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THE POSSIBILITY OF THE CLAUSES IN THE CONDITIONAL AND DISJUNCTION[1]

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Abstract

The theory of mental models gives an account of how human beings infer conclusions. That account is not coherent with classical logic. It admits inferences that are incorrect in that logic. Two of those inferences are addressed here. One of them allows deriving that the clauses of a conditional are possible if that conditional is true. The other one enables to deduce that the disjuncts of a disjunction are possible if that disjunction is true. Resorting to the way Chrysippus of Soli considers conditional relations, the present paper offers two axioms capturing the structures of these two inferences. The idea is that those axioms could be included in a hypothetical axiomatic system attempting to reproduce how human inferential processes are.

Keywords: *Chrysippus of Soli; Conditional; Disjunction; Mental models; Reasoning.*

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LA POSIBILIDAD DE LAS CLÁUSULAS EN EL CONDICIONAL Y LA DISYUNCIÓN

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Resumen

La teoría de los modelos mentales ofrece una explicación de cómo los seres humanos infieren conclusiones. Tal explicación no es coherente con la lógica clásica. Admite inferencias que son incorrectas en dicha lógica. Dos de esas inferencias son consideradas aquí. Una de ellas permite derivar que las cláusulas de un condicional son posibles si ese condicional es verdadero. La otra autoriza a deducir que las cláusulas de una disyunción son posibles si esa disyunción es verdadera. Recurriendo al modo que en Crisipo de Solos entiende las relaciones condicionales, este trabajo propone dos axiomas que describen las estructuras de esas dos inferencias. La idea es que tales axiomas podrían ser incluidos en un hipotético sistema axiomático que intentara reproducir cómo son los procesos inferenciales humanos.

Palabras Clave: *Crisipo de Solos; condicional; disyunción; modelos mentales; razonamiento.*

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THE POSSIBILITY OF THE CLAUSES IN THE CONDITIONAL AND DISJUNCTION

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I. Introduction

It is possible to understand human reasoning as an analysis of models. That is what the theory of mental models does. It proposes that reasoning is thinking about the possibilities that hold when a particular sentence is true (e.g., Johnson-Laird & Ragni, 2019; more information on the way this theory works is given below). This theory has allowed explaining many of the problems the idea that human reasoning is compatible with classical logic causes (see also, e.g., Orenes & Johnson-Laird, 2012). Thus, the theory of mental models has shown that the manner human beings think does not respond to the requirements of that logic in several points (see also, e.g., Johnson-Laird et al., 2015).

Some of those points have to do with deductions that are correct in propositional calculus and that people often deem as incorrect inferences. A case of those deductions is (1).

(1) $p \therefore p \vee q$

Where '$\therefore$' is the symbol for logical deduction and '$\vee$' represents disjunction.

Inference (1) is correct in classical propositional logic. However, as explained by proponents of the theory of mental models (Orenes & Johnson-

Laird, 2012), people do not always make inferences with this structure. A similar case is (2).

(2) $q \therefore p \rightarrow q$

Where '→' stands for the material conditional.

This second inference is correct in classical logic, too. Nevertheless, as also shown by the theory of mental models (Orenes & Johnson-Laird, 2012), individuals often think that it is not right.

Besides, there are cases in which, according to the theory of mental models, people make inferences that are not admitted in standard logic. An inference of this last type is (3).

(3) $p \rightarrow q \therefore \Diamond p \wedge \Diamond q$

Where '◊' represents the modal operator of possibility and '∧' expresses conjunction.

None of the usual normal modal logics can accept (3). But the theory of mental models can account for the reasons why individuals can admit it (e.g., Espino et al., 2020).

A case akin to (3) is (4).

(4) $p \vee q \therefore \Diamond p \wedge \Diamond q$

Inference (4) is wrong in every usual normal modal logic as well. Nonetheless, the theory of mental models can also explain why people can accept it (e.g., Johnson-Laird et al., 2021).

To build an axiomatic system as close to the theory of mental models as possible, it is necessary, at least, to solve the problems that (1) to (4) present. This is because (1) to (4) are examples of the characteristics moving the theory away from classical logic. To remove the difficulties associated to (1) and (2) may not be hard. It may suffice to eliminate or limit the situations in which (5) and (6) can be used.

(5) $p \rightarrow (p \vee q)$

(6) $q \rightarrow (p \rightarrow q)$

There are theories that have done something similar. For instance, there is a theory claiming that the human mind follows a special mental logic. It is a mental logic not accepting all the rules of propositional calculus. The theory is the mental logic theory (e.g., O'Brien, 2014). According to this theory, there are a number of schemata valid in classical logic people use, but individuals do not apply all the schemata classical logic enables. In addition, the use of the schemata allowed is limited in some cases (Braine & O'Brien, 1998a). It considers the inference corresponding to (5) to be incorrect (e.g., Braine & O'Brien, 1998a; for an analysis of the problems of the mental logic theory with this inference, see also, e.g., López-Astorga, 2017). On the other hand, it restricts the application of the inference corresponding to (6) (see also, e.g., Braine & O'Brien, 1998b).

