k) \tag{4.18}$$

In this step, equations (4.4), (3.20) and (3.11) are modified, and equations (3.9) and (3.10) are eliminated. The modified constraints are shown next:

$$\begin{aligned}MIN\quad &\sum_{k\in K}(CH)HMAX_k+\sum_{k\in K}\sum_{t\in T}(CH_{kt})h_{kt}+\sum_{i\in I}CF_ja_j+\sum_{t\in T}\sum_{k\in K}\sum_{p\in P}C_{kt}^py_{kt}^p\\&+\sum_{j\in JI(i)}\sum_{a\in AIJ(i,j)}NC_i^as_{ij}^a+\sum_{j\in JI(i)}\sum_{a\in AIJ(i,j)}N\left(CI_j^aH\right)\left[L_{ij}^a+(FCI)F_{ij}^a+FIS_j^a\sqrt{L_{ij}^a}\right]s_{ij}^a\\&+\sum_{k\in K}\sum_{p\in P}NC_k^pd_k^p+\sum_{k\in KJ(j)}\sum_{p\in PJK(j,k)}N\left(CI_k^pH\right)\left[L_{jk}^p+(FCI)F_{jk}^p+FIS_k^p\sqrt{L_{jp}^p}\right]x_{jk}^p+\\&\sum_{k\in K}\sum_{t\in T}\sum_{p\in P}CM^p(1-\phi^p)y_{kt}^p+\sum_{k\in K}\sum_{t\in T}(CC_{kt})z_{kt}+\sum_{k\in K}CF_k\end{aligned} \tag{4.19}$$

In this objective function the production and maintenance costs were customized.

Expression for stochastic demand constraint:

$$\sum_{p\in P}\left[\sum_{t=1}^{N-\pi^p}\theta^p\phi^p\pi^py_{kt}^p+\sum_{l=0}^{\pi^p-1}\theta^p\phi^p\left(\pi^p-l\right)y_{k,N-\pi^p+1+l}^p\right]\geq z_{1-\alpha}\sigma\left(\Theta_k(\xi)\right)+E\left(\Theta_k(\xi)\right)\quad k\in K \tag{4.20}$$

Expression for robot flow bounds at a given DZ:

$$GMIN_k^p\leq y_{kt}^p\leq GMAX_k^p\quad k\in K, t\in T,\ p\in PK(k) \tag{4.21}$$

Once the problems are solved, the objective function bounds can be determined

Minimum bound –*OFMIN*–*: Min {Model 1}*

Maximum bound –*OFMAX*–*: Min {Model 2}*

The valid constraint obtained is incorporated to the original model and presented below:

$$\begin{aligned}OFMIN\quad\leq&\sum_{k\in K}(CH)HMAX_k+\sum_{k\in K}\sum_{t\in T}(CH_{kt})h_{kt}+\sum_{i\in I}CF_ja_j+\sum_{k\in K}\sum_{t\in T}\sum_{p\in P}\sum_{e\in E(p)}C_{kt}^{pe}\psi_{kt}^{pe}\\&+\sum_{j\in JI(i)}\sum_{a\in AIJ(i,j)}NC_i^as_{ij}^a+\sum_{j\in JI(i)}\sum_{a\in AIJ(i,j)}N\left(CI_j^aH\right)\left[L_{ij}^a+(FCI)F_{ij}^a+FIS_j^a\sqrt{L_{ij}^a}\right]s_{ij}^a\\&+\sum_{k\in K}\sum_{p\in P}NC_k^pd_k^p+\sum_{k\in KJ(j)}\sum_{p\in PJK(j,k)}N\left(CI_k^pH\right)\left[L_{jk}^p+(FCI)F_{jk}^p+FIS_k^p\sqrt{L_{jp}^p}\right]x_{jk}^p+\\&\sum_{k\in K}\sum_{t\in T}\sum_{p\in P}\sum_{e\in E(p)}CM^p(1-\phi^p)\psi_{kt}^{pe}+\sum_{k\in K}\sum_{t\in T}(CC_{kt})z_{kt}+\sum_{k\in K}CF_k\leq OFMAX\end{aligned} \tag{4.22}$$

Summarizing, it can be said that, as a result of the solution procedure, an MIP (and therefore a more treatable) formulation of the original problem is attained. Such formulation is made up of equations: 3.2, 3.3, 3.5 to 3.14, 3.16, 3.17, 3.19, 4.2 to 4.4, 4.8 to 4.14, and equation 4.22, together with its associated objective function.

5. Solution Procedure 2

If the production scale constraints (eq: 3.9, 3.10 and 3.11) are not directly included in the MNMIP model, but are contemplated later, once the new relaxed MIP problem has been solved, a new solution procedure can be obtained, as presented next.

5.1. Step 1: **solving the relaxed MIP problem**: the solution of the problem is expressed by constraints (equations) 3.2, 3.3, 3.5 to 3.8, 3.12 to 3.14, 3.16, 3.17, 3.19, 4.2 to 4.4, 4.11 to 4.14, 4.20, 4.21, and a modified objective function (eq. 4.19) from which the production cost has been removed, as shown below:

$$\begin{aligned}MIN\quad &\sum_{k\in K}(CH)HMAX_k+\sum_{k\in K}\sum_{t\in T}(CH_{kt})h_{kt}+\sum_{i\in I}CF_ja_j\\&+\sum_{j\in JI(i)}\sum_{a\in AIJ(i,j)}NC_i^as_{ij}^a+\sum_{j\in JI(i)}\sum_{a\in AIJ(i,j)}N\left(CI_j^aH\right)\left[L_{ij}^a+(FCI)F_{ij}^a+FIS_j^a\sqrt{L_{ij}^a}\right]s_{ij}^a\\&+\sum_{k\in K}\sum_{p\in P}NC_k^pd_k^p+\sum_{k\in KJ(j)}\sum_{p\in PJK(j,k)}N\left(CI_k^pH\right)\left[L_{jk}^p+(FCI)F_{jk}^p+FIS_k^p\sqrt{L_{jp}^p}\right]x_{jk}^p+\\&\sum_{k\in K}\sum_{t\in T}\sum_{p\in P}CM^p(1-\phi^p)y_{kt}^p+\sum_{k\in K}\sum_{t\in T}(CC_{kt})z_{kt}+\sum_{k\in K}CF_k\end{aligned} \tag{5.1}$$

5.2. ***Step 2**: **integrating the production cost:*** In order to integrate the production cost into the model, the production scale constraints must be included. This allows to incorporate the algorithm below, starting from the optimal values of y_{kt}^p obtained in the first step, and noted here as Ψ_{kt}^p

Begin
$S \leftarrow 0$
For k = 1 to $N(k)$
  For t = 1 to $N(t)$
    For p = 1 to $N(p)$
      For e = 1 to $N(e)$
        If $\Psi_{kt}^{p} > 0$ and $GMIN_k^{pe} \leq \Psi_{kt}^{p} \leq GMAX_k^{pe}$
          Then $\Xi \leftarrow \{k, t, p, e\}$
            $S \leftarrow S + c_{kt}^{pe} \Psi_{kt}^{p}$
        End If
      End For
    End For
  End For
End For
End

Where Ξ is the set of indexes associated to the positive optimal flows Ψ_{kt}^{p}, and S is the supply chain production cost. Finally, the production cost is added to the optimal solution of the objective function (eq. 4.19).

