A PROPOSAL FOR A SYNTACTIC SOLUTION OF THE PROBLEMS OF DISJUNCTION IN HUMAN THOUGHT

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Abstract

The mental logic theory does not accept a rule correct in standard propositional calculus: the disjunction introduction rule (that is, the rule that allows inferring, for example, p or q from p). This is a problem because that very theory admits another schema in which the rule is really involved. It is true that, as shown by López-Astorga, the mental logic theory can be updated following recent empirical results and that such an update can help the theory solve some of its difficulties. However, López-Astorga's update does not address the challenges directly raised by the disjunction introduction rule, and this paper is intended to do so. In particular, my thesis here is that all of the difficulties of the aforementioned rule can disappear if it is considered to be not a Core Schema of the human syntax of thought, but a Feeder Schema of it.

Keywords: *disjunction; mental logic; schema; syntax; thought.*

Una propuesta de solución sintáctica para los problemas de la disyunción en el pensamiento humano

Resumen

La teoría de la lógica mental no acepta una regla correcta en el cálculo proposicional estándar: la regla de introducción de la disyunción (esto es, la regla que permite inferir, por ejemplo, p o q de p). Esto es un problema porque esa misma teoría admite otro esquema en el que la regla está realmente implicada. Es cierto que, como ha mostrado López-Astorga, la teoría de la lógica mental puede ser actualizada de acuerdo con resultados empíricos recientes y que tal actualización puede ayudar a la teoría a resolver algunas de sus dificultades. No obstante, la actualización de López-Astorga no afronta los desafíos directamente planteados por la regla de introducción de la disyunción y este trabajo pretende proceder en esa dirección. En concreto, mi tesis aquí es que todas las dificultades de la mencionada regla pueden desaparecer si no es considerada un Esquema Núcleo de la sintaxis del pensamiento humano, sino un Esquema Alimentador de la misma.

Palabras clave: *disyunción; lógica mental; esquema; sintaxis; pensamiento.*

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Introduction

López-Astorga (2016a) has shown that the mental logic theory (e.g., Braine & O'Brien, 1998a; O'Brien, 2009, 2014; O'Brien & Li, 2013; O'Brien & Manfrinati, 2010) has an important problem that needs to be solved. As it is well known, that theory does not accept all of the rules that are valid or correct in standard propositional calculus or Gentzen's (1935) framework (see, e.g., López-Astorga, 2016a, 196-197; or López-Astorga, 2016b, 14), and, in this way, one of the rules of that calculus that the theory rejects is the disjunction introduction rule (from now on, DIR), i.e., the rule enabling to deduce a formula such as [p v q], where 'v' represents disjunction, from a formula such as [p] (see, e.g., López-Astorga, 2015a, 143; or López-Astorga, 2016a, 197). The theory rejects DIR because empirical data suggest that people do not usually apply it. However, the problem is that, as indicated by López-Astorga (2016a, 199), the mental logic theory does admit another rule that is more complex and that, in addition, seems to require the previous use of DIR to be applied. That rule is Schema 2 in the version of this last theory presented by Braine and O'Brien (1998b, 80; Table 6.1), and the formal structure assigned to it by López-Astorga (2016a, 199) is akin to this:

$(p_1 \text{ v}\ldots\text{v } p_n) \to q$

p_i

Therefore, q

Where '->' stands for the conditional.

It is absolutely true that López-Astorga (2016b) has also proposed an update of the original mental logic theory that takes into account certain experimental results that are to be found in the literature of cognitive science, modifies some aspects of the theory, and, by virtue of those modifications, allows it to solve some of its more important difficulties. Nevertheless, that update, which is called by him the Reasoning Formal Theory or RFT (from now on, I will use these acronyms to refer to his update too), does not directly deal with the problem related to DIR raised by him in López-Astorga (2016a). So, the main goal of this paper is to try to address that problem and offer a possible solution of it based on the essential theses of RFT.

In particular, my proposal, which, given that it does not ignore the central ideas of RFT, is obviously syntactic, resorts to a distinction coming from the original mental logic theory that RFT assumes as well. That distinction is that between Core Schemata and Feeder Schemata. Indeed, both the mental logic theory and RFT claim that all of the schemata used by the human mind are not of the same kind. There are different types of schemata, and two of them are those mentioned. In this way, Core Schemata are schemata that "… are used without restriction whenever they are applicable" (Braine & O'Brien, 1998b, 79-83; see also, e.g., López-Astorga, 2016b, 14). On the other hand, Feeder Schemata are schemata that "… are used only when their output feeds another schema …" (Braine & O'Brien, 1998b, 83; see also, e.g., López-Astorga, 2016b, 15). Thus, my idea is that, while it is clear that DIR cannot be a Core Schema, it can be accepted as a Feeder Schema, and that, although it is not usually applied, it does be used when its application can lead to other conclusions or enable the use of other schemata.