6. Sensibility Analysis

The sensibility analysis was performed using solution procedure 1, with the aid of a commercial LINGO 10™ software package. The computing of the scenarios was carried out on a Pentium-4 2.8 Ghz, 1GB RAM equipment, applying a Win XP-SP2 operational program. The problem comprises 20 suppliers handling 2 components each, 10 possible plant locations, 2 products in two production scales, and 20 Gaussian DZs. All possible combinations of logistic procurement connections were admitted, together with a distribution network featuring three DCs per production plant. Finally, a 5% Gaussian significance was used for the demand chance constraint. The used data can be seen in the Appendix 2. These problems have 14048 constraints and 4171 variables, of which 1241 are binary. The parameters under scrutiny were: area size, helicopter capacity and robot speed performance. Instance solutions take 500 seconds in average, with a maximum of 900 seconds. All the program runs were conducted with a maximum gap of 0.001, and most of the results were probably optimal.

The following table presents the percentage ranking of the average unitary costs that are part of the objective function:

TABLE 1. Average unitary cost percentage structure of the supply chain

CH_{kt}	CF_j	C_{kt}^{pe}	C_i^a	CI_j^a	C_k^p	CI_k^p	CM^p	CC_k^t
64	35.62	0,0725	0.0534	0,0312	0,0044	0,04025	0,05	0,0001

Fuente: elaboración propia.

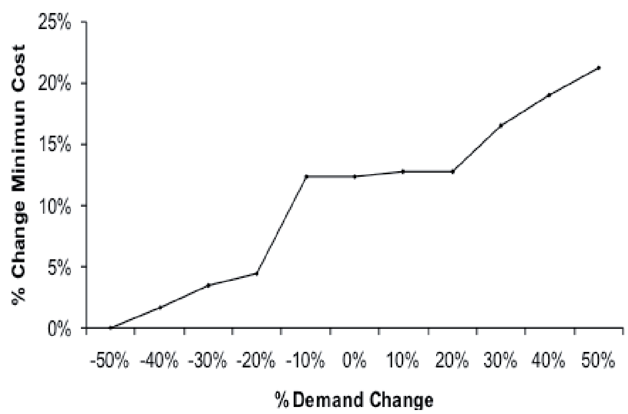
As it can be observed, the highest average unitary costs correspond to the strategic aspects, which are infrastructure distribution, and plant and distribution centre installation.

The sensibility analysis is applied by modifying the parameter percentage values and monitoring their impacts on the total cost of the supply chain.

6.1. Demand effect on minimum cost

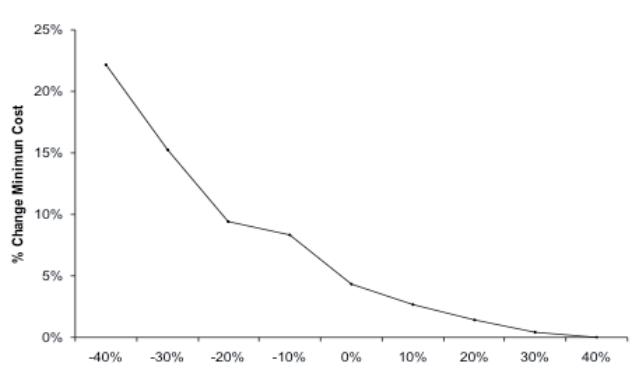
Although the costs rise in direct relation to the expansion of the demand area, the non-linear nature of the problem becomes evident when changes in production scale and distribution infrastructure occur (e.g. more helicopters are required in order to satisfy a greater demand for robots). The smooth behaviour of the costs in some sections of Figure 1 is due to tactical changes in the supply chain. The slope remains unchanged as long as no adjustments in production or distribution infrastructure are undertaken in order to satisfy the demand, as can be observed in Figure 1.

FIGURE 1. Effect of demand on minimum cost



6.2. Effect of robot scanning speed on minimum cost

FIGURE 2. Effect of robot scanning speed on minimum cost

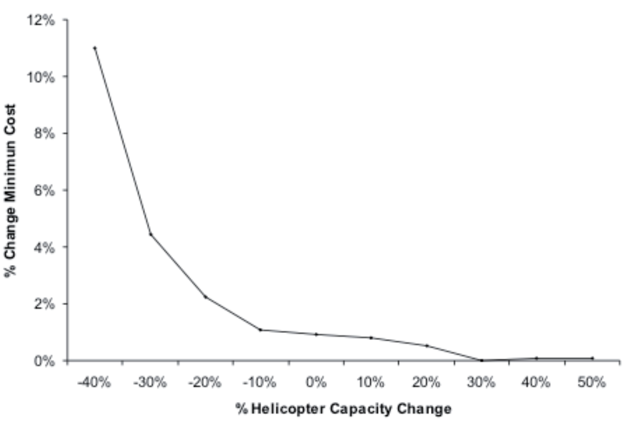


Robot scanning speed appears to be the most influential cost factor, since it constitutes the link between inventory, production and distribution activities in both strategic and tactical contexts. On one hand, the costs associated to production and distribution, diminish as scanning speed increases. On the other hand, inventory costs rise when there are an increasing number of unused robots during long time periods, as a consequence of the scanning speed increase. These two tendencies determine a general behaviour, according to which there is an overall cost percentage drop, marginally decreasing as the scanning speed is raised, due to the existence of counter-balance costs within the same objective function. (See figure 2).

6.3. Effect of helicopter capacity on minimum cost

Due to the fact that helicopters offer multiple usage possibilities, it is important to analyze their potential payload percentage per trip. As shown in figure 4, helicopter payload capacity has a significant impact on distribution costs, and therefore, on overall costs. Variation in total costs as a result of changes in helicopter payload capacity is similar to that shown in figure 2, resulting from robot scanning speed, as far as both parameters have a similar effect on distribution costs at DZs.

FIGURE 3. Effect of helicopter capacity on minimum cost



Conclusions

The current paper presents an MIP and an MNMIP models for anti-personal landmine disposal by means of robots that –following the recommendations found in the cited references– include and integrate strategic and tactical relevant aspects of the supply chain. For that reason the model includes certain considerations, such as uncertain procurement, stochastic inventories in plants, production scales, supply-production-distribution capacities, particular distribution-production infrastructure, location-allocation considerations, stochastic demand, and BOM. Additionally, the models detail optimal helicopter operation by considering each period's trip frequency during the planning horizon.

For their solution, two procedures are suggested, allowing converting the original (and relatively hard) MINLP into a not so hard MIP. The efficiency of the second procedure tends to improve when the operation scales are considerable enough; otherwise, it lessens. On the other hand, the first procedure is advantageously explicit, in contrast with the second one, where the production cost is algorithmically induced.

Finally, the impact of parameter percentage variations on the overall cost is analyzed. The costs do not substantially differ if percentage variations do not entail changes in plant operation level or in the number of helicopters used in the distribution. The suggested solution procedure is considered satisfactory in terms of time for the analyzed example.

Among the new research possibilities opened up by this paper are the development of new solution procedures (e.g., decomposition methods) that allow the application of the model in larger scales of optimization, and the consideration of qualitative aspects that can be relevant in these types of organizations, such as transaction costs,. The transaction costs are important because have a significant impact on the total costs, nevertheless these cannot be measured directly and require of new developments (like to IAM; (García *et al*., 2009) and so, can be included in the optimization of the supply chain.

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Appendix 1

The chance approximation of the stochastic demand constraint is presented in the following equation:

$$P\left\{\sum_{p\in P}\sum_{e\in E(p)}\left[\sum_{t=1}^{N-\pi^p}\theta^p\phi^p\pi^p y_{kt}^p w_{kt}^{pe}+\sum_{l=0}^{\pi^p-1}\theta^p\phi^p\left(\pi^p-l\right)y_{k,N-\pi^p+1+l}^p\, w_{k,N-\pi^p+1+l}^{pe}\right]\geq\Theta_k^p(\xi)\right\}\geq 1-\alpha \quad k\in K \tag{A1}$$

$$P\left\{\frac{A_k(\xi)-E\left(\Theta_k(\xi)\right)}{\sigma\left(\Theta_k(\xi)\right)}\leq\frac{\sum_{p\in P}\sum_{e\in E(p)}\left[\sum_{t=1}^{N-\pi^p}\theta^p\phi^p\pi^p y_{kt}^p w_{kt}^{pe}+\sum_{l=0}^{\pi^p-1}\theta^p\phi^p\left(\pi^p-l\right)y_{k,N-\pi^p+1+l}^p\, w_{k,N-\pi^p+1+l}^{pe}\right]-E\left(\Theta_k(\xi)\right)}{\sigma\left(\Theta_k(\xi)\right)}\right\}\geq 1-\alpha \quad k\in K \tag{A2}$$

$$F\left(\frac{\sum_{p\in P}\sum_{e\in E(p)}\left[\sum_{t=1}^{N-\pi^p}\theta^p\phi^p\pi^p y_{kt}^p w_{kt}^{pe}+\sum_{l=0}^{\pi^p-1}\theta^p\phi^p\left(\pi^p-l\right)y_{k,N-\pi^p+1+l}^p\, w_{k,N-\pi^p+1+l}^{pe}\right]-E\left(\Theta_k(\xi)\right)}{\sigma\left(\Theta_k(\xi)\right)}\right)\geq 1-\alpha \quad k\in K \tag{A3}$$

Under the normal demand assumption

$$\frac{\sum_{p\in P}\sum_{e\in E(p)}\left[\sum_{t=1}^{N-\pi^p}\theta^p\phi^p\pi^p y_{kt}^p w_{kt}^{pe}+\sum_{l=0}^{\pi^p-1}\theta^p\phi^p\left(\pi^p-l\right)y_{k,N-\pi^p+1+l}^p\, w_{k,N-\pi^p+1+l}^{pe}\right]-E\left(\Theta_k(\xi)\right)}{\sigma\left(\Theta_k(\xi)\right)}\geq z_{1-\alpha}\quad k\in K \tag{A4}$$

The following is an approximate deterministic constraint of equation (A1).

$$\sum_{\substack{p\in P \\ k\in K}}\sum_{e\in E(p)}\left[\sum_{t=1}^{N-\pi^p}\theta^p\phi^p\pi^p y_{kt}^p w_{kt}^{pe} + \sum_{l=0}^{\pi^p-1}\theta^p\phi^p\left(\pi^p - l\right) y_{k,N-\pi^p+1+l}^p \; w_{k,N-\pi^p+1+l}^{pe}\right] \geq z_{1-\alpha}\sigma\left(\Theta_k(\xi)\right) + E\big(\big(\tag{A5}$$

Appendix 2

DCs (k): 20

Plants (j): 10

Raw materials (a): 2

Scales (e): 2

Suppliers (i): 20

Time periods (t): 10

Types of robots (p): 2

Parameters

CH = 3000000

H = 0.8

FCI = 0.9

$(\boldsymbol{\Theta}_k(\epsilon)) = 0, k \in K$

TABLE 2.1. Values of parameter $PROB_{ij}^a$

i	j =1		j = 2		j = 3		j = 4		j = 5		j = 6		j = 7		j = 8		j = 9		j = 10	
	a=1	a=2	a=1	a=2	a=1	a=2	a=1	a=2	a=1	a=2	a=1	a=2	a=1	a=2	a=1	a=2	a=1	a=2	a=1	a=2
1	0,98	1	0,95	0,92	0,92	1	0,95	1	1	1	0,95	0,95	1	1	1	1	1	0,95	1	1
2	0,95	1	0,94	0,98	0,98	1	0,91	0,99	1	1	0,91	0,91	1	1	1	1	1	0,91	0,99	1
3	0,94	1	0,93	0,99	0,99	1	0,94	0,96	1	1	1	1	1	1	1	1	1	0,94	0,96	1
4	0,93	1	0,92	0,96	0,96	1	0,95	0,95	1	1	1	1	1	1	0,96	0,92	1	0,95	0,95	1
5	0,92	1	0,98	0,95	0,95	1	0,96	1	1	1	1	1	1	1	0,98	0,98	1	0,96	1	1
6	0,98	1	0,99	0,95	0,95	1	0,95	1	1	1	1	1	0,92	0,92	0,99	0,99	1	0,95	1	1
7	0,99	1	0,96	0,91	0,91	0,96	0,99	0,94	1	1	0,92	0,92	0,98	0,98	0,96	0,96	0,96	0,99	0,94	1
8	0,96	1	0,98	0,94	0,94	0,95	0,96	0,93	1	1	0,98	0,98	0,99	0,99	0,95	0,95	0,95	0,96	0,93	1
9	0,95	1	0,99	0,93	0,93	0,95	0,98	0,92	0,95	0,95	0,99	0,99	0,96	0,96	0,95	0,95	0,95	0,98	0,92	0,95
10	0,95	1	0,96	0,92	0,92	0,91	0,99	0,98	0,95	0,95	0,96	0,96	0,95	0,95	1	0,91	0,91	0,99	0,98	0,95
11	0,91	1	0,95	0,98	0,98	0,97	0,96	0,99	0,92	0,92	0,95	0,95	0,95	0,95	1	0,94	0,97	0,96	0,99	0,92
12	0,97	1	0,95	0,99	0,99	0,99	0,95	0,96	0,98	0,98	0,95	0,95	0,91	0,91	1	0,93	0,99	0,95	0,96	0,98
13	0,99	1	0,91	0,96	0,96	0,96	0,95	0,95	0,99	0,99	0,91	0,91	0,94	0,94	1	0,92	0,96	0,95	0,95	0,99
14	0,96	1	0,94	0,96	0,96	0,99	0,92	0,95	0,96	0,96	0,94	0,94	0,93	0,93	0,95	0,98	0,99	0,92	0,95	0,96
15	0,97	1	0,95	0,95	0,95	0,96	0,98	0,91	0,92	0,92	1	1	0,92	0,92	0,92	0,99	0,96	0,98	0,91	0,92
16	0,98	1	0,96	0,95	0,95	0,92	0,99	0,97	0,98	0,98	1	1	0,98	0,98	0,98	0,96	0,92	0,99	0,97	0,98
17	0,94	1	0,95	0,91	0,91	0,98	0,96	1	0,99	0,99	1	1	0,99	0,99	0,99	0,96	0,98	0,96	1	0,99
18	0,95	1	0,99	0,97	0,97	0,99	0,98	1	0,96	0,96	1	1	0,96	0,96	0,96	0,95	0,99	0,98	1	0,96
19	0,96	1	0,96	0,99	0,99	0,96	0,95	1	0,96	0,96	1	1	0,96	0,96	0,98	1	0,96	0,95	1	0,96
20	0,95	1	1	0,96	0,96	1	0,99	1	0,95	0,95	0,95	0,95	0,95	0,95	0,98	1	1	0,99	1	0,95