To show all of this, I will firstly describe in more detail the problem regarding DIR that López-Astorga (2016a) sees in the mental logic theory. Then, I will explain the changes that RFT introduces in the theory and why DIR continues to be a problem in that update. Finally, I will propose my possible solution that considers DIR to be a Feeder Schema and that respects the general lines of both the mental logic theory and RFT.

DIR and Schema 2 in the mental logic theory

It is also very known that the mental logic theory claims that there is a clear link between human thought and syntax. In fact, the theory assumes the hypothesis that a syntax of thought exists, which in turn is related to approaches such those of Fodor (1975) or Macnamara (1986) and supported by the proponents of the mental logic theory in different papers, chapters, and books (e.g., Braine & O'Brien, 1998c; O'Brien & Li, 2013; this point is also commented on in papers such as, for instance, López-Astorga, 2016b, 12-13). In this way, following the theory, the syntax of thought seems to include a schemata set in which, as said, DIR is not. Nonetheless, as also indicated, a schema such as Schema 2 does be a part of that set, and this circumstance, according to López-Astorga (2016a), is, as also stated, a problem.

The reason is that, as López-Astorga claims in other work as well (López-Astorga, 2015b, 147), Schema 2 is just a version of another Core Schema of the theory in which one of the clauses of one of its premises is a disjunction (see also López-Astorga, 2016a, 200). In particular, the argument is that Schema 2 is just a version of Schema 7 in Braine and O'Brien (1998b, 80; Table 6.1), which has a logical form similar to the following:

p -> q
p

Therefore, q

It is evident that Schema 7 is in turn a version of an ancient rule, *Modus Ponendo Ponens*, which, according to Diogenes Laërtius (*Vitae Philosophorum* 7, 80) was first provided by Chrysippus of Soli (see also, e.g., Boeri & Salles, 2014, 217 & 228; Fragment 9.7). According to Diogenes, *Modus Ponendo Ponens* is a *ἀναπόδεικτος* (*indemonstrable*) of Stoic logic and consists of two premises and a conclusion. The first premise is a *συνημμένον* (conditional) and the second premise is the *ἡγούμενον* (*εἰ* -in English, if- clause or antecedent) of that very *συνημμένον*. Thus, the conclusion is the *λῆγον* (the other clause, i.e., the consequent) of the *συνημμένον*.

From this perspective, there is no doubt that, indeed, Schema 2 is a version of *Modus Ponendo Ponens* (i.e., of Schema 7). Continuing with Greek words, the only difference between the two schemata is that in the former the *ἡγούμενον* is a *διεζευγμένον* (disjunction), which means that, whenever that *διεζευγμένον* is true, the *λῆγον* is true too. In this way, if the fact that the second premise of Schema 2 is just one of the disjuncts of the

διεζευγμένον, and not the *διεζευγμένον* in entirety, is taken into account, it is obvious that the application of Schema 2 involves the use of DIR.

This is exactly the argument given by López-Astorga (2016a, 200-201). It is only possible to draw [q] from premises such as $[(p_1 \, v \ldots v \, p_i \, v \ldots v \, p_n) \rightarrow q]$ and $[p_i]$ if the individual is able to note that, if $[p_i]$ is true, $[p_1 \, v \ldots v \, p_i \, v \ldots v \, p_n]$ is true as well. Thus, what really occurs is that the individual derives $[p_1 \, v \ldots v \, p_i \, v \ldots v \, p_n]$ from $[p_i]$, and once this has been done, it is deduced [q] from $[(p_1 \, v \ldots v \, p_i \, v \ldots v \, p_n) \rightarrow q]$ and $[p_1 \, v \ldots v \, p_i \, v \ldots v \, p_n]$. Therefore, Schema 2 is actually the combination of two schemata: DIR and *Modus Ponendo Ponens* (Schema 7). This is son because, as said, it appears that Schema 2 can only be applied if DIR is previously applied to the second premise.