Fuente: elaboración propia.

TABLE 2.2. Values of parameters PRO_j^p, CAP_j^p, CF_j

		j =1	j = 2	j = 3	j = 4	j = 5	j = 6	j = 7	j = 8	j = 9	j = 10
PRO_j^p	p=1	0,95	0,96	0,94	0,92	0,95	0,94	0,96	0,92	0,97	0,96
	p=2	0,95	0,96	0,94	0,92	0,95	0,94	0,96	0,92	0,97	0,96
CAP_j^p	p=1	100	100	100	100	100	100	100	100	100	100
	p=2	80	80	80	80	80	80	80	80	80	80
CF_j		883350	1070409	1382117	1208924	1313932	858596	987886	1280984	1441017	1245165

Fuente: elaboración propia.

TABLE 2.3. Values of parameters Q^{ap}, O^p, β^p, θ^p, φ^p, π^p, CM

	Q^{ap}		O^p	β^p	θ^p	φ^p	π^p	CM
	a=1	a=2						
p = 1	2	3	1	5	0,0001	0,1	4	1500
p = 2	1	2	1	4	0,0002	0,12	5	2000

Fuente: elaboración propia.

TABLE 2.4. Values of parameters CI_j^a, FIS_i^a

	FIS_j^a		CI_j^a	
i	a=1	a=2	a=1	a=2
1	0.28571429	0.76190476	1000	1500
2	0.28571429	0.47619048	1000	1500
3	0.71428571	0.71428571	1000	1500
4	0.57142857	0.23809524	1000	1500
5	0.28571429	0.9047619	1000	1500
6	0.95238095	0.38095238	1000	1500
7	0.38095238	0.28571429	1000	1500
8	0.42857143	0.38095238	1000	1500
9	0.28571429	0.66666667	1000	1500
10	0.47619048	0.5	1000	1500

Fuente: elaboración propia.

TABLE 2.5. Values of parameters $SMIN_i^a$, $SMAX_i^a$

	$SMIN_i^a$		$SMAX_i^a$		C_i^a	
i	a=1	a=2	a=1	a=2	a=1	a=2
1	10	10	500	400	6476	11591
2	20	1	100	50	5662	8240
3	20	20	500	400	5713	9681
4	10	15	300	200	5726	11257
5	25	10	500	400	5200	9978
6	30	20	800	700	5946	11859
7	40	30	500	400	5096	9634
8	60	15	200	100	6098	10675
9	50	10	400	300	5667	10370
10	10	15	600	500	5274	10578
11	1	15	300	200	5446	10166
12	20	20	500	400	5725	10346
13	30	20	400	300	5310	10993
14	15	15	200	100	6326	11878
15	20	10	100	10	5435	9762
16	20	10	100	10	5310	9203
17	15	10	200	100	6474	10033
18	10	1	250	150	5893	11079
19	20	10	300	200	5023	10669
20	10	10	250	150	5712	8589

Fuente: elaboración propia.

TABLE 2.6. Values of parameters $GMIN_k^{pe}$, $GMAX_k^{pe}$, $ZMAX_k$, HE_k, $E(\Theta_k(\epsilon))$, FIS_k^p

	$GMIN_k^{pe}$				$GMAX_k^{pe}$				C_k^p		CI_k^p		$ZMAX_k$	HE_k	FIS_k^1	FIS_k^2	$E(\Theta_k(\epsilon))$
	p = 1		p = 2		p = 1		p = 2		p = 1	p = 2	p = 1	p = 2					
k	e = 1	e = 2	e = 1	e = 2	e = 1	e = 2	e = 1	e = 2									
1	0	0	0	0	50	50	50	50	131	150	1000	1500	15	2	0.28571429	0.76190476	60917
2	0	0	0	0	50	50	50	50	116	166	1000	1500	12	4	0.28571429	0.47619048	47480
3	0	0	0	0	50	50	50	50	144	154	1000	1500	20	3	0.71428571	0.71428571	33919
4	0	0	0	0	50	50	50	50	122	165	1000	1500	10	3	0.57142857	0.23809524	40101
5	0	0	0	0	50	50	50	50	118	128	1000	1500	16	2	0.28571429	0.9047619	45636
6	0	0	0	0	50	50	50	50	148	151	1000	1500	12	4	0.95238095	0.38095238	36463
7	0	0	0	0	50	50	50	50	103	150	1000	1500	5	4	0.38095238	0.28571429	42538
8	0	0	0	0	50	50	50	50	120	156	1000	1500	6	1	0.42857143	0.38095238	30097
9	0	0	0	0	50	50	50	50	157	153	1000	1500	14	1	0.28571429	0.66666667	43618
10	0	0	0	0	50	50	50	50	149	138	1000	1500	9	3	0.47619048	0.5	46170
11	0	0	0	0	50	50	50	50	111	159	1000	1500	10	4	0.47936508	0.62	33733
12	0	0	0	0	50	50	50	50	122	132	1000	1500	11	4	0.71428571	0.08	47173
13	0	0	0	0	50	50	50	50	111	126	1000	1500	9	3	0.23809524	0.97	46494
14	0	0	0	0	50	50	50	50	140	148	1000	1500	19	4	0.9047619	0.98	32479
15	0	0	0	0	50	50	50	50	137	171	1000	1500	7	1	0.38095238	0.32	47545
16	0	0	0	0	50	50	50	50	145	135	1000	1500	7	4	0.28571429	0.94	43509
17	0	0	0	0	50	50	50	50	124	149	1000	1500	15	2	0.71428571	0.17	43749
18	0	0	0	0	50	50	50	50	117	162	1000	1500	11	1	0.57142857	0.14	39292
19	0	0	0	0	50	50	50	50	126	168	1000	1500	17	1	0.28571429	0.52	40267
20	0	0	0	0	50	50	50	50	121	163	1000	1500	12	1	0.95238095	0.11	48242

Fuente: elaboración propia.

TABLE 2.7. Values of parameter $XMAX_{jk}^p$

j	1		2		3		4		5		6		7		8		9		10	
k \ p	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1	3	7	5	6	4	4	5	4	4	6	5	4	3	7	4	3	5	3	4	4
2	3	4	7	4	6	3	4	6	6	5	6	5	5	3	4	6	4	7	7	7
3	6	3	6	5	4	7	6	7	3	7	7	3	5	3	3	4	3	3	7	4
4	5	4	4	6	4	6	5	4	6	5	7	7	6	4	7	6	5	7	3	5
5	6	4	7	5	3	7	6	4	4	4	7	6	7	6	3	7	5	7	7	6
6	5	5	3	7	5	4	4	6	5	3	3	7	4	7	5	4	5	7	3	3
7	3	6	5	3	3	7	6	7	3	5	7	4	7	7	3	7	7	3	4	3
8	6	4	6	7	5	4	7	7	5	6	4	5	4	3	3	7	5	6	6	7
9	5	3	6	3	3	4	4	4	3	3	7	7	3	5	3	4	6	5	4	4
10	6	5	7	3	3	6	7	5	3	3	4	6	5	4	6	5	7	3	6	5
11	7	7	7	5	5	6	3	3	7	5	4	3	6	3	3	3	5	4	5	7
12	5	5	7	3	5	3	6	7	7	6	7	3	3	7	3	7	5	5	7	3
13	7	5	3	5	4	4	4	7	7	5	3	5	7	5	6	7	7	5	5	5
14	6	5	6	7	3	3	4	7	7	4	4	7	4	6	7	7	4	5	5	3
15	5	7	7	6	5	5	7	3	4	7	3	3	4	7	6	3	6	6	7	7
16	6	4	4	4	4	6	4	6	3	3	4	7	3	3	4	5	4	7	6	4
17	4	5	7	5	6	7	3	4	3	5	6	7	3	7	5	4	7	7	6	5
18	3	6	7	3	3	7	7	7	4	3	7	5	5	4	6	6	3	3	7	7
19	6	7	5	6	7	7	5	6	3	5	3	7	3	5	7	5	3	6	6	6
20	5	4	3	5	3	5	6	7	5	4	4	5	4	5	5	4	4	5	4	3

Fuente: elaboración propia.

TABLE 2.8. Values of parameter L_{ij}^{a}

	j	1		2		3		4		5		6		7		8		9		10	
i	*a*	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1		0.11	0.69	0.9	0.05	0.67	0.03	0.26	0.75	0.95	0.16	0.3	0.78	0.16	0.47	0.82	0.64	0.49	0.06	0.42	0.25
2		0.33	0.61	0.49	0.71	0.11	0.9	0.35	0.21	0.31	0.55	0.02	0.66	0.24	0.28	0.28	0.16	0.81	0.36	0.38	0.34
3		0.88	0.93	0.98	0.2	0.06	0.25	0.46	0.12	0.5	0.93	0.2	0.91	0.35	0.76	0.62	0.81	0.91	0.46	0.96	0.8
4		0.47	0.87	0.88	0.05	0.84	0.07	0.44	0.71	0.97	0.86	0.58	0.57	0.77	0.52	0.59	0.72	0.32	0.28	0.98	0.86
5		0.09	0.22	0.17	0.06	0.33	0.91	0	0.46	0.17	0.32	0.68	0.36	0.32	0.5	0.46	0.15	0.22	0.38	0.62	0.97
6		0.06	0.15	0.6	0.6	0.99	0.96	0.35	1	0.95	0.87	0.13	0.54	0.83	0.85	0.44	0.15	0.68	0.87	0.38	0.96
7		0.21	0.12	0.02	0.88	0.78	0.61	0.86	0.62	0.29	0.67	0.84	0.24	0.73	0.04	0.38	0.81	0.02	0.15	0.58	0.71
8		0.75	0.16	0.89	0.46	0.27	0.82	0.44	0.98	0.89	0.11	0.22	0.29	0.14	0.27	0.54	0.06	0.6	0.83	0.78	0.75
9		0.43	0.32	0.58	0.52	0.13	0.39	0.77	0.49	0.3	0.99	0.54	0.95	0.86	0.4	0.97	0.35	0.73	0.89	0.22	0.51
10		0.01	0.8	0.72	0.52	0.77	0.17	0.76	0.76	0.45	0.57	0.63	0.16	0.77	0.24	0.11	0.45	0.99	0.75	0.19	0.3
11		0.55	0.17	0.7	0.02	0.1	0.27	0.59	0.52	0.63	0.83	0.3	0.52	0.59	0.96	0.37	0.72	0.74	0.41	0.03	0.99
12		0.87	0.24	1	0.44	0.35	0.38	0.86	0.78	0.53	0.37	0.73	0.12	0.85	0.97	0.46	0.79	0.93	0.01	0.02	0.25
13		0.15	0.88	0.24	0.64	0.48	0.44	0.95	0.29	0.33	0.7	0.76	0.03	0.47	0.55	0.55	0.5	0.44	0.11	0.39	0.74
14		0.84	0.9	0.81	0.93	0.31	0.73	0.21	0.89	0.39	0.99	0.98	1	0.51	0.73	0.41	0.4	0.76	0.38	0.22	0.45
15		0.18	0.35	0.63	0.36	0.68	0.67	0.38	0.66	0.87	0.8	0.27	0.49	0.5	0.99	0.14	0.49	0.88	0.15	0.8	0.59
16		0.97	0.82	0.99	0.95	0.44	0.62	0.83	0.45	0.53	0.18	0.63	0.39	0.54	0.19	0.11	0.16	0.2	0.07	0.23	0.01
17		0.9	0.01	0.85	0.1	0.03	0.06	0.17	0.08	0.5	0.52	0.85	0.36	0.5	0.58	0.91	0.47	0.61	0.24	0.94	0.05
18		0.7	0.27	0.59	0.28	0.09	0.78	0.87	0.37	0.23	0.41	0.73	0.75	0.19	0.26	0.37	0.17	0.91	0.54	0.9	0.24
19		0.88	0.66	0.2	0.61	0.96	0.8	0.54	0.31	0.17	0.9	0.18	0.8	0.71	0.19	0.6	0.29	0.96	0.33	0.49	0.73
20		0.36	0.38	0.42	0.34	0.71	0.14	0.65	0.37	0.5	0.04	0.82	0.38	0.7	0.73	0.07	0.34	0.01	0.01	0.23	0.66

Fuente: elaboración propia.