However, as also indicated, DIR is not accepted by the mental logic theory. Thus, this is a very important point that needs to be clarified, especially because Schema 2 seems to be one of the great strengths of this last theory. True, as shown by López-Astorga (2015b, 148-150), the fact that, in experiments such as those carried out by Braine, Reiser, and Rumain (1998), Schema 2 has a percentage of error of 0% is hard to explain for other rival theories such as, for example, the mental models theory (e.g., Johnson-Laird, 2012, 2015; Johnson-Laird, Khemlani, & Goodwin, 2015; Khemlani, Lotstein, Trafton, & Johnson-Laird, 2015; Orenes & Johnson-Laird, 2012; Ragni, Sonntag, and Johnson-Laird, 2016), and is easier to account for if a syntax of thought and a direct relationship between that syntax and the schema are assumed (if there is no a syntax of thought, it is difficult to understand how individuals make inferences such as those with the structure of Schema 2, which, because they include several elements –a disjunction, a conditional, and an indefinite number of propositions-, are very complex). Nevertheless, the problem or internal contradiction that, following López-Astorga's (2016a) arguments, the mental logic theory seems to have regarding DIR appears to undermine this clear support that empirical evidence provides to the formal theories. It hence is important to overcome such difficulties, and I will try to do that below.

RFT and Schema 2

As said, RFT is just the update of the mental logic theory presented by López-Astorga (2016b). Really, his intention is to solve some of the problems that the theory has at present by taking recent experimental data into account. In this way, while RFT keeps relevant aspects of the mental logic theory, including, as also mentioned, the distinction between Core and Feeder Schemata, it also modifies other points of the theory that have certain importance. For example, the schemata accepted by RFT are not exactly the

same as the mental logic theory. RFT considers some of the Core Schemata of the mental logic theory to be trivial and, therefore, eliminates them. In the same way, paying attention to the literature, RFT also adds some Feeder Schemata that are not in the original version of the theory.

Nonetheless, what is interesting for this paper is the role that Schema 2 plays in RFT. In this regard, it can be said that Schema 2 really plays no role in RFT, since it is removed from the theory. The reason is that, according to López-Astorga (2016b, 20), by virtue of the syntax of thought, people usually understand what a disjunction means, that is, that if only one of the disjuncts is true, the disjunction in entirety is true. Thus, if we know, for instance, that $[p_i]$ is true, we know that $[p_1 \text{ v}\ldots\text{v } p_i \text{ v}\ldots\text{v } p_n]$ is true too.

But it is obvious that this elimination of the schema does not remove the problem, since, following the explanation given in the previous section, it can be stated that López-Astorga's (2016b) account clearly supposes that individuals in some way apply DIR. So, the problem continues to be the same: if people have the ability to make complex inferences in which DIR is involved, it is necessary to explain why they do not always use that rule. I think, as indicated above, that the cause is that, although DIR does be a rule of the human syntax of thought, it is not a Core Schema, but just a Feeder Schema. The next section develops this argument.

DIR as a Feeder Schema

There is no doubt that the problem of DIR is difficult and López-Astorga has actually addressed it in several works. For example, in López-Astorga (2015a) he studies cases in which people appear to use it and comes to the conclusion that individuals do not really apply the rule in those cases, since the disjunctions in them are not true disjunctions. He analyzes tasks such as this:

"Lucia wore jewelry.
Therefore, Lucia wore the bracelet or she wore jewelry" (Orenes & Johnson-Laird, 2012, 363; see also López-Astorga, 2015a, 145).

In these tasks, people often consider the conclusion to be correct. So, they appear to apply DIR. However, resorting to truth tables, López-Astorga (2015a, 146-147) argues that the disjunctions of conclusions such as that of the aforementioned task are not real disjunctions, that they do not always enable situations in which only one of the disjuncts is true (as it can be noted, in the example indicated, if the first disjunct is true, i.e., if Lucia wore the

bracelet, the second disjunct is also true, i.e., Lucia wore jewelry, since it is not possible to wear a bracelet and not to wear jewelry), and that they actually correspond to a kind of conditionals (what the conclusion of the problem seems to provide is that, "both if Lucia wears the bracelet and if she does not wear the bracelet, she wears jewelry", López-Astorga, 2015a, p. 146).

It is clear that the account given by López-Astorga (2015a) is very important to formal theories such as the mental logic theory or RFT, since other frameworks that are not formal such as that of the mental models theory can easily explain the majority responses in tasks such as the one indicated (see Orenes & Johnson-Laird's, 2012, paper for the explanation that the mental models theory offers about such responses), and the formal theories need to do that as well. Nevertheless, that account is not useful for the goals of this paper. In the literature, the disjunctions included in the antecedent of the first premise of Schema 2 do not use to be false disjunctions such as those detected by López-Astorga (2015a). In fact, they were not so in the experiments carried out by Braine et al. (1998), which, as said, showed that people do not make mistakes with that schema. Therefore, it is necessary a different solution that, respecting the basic assumptions of the mental logic theory and RFT, enables to overcome the problem.