TABLE 2.9. Values of parameter F_{ij}^{a}

	j	1		2		3		4		5		6		7		8		9		10	
i	*a*	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1		0,16	0,08	0,11	0,94	0,02	0,13	0,8	0,56	0,38	0,76	0,88	0,5	0,88	0,22	0,76	0,88	0,95	0,81	0,04	0,37
2		0,1	0,66	0,19	0,02	0,62	0,78	0,9	0,08	0,42	0,89	0,8	0,76	0,69	0,08	0,68	0,29	0,85	0,53	0,32	0,51
3		0,39	0,16	0,3	0,05	0,09	0,05	0,66	0,47	0,73	0,62	0,53	0,71	0,88	0,67	0,88	0,63	0,74	0,41	0,53	0,83
4		0,38	0,33	0,58	0,22	0,88	0,43	0,13	0,14	0,26	0,08	0,3	0,94	0,6	0,16	0,24	0,71	0,92	0,43	0,4	0,59
5		0,69	0,56	0,92	0,39	0,82	0,27	0,51	0,92	0,5	0,97	0,9	0,3	0,93	0,03	0,89	0,7	0,81	0,23	0,34	0,2
6		0,9	0,44	0,31	0,09	0,42	0,01	0,18	0,53	0,27	0,98	0,08	0,99	0,3	0,32	0,78	0,61	0,07	0,72	0,25	0,76
7		0,11	0,11	0,23	0,98	0,19	0,85	0,26	0,29	0,82	0,32	0,45	0,29	0,03	0,68	0,56	0,74	0,27	0,31	0,42	0,79
8		0,72	0,74	0,48	0,68	0,86	0,15	0,12	0,68	0,62	0,94	0,95	0,79	0,96	0,66	0,87	0,7	0,91	0,58	0,53	0,41
9		0,21	0,37	0,83	0,54	0,12	0,17	0,83	0,24	0,52	0,17	0,35	0,96	0,5	0,92	0,86	0,56	0,98	0,43	0,48	0,1
10		0,05	0,4	0,14	0,94	0,15	0,29	0,51	0,92	0,11	0,14	0,75	0,84	0,66	0,82	0,6	0,46	0,75	0,53	0,98	0,51
11		0,82	0,24	0,44	0,5	0,32	0,81	0,56	0,23	0,37	0,52	0,73	0,57	0,28	0,44	0,81	0,8	0,37	0,72	0,93	0,99
12		0,14	0,2	0,13	0,53	0,69	0,07	0,11	0,79	0,34	0,11	0,7	0,89	0,97	0,33	0,88	0,08	0,64	0,27	0,21	0,18
13		0,94	0,06	0	0,69	0,13	0,72	0,8	0,28	0,52	0,83	0,77	0,15	0,26	0,44	0,58	0,33	0,02	0,62	0,12	0,58
14		0,22	0,19	0,06	0,35	0,12	0,25	0,28	0,08	0,79	0,62	0,85	0,83	0,95	0,36	0,76	0,79	0,61	0,66	0,56	0,27
15		0,45	0,22	0,29	0,71	0,56	0,38	0,13	0,2	0,84	0,75	0,32	0,51	0,63	0,06	0,1	0,51	0,61	0,46	0,39	0,59
16		0,39	0,19	0,18	0,84	0,29	0,05	0,21	0,41	0,48	0,88	0,75	0,12	0,51	0,61	0,01	0,15	0,7	0,66	0,09	0,74
17		0,89	0,47	0,65	0,66	0,05	0,78	0,4	0,52	0,75	0,72	0,88	0,21	0,68	0,09	0,45	0,41	0,32	0,6	0,56	0,01
18		0,29	0,59	0,81	0,23	0,42	0,08	0,76	0,69	0,38	0,31	0,17	0,11	0,87	0,1	0,94	0,4	0,47	0,26	0,53	0,25
19		0,09	0,73	0,44	0,72	0,74	0,83	0,99	0,73	0,52	0,95	0,99	0,6	0,6	0,45	0,7	0,96	0,11	0,35	0,5	0,44
20		0,83	0,04	1	0,17	0,67	0,67	0,68	0,99	0,95	0,71	0,89	0,48	0,11	0,02	0,37	0,53	0,15	0,86	0,08	0,13

Fuente: elaboración propia.

TABLE 2.10. Values of parameter L_{jk}^{p}

	j	1		2		3		4		5		6		7		8		9		10	
k	*p*	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1		0,11	0,69	0,9	0,05	0,67	0,03	0,26	0,75	0,95	0,16	0,3	0,78	0,16	0,47	0,82	0,64	0,49	0,06	0,42	0,25
2		0,33	0,61	0,49	0,71	0,11	0,9	0,35	0,21	0,31	0,55	0,02	0,66	0,24	0,28	0,28	0,16	0,81	0,36	0,38	0,34
3		0,88	0,93	0,98	0,2	0,06	0,25	0,46	0,12	0,5	0,93	0,2	0,91	0,35	0,76	0,62	0,81	0,91	0,46	0,96	0,8
4		0,47	0,87	0,88	0,05	0,84	0,07	0,44	0,71	0,97	0,86	0,58	0,57	0,77	0,52	0,59	0,72	0,32	0,28	0,98	0,86
5		0,09	0,22	0,17	0,06	0,33	0,91	0	0,46	0,17	0,32	0,68	0,36	0,32	0,5	0,46	0,15	0,22	0,38	0,62	0,97
6		0,06	0,15	0,6	0,6	0,99	0,96	0,35	1	0,95	0,87	0,13	0,54	0,83	0,85	0,44	0,15	0,68	0,87	0,38	0,96
7		0,21	0,12	0,02	0,88	0,78	0,61	0,86	0,62	0,29	0,67	0,84	0,24	0,73	0,04	0,38	0,81	0,02	0,15	0,58	0,71
8		0,75	0,16	0,89	0,46	0,27	0,82	0,44	0,98	0,89	0,11	0,22	0,29	0,14	0,27	0,54	0,06	0,6	0,83	0,78	0,75
9		0,43	0,32	0,58	0,52	0,13	0,39	0,77	0,49	0,3	0,99	0,54	0,95	0,86	0,4	0,97	0,35	0,73	0,89	0,22	0,51
10		0,01	0,8	0,72	0,52	0,77	0,17	0,76	0,76	0,45	0,57	0,63	0,16	0,77	0,24	0,11	0,45	0,99	0,75	0,19	0,3
11		0,55	0,17	0,7	0,02	0,1	0,27	0,59	0,52	0,63	0,83	0,3	0,52	0,59	0,96	0,37	0,72	0,74	0,41	0,03	0,99
12		0,87	0,24	1	0,44	0,35	0,38	0,86	0,78	0,53	0,37	0,73	0,12	0,85	0,97	0,46	0,79	0,93	0,01	0,02	0,25
13		0,15	0,88	0,24	0,64	0,48	0,44	0,95	0,29	0,33	0,7	0,76	0,03	0,47	0,55	0,55	0,5	0,44	0,11	0,39	0,74
14		0,84	0,9	0,81	0,93	0,31	0,73	0,21	0,89	0,39	0,99	0,98	1	0,51	0,73	0,41	0,4	0,76	0,38	0,22	0,45
15		0,18	0,35	0,63	0,36	0,68	0,67	0,38	0,66	0,87	0,8	0,27	0,49	0,5	0,99	0,14	0,49	0,88	0,15	0,8	0,59
16		0,97	0,82	0,99	0,95	0,44	0,62	0,83	0,45	0,53	0,18	0,63	0,39	0,54	0,19	0,11	0,16	0,2	0,07	0,23	0,01
17		0,9	0,01	0,85	0,1	0,03	0,06	0,17	0,08	0,5	0,52	0,85	0,36	0,5	0,58	0,91	0,47	0,61	0,24	0,94	0,05
18		0,7	0,27	0,59	0,28	0,09	0,78	0,87	0,37	0,23	0,41	0,73	0,75	0,19	0,26	0,37	0,17	0,91	0,54	0,9	0,24
19		0,88	0,66	0,2	0,61	0,96	0,8	0,54	0,31	0,17	0,9	0,18	0,8	0,71	0,19	0,6	0,29	0,96	0,33	0,49	0,73
20		0,36	0,38	0,42	0,34	0,71	0,14	0,65	0,37	0,5	0,04	0,82	0,38	0,7	0,73	0,07	0,34	0,01	0,01	0,23	0,66

Fuente: elaboración propia.

TABLE 2.11. Values of parameters CC_{kt}, CH_{kt}

	CC_{kt}										CH_{kt}									
k \ t	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
1	3	5	5	5	3	3	4	4	3	4	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
2	5	3	5	5	4	4	5	4	4	3	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
3	5	3	4	5	5	4	5	3	4	3	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
4	4	3	3	3	4	4	5	4	3	4	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
5	3	4	5	5	5	5	4	5	4	4	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
6	3	3	3	4	4	3	3	4	3	5	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
7	5	3	4	3	5	4	3	5	5	4	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
8	3	5	3	4	4	3	4	4	5	3	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
9	3	5	4	5	4	3	3	3	3	5	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
10	4	4	3	5	3	5	4	5	4	3	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
11	4	5	3	3	4	5	3	4	3	5	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
12	3	4	5	3	4	3	3	3	3	3	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
13	4	4	3	5	5	4	3	3	3	3	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
14	5	4	5	4	4	4	4	3	5	3	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
15	3	4	5	5	5	4	4	5	4	4	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
16	3	3	5	3	4	5	3	4	3	4	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
17	3	5	3	5	5	5	4	4	5	5	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
18	4	4	5	3	5	4	5	3	5	5	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
19	5	5	5	5	5	5	5	4	5	3	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600
20	5	4	4	5	4	5	5	3	3	3	1610300	1659300	1802900	1898900	2018200	2191600	2303100	2480100	2354100	2583600

Fuente: elaboración propia.

TABLE 2.12. Values of parameter C_{kt}^{pe}

t	C_{kt}^{11}										C_{kt}^{12}									
k	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
1	16103	16593	18029	18989	20182	21916	23031	24801	23541	25836	14493	14934	16226	17090	18164	19724	20728	22321	21187	23252
2	16610	17763	18859	20881	19693	22306	21274	22435	24568	24444	14949	15987	16973	18793	17724	20075	19147	20192	22111	22000
3	17327	18202	19343	18562	21374	22500	21897	24901	25056	24977	15594	16382	17409	16706	19237	20250	19707	22411	22550	22479
4	15995	17551	17008	19278	19034	20439	23497	23183	25642	25405	14396	15796	15307	17350	17131	18395	21147	20865	23078	22865
5	16480	16657	18600	18511	21926	20086	23892	23992	24522	25361	14832	14991	16740	16660	19733	18077	21503	21593	22070	22825
6	16150	17733	19921	18542	21405	20145	23232	23420	25293	26710	14535	15960	17929	16688	19265	18131	20909	21078	22764	24039
7	16542	18647	17017	19358	20499	22349	22156	24403	24645	25779	14888	16782	15315	17422	18449	20114	19940	21963	22181	23201
8	16266	16084	18013	18784	20348	21794	21300	23212	23304	26078	14639	14476	16212	16906	18313	19615	19170	20891	20974	23470
9	15407	17922	19795	20587	21667	20665	21029	24224	24155	26042	13866	16130	17816	18528	19500	18599	18926	21802	21740	23438
10	15972	18052	19442	20565	21455	21591	22440	22495	25171	25386	14375	16247	17498	18509	19310	19432	20196	20246	22654	22847
11	17499	18437	19593	20513	19878	22289	22263	22470	23550	25072	15749	16593	17634	18462	17890	20060	20037	20223	21195	22565
12	17101	17729	18120	20998	20572	22029	22383	24676	25575	25707	15391	15956	16308	18898	18515	19826	20145	22208	23018	23136
13	17740	16397	17773	20969	20135	22446	23476	24851	23799	25295	15966	14757	15996	18872	18122	20201	21128	22366	21419	22766
14	16569	17039	19074	18226	20751	22796	23301	22177	23710	24642	14912	15335	17167	16403	18676	20516	20971	19959	21339	22178
15	16825	16213	17022	18584	20474	21997	22406	22786	23224	26771	15143	14592	15320	16726	18427	19797	20165	20507	20902	24094
16	16753	16492	19639	18586	21749	22216	23979	22586	24313	26086	15078	14843	17675	16727	19574	19994	21581	20327	21882	23477
17	16917	18873	19579	20484	19439	20398	22180	22372	23593	25721	15225	16986	17621	18436	17495	18358	19962	20135	21234	23149
18	16719	16884	17436	19420	19537	22796	22786	23209	23898	25474	15047	15196	15692	17478	17583	20516	20507	20888	21508	22927
19	16708	17163	19314	20805	21480	21970	21598	22044	25778	25284	15037	15447	17383	18725	19332	19773	19438	19840	23200	22756
20	17039	18742	19073	20148	21962	20445	22296	23173	24861	24380	15335	16868	17166	18133	19766	18401	20066	20856	22375	21942