In principle, it can be stated that, in general, people do not apply DIR. They only seem to use it when, as in Schema 2, allows transforming a formula into another one that in turn enables to apply another schema (in the case of Schema 2, as explained, *Modus Ponendo Ponens*), that is, when its "… use can allow drawing conclusions" (López-Astorga, 2016c, 124; see also López-Astorga, 2016b, 15). However, as mentioned above, this is exactly the main characteristic of Feeder Schemata. From the literature on the mental logic theory, it can be deduced that Feeder Schemata are not always applied because they could cause infinite processes (see, e.g., Braine & O'Brien, 1998b, 83). For instance, a Feeder Schema can be this:

p
q
------
p · q
Where '·' stands for conjunction.

This schema is akin to schema 8 in Braine and O'Brien (1998b, 80; Table 6.1) and Feeder Schema 6 in RFT (López-Astorga, 2016b, 23). The reason why it is a Feeder Schema is obvious: it can enable infinite processes such as the following:

p
q
p · q
p · q · q
p · q · q · q
p · q · q · q · q
… and so on

In this way, its use is restricted to the situations in which it can really allow continuing with an inference (other examples of infinite processes of derivation to which the use of Feeder Schemata of the mental logic theory can lead can be found in texts such as that of López-Astorga, 2016d, 206).

But all of this can cause one to think that what the literature shows is that something similar happens with DIR and that a possible solution to the problem of this paper can be to consider it to be a Feeder Schema. This idea seems correct for several reasons. The first one is that it appears to be consistent with the literature, since, apparently, what is really indicated by researches such as that of Braine et al. (1998) is that DIR is not used in any time, but only, as said, when it enables to derive more conclusions, and that it hence has the main characteristic of Feeder Schemata, i.e., the one of being used only when useful. On the other hand, this is not a problem for RFT. On the contrary, it even improves the theory. As commented, RFT does not take Schema 2 as an essential schema of human thought, but it does accept that people are able to execute correctly inferences with the formal structure of that schema. So, to add DIR as a Feeder Schema to RFT is not only coherent with its theses, but it also leads to better explain what truly occurs when individuals make inferences with a structure akin to that of Schema 2.

In addition, if the possibility of infinite processes is taken into account again, it is also evident why the correct status of DIR should be the one of a Feeder Schema. Indeed, if DIR were not a schema of that kind, it would be possible to make deductions such as this one:

p
p : q
p : q : r
p : q : r : s
p : q : r : s : t
… and so on

Where ':' represents disjunction (it is the symbol used in RFT for disjunction, which is different from that of standard logic).

So, ultimately, DIR fulfills the characteristics that a Feeder Schema should have. It is not applied whenever possible, but only when necessary to the use of other schemata. If the latter is not the case, it is not applied, since it could lead to endless processes.

Conclusions

All of the theories that are nowadays trying to account for human reasoning have important challenges to face. Overall, they do that and overcome their difficulties, but there are also always little details that are problematic and hard to explain. Neither the mental logic theory nor RFT are exceptions in this regard. In this paper, I have tried to address one of the little details that cause problems to these formal approaches.

Both the mental logic theory and the update of it offered by RFT reject DIR. However, people make inferences with the formal structure of Schema 2 in a correct way (as said, the percentage of error is 0%). It is true that it can be argued, as López-Astorga (2015a) does, that many cases in which DIR appears to be applied refer to sentences that just seem to be disjunctions, not being actual disjunctions. Nevertheless, if we pay attention to the experiments presented in the literature, we can note that this argument cannot be used to explain what happens in the tasks in which Schema 2 is involved. In those tasks, the disjunctions included in the antecedents of the conditionals are clearly real disjunctions. So, another solution is necessary.

The one that I have proposed here is that DIR should be considered to be a schema of RFT, but not a Core Schema of it, but just a Feeder Schema of that framework. In my view, this idea is absolutely supported by the literature, since the experiments about human reasoning show that DIR is actually used only when its result can help the reasoner obtain a conclusion that cannot be inferred without it. In spite of this, it is also necessary to acknowledge that, although my idea takes the general theses of RFT into account and, at the same time, is, as far as I know, as said, compatible with the empirical results reported so far, it is obvious that further research can be required in order to better support it. In this way, it can be thought that, for the time being, the idea is only a hypothesis. However, it is also evident that, even if it is thought so, that does not prevent that the idea is used as a provisional solution that enables to continue to work from theories such as RFT

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