t	C_{kt}^{21}										C_{kt}^{22}									
k	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
1	22544	23230	25241	26585	28255	30682	32243	34721	32957	36170	20290	20907	22717	23927	25430	27614	29019	31249	29661	32553
2	23254	24868	26403	29233	27570	31228	29784	31409	34395	34222	20929	22381	23763	26310	24813	28105	26806	28268	30956	30800
3	24258	25483	27080	25987	29924	31500	30656	34861	35078	34968	21832	22935	24372	23388	26932	28350	27590	31375	31570	31471
4	22393	24571	23811	26989	26648	28615	32896	32456	35899	35567	20154	22114	21430	24290	23983	25754	29606	29210	32309	32010
5	23072	23320	26040	25915	30696	28120	33449	33589	34331	35505	20765	20988	23436	23324	27626	25308	30104	30230	30898	31955
6	22610	24826	27889	25959	29967	28203	32525	32788	35410	37394	20349	22343	25100	23363	26970	25383	29273	29509	31869	33655
7	23159	26106	23824	27101	28699	31289	31018	34164	34503	36091	20843	23495	21442	24391	25829	28160	27916	30748	31053	32482
8	22772	22518	25218	26298	28487	30512	29820	32497	32626	36509	20495	20266	22696	23668	25638	27461	26838	29247	29363	32858
9	21570	25091	27713	28822	30334	28931	29441	33914	33817	36459	19413	22582	24942	25940	27301	26038	26497	30523	30435	32813
10	22361	25273	27219	28791	30037	30227	31416	31493	35239	35540	20125	22746	24497	25912	27033	27204	28274	28344	31715	31986
11	24499	25812	27430	28718	27829	31205	31168	31458	32970	35101	22049	23231	24687	25846	25046	28085	28051	28312	29673	31591
12	23941	24821	25368	29397	28801	30841	31336	34546	35805	35990	21547	22339	22831	26457	25921	27757	28202	31091	32225	32391
13	24836	22956	24882	29357	28189	31424	32866	34791	33319	35413	22352	20660	22394	26421	25370	28282	29579	31312	29987	31872
14	23197	23855	26704	25516	29051	31914	32621	31048	33194	34499	20877	21470	24034	22964	26146	28723	29359	27943	29875	31049
15	23555	22698	23831	26018	28664	30796	31368	31900	32514	37479	21200	20428	21448	23416	25798	27716	28231	28710	29263	33731
16	23454	23089	27495	26020	30449	31102	33571	31620	34038	36520	21109	20780	24746	23418	27404	27992	30214	28458	30634	32868
17	23684	26422	27411	28678	27215	28557	31052	31321	33030	36009	21316	23780	24670	25810	24494	25701	27947	28189	29727	32408
18	23407	23638	24410	27188	27352	31914	31900	32493	33457	35664	21066	21274	21969	24469	24617	28723	28710	29244	30111	32098
19	23391	24028	27040	29127	30072	30758	30237	30862	36089	35398	21052	21625	24336	26214	27065	27682	27213	27776	32480	31858
20	23855	26239	26702	28207	30747	28623	31214	32442	34805	34132	21470	23615	24032	25386	27672	25761	28093	29198	31325	30719

Fuente: elaboración propia.

TABLE 2.13. Values of parameter F_{jk}^{p}

k \ j	F_{jk}^{1}																			
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	0.16	0.1	0.39	0.38	0.69	0.9	0.11	0.72	0.21	0.05	0.82	0.14	0.94	0.22	0.45	0.39	0.89	0.29	0.09	0.83
2	0.11	0.19	0.3	0.58	0.92	0.31	0.23	0.48	0.83	0.14	0.44	0.13	0	0.06	0.29	0.18	0.65	0.81	0.44	1
3	0.02	0.62	0.09	0.88	0.82	0.42	0.19	0.86	0.12	0.15	0.32	0.69	0.13	0.12	0.56	0.29	0.05	0.42	0.74	0.67
4	0.8	0.9	0.66	0.13	0.51	0.18	0.26	0.12	0.83	0.51	0.56	0.11	0.8	0.28	0.13	0.21	0.4	0.76	0.99	0.68
5	0.38	0.42	0.73	0.26	0.5	0.27	0.82	0.62	0.52	0.11	0.37	0.34	0.52	0.79	0.84	0.48	0.75	0.38	0.52	0.95
6	0.88	0.8	0.53	0.3	0.9	0.08	0.45	0.95	0.35	0.75	0.73	0.7	0.77	0.85	0.32	0.75	0.88	0.17	0.99	0.89
7	0.88	0.69	0.88	0.6	0.93	0.3	0.03	0.96	0.5	0.66	0.28	0.97	0.26	0.95	0.63	0.51	0.68	0.87	0.6	0.11
8	0.76	0.68	0.88	0.24	0.89	0.78	0.56	0.87	0.86	0.6	0.81	0.88	0.58	0.76	0.1	0.01	0.45	0.94	0.7	0.37
9	0.95	0.85	0.74	0.92	0.81	0.07	0.27	0.91	0.98	0.75	0.37	0.64	0.02	0.61	0.61	0.7	0.32	0.47	0.11	0.15
10	0.04	0.32	0.53	0.4	0.34	0.25	0.42	0.53	0.48	0.98	0.93	0.21	0.12	0.56	0.39	0.09	0.56	0.53	0.5	0.08
k \ j	F_{jk}^{2}																			
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	0.08	0.66	0.16	0.33	0.56	0.44	0.11	0.74	0.37	0.4	0.24	0.2	0.06	0.19	0.22	0.19	0.47	0.59	0.73	0.04
2	0.94	0.02	0.05	0.22	0.39	0.09	0.98	0.68	0.54	0.94	0.5	0.53	0.69	0.35	0.71	0.84	0.66	0.23	0.72	0.17
3	0.13	0.78	0.05	0.43	0.27	0.01	0.85	0.15	0.17	0.29	0.81	0.07	0.72	0.25	0.38	0.05	0.78	0.08	0.83	0.67
4	0.56	0.08	0.47	0.14	0.92	0.53	0.29	0.68	0.24	0.92	0.23	0.79	0.28	0.08	0.2	0.41	0.52	0.69	0.73	0.99
5	0.76	0.89	0.62	0.08	0.97	0.98	0.32	0.94	0.17	0.14	0.52	0.11	0.83	0.62	0.75	0.88	0.72	0.31	0.95	0.71
6	0.5	0.76	0.71	0.94	0.3	0.99	0.29	0.79	0.96	0.84	0.57	0.89	0.15	0.83	0.51	0.12	0.21	0.11	0.6	0.48
7	0.22	0.08	0.67	0.16	0.03	0.32	0.68	0.66	0.92	0.82	0.44	0.33	0.44	0.36	0.06	0.61	0.09	0.1	0.45	0.02
8	0.88	0.29	0.63	0.71	0.7	0.61	0.74	0.7	0.56	0.46	0.8	0.08	0.33	0.79	0.51	0.15	0.41	0.4	0.96	0.53
9	0.81	0.53	0.41	0.43	0.23	0.72	0.31	0.58	0.43	0.53	0.72	0.27	0.62	0.66	0.46	0.66	0.6	0.26	0.35	0.86
10	0.37	0.51	0.83	0.59	0.2	0.76	0.79	0.41	0.1	0.51	0.99	0.18	0.58	0.27	0.59	0.74	0.01	0.25	0.44	0.13

Fuente: elaboración propia.

TABLE 2.14. Set $RJK(j,k)$

k \ j	$RJK(j,k)$																			
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1

Fuente: elaboración propia.