Inferences (3) and (4) might be a greater challenge. This paper will deal with that challenge. The paper will not offer a new axiomatic system working in a manner compatible with the theory of mental models. It will show only how to introduce two axioms related, respectively, to (3) and (4) in a hypothetical axiomatic system. The intention is to attempt to bring that axiomatic system together with the way people derive conclusions according to the theory of mental models. To do that, the paper will resort to the criterion Chrysippus of Soli presented to interpret the conditional. The reason for this election is that Stoic logic has shown to be useful to deal with different cognitive problems (e.g., López-Astorga, 2021a).

The first section will be devoted to general important theses of the theory of mental models and the way models work within it. Then, it will be explained why, following that theory, people tend not to accept (1). Third, the reasons why, from the perspective of that very theory, individuals also usually reject (2) will be indicated. The next section will address (3). It will present the arguments of the theory of mental models to accept it. Fifth, a similar account for (4) will be offered. The sixth section will develop the manner Chrysippus understood the relation antecedent-consequent in conditional sentences. The last section will describe the way an axiom capturing (3) and an axiom capturing (4) can be introduced. The introduction of the axioms will be done by virtue of Chrysippus' interpretation.

II. Models as possibilities in the theory of mental models

The theory of mental models claims that the human mind links sentential connectives to models (see also, e.g., Khemlani et al., 2018). Those models are understood as possibilities (see also, e.g., Byrne & Johnson-Laird, 2020). The possibilities are joined by means of conjunctions, prompting 'conjunctions of possibilities' (see also, e.g., Khemlani et al., 2017). The models or possibilities that the theory attributes to inclusive disjunctions are those in (7) (see also, e.g., Quelhas et al., 2019; the symbols this paper will use to express models are the same as those in works such as López-Astorga, 2021b).

(7) Possible (p & q) & Possible (p & ¬q) & Possible (¬p & q)

Where 'Possible (x)' means that 'x is possible broadly speaking' (not with the meaning it has in modal logic), '&' is conjunction (the symbol '∧' is not used here because models should to be differentiated from logical formulae; see, e.g., Johnson-Laird, 2010), and '¬' denotes negation (conjunction of possibilities (7) is conjunction of possibilities (2) in López-Astorga, 2021b; p and q are the disjuncts of the inclusive disjunction).

On the other hand, the models the theory assigns to the conditional are those in (8) (see also, e.g., Goodwin & Johnson-Laird, 2018).

(8) Possible (p & q) & Possible (¬p & q) & Possible (¬p & ¬q)

(Conjunction of possibilities (8) is conjunction of possibilities (6) in López-Astorga, 2021b; p is the antecedent of the conditional and q is its consequent).

It is important to note that, both in (7) and in (8), the connective binding the possibilities is conjunction. So, the possibilities are not rows in truth tables (e.g., Johnson-Laird & Ragni, 2019).

Furthermore, the differences between the theory of mental models and classical standard logic are various. For example, the theory of mental models

> […] distinguishes between two systems of reasoning-an idea due to the late Peter Wason (e.g., Johnson-Laird & Wason, 1970), but that the model theory has always maintained (cf. Evans, 2008). System 1, the intuitive system, relies on models that represent only what is true in each possibility (Johnson-Laird et al., 2021, p. 957).

Thus, system 1 does not allow considering all the entire possibilities, since that activity requires deliberation. This means that system 1 does not allow considering all the entire possibilities in (7) and (8), but only the clauses that are not negated in those possibilities, that is, what is most intuitive. In the case of (7), that implies that, with system 1, individuals can realize that p and q are possible at the same time (first possibility), that p is possible (second possibility), and that q is possible as well (third possibility). They cannot become aware of that q can be false when p is true (second possibility), or that p can be false when q is true (third possibility). Something similar happens with (8). System 1 leads to take only its first possibility into account (i.e., the possibility of p and q being true at once). The other two possibilities, that in which p is not true but q is (second possibility) and that in which none of the two clauses is true (third possibility), are ignored. To note (7) and (8) as expressed above, other system is necessary. This is because "A deliberative process of reasoning, system 2, can construct explicit models that also represent an exhaustive conjunction of default possibilities. In each possibility they represent what is true and also what is false, using true negations to do so" (Johnson-Laird et al., 2021, pp. 957-958).

Another important component differentiating the theory of mental models from standard logic is modulation. Modulation is a process in which

"The meaning of words, knowledge, and the conversational context can block the construction of models of possibilities, and they can add causal, spatiotemporal, and other relations between elements in models. Experiments have corroborated these effects" (Khemlani et al., 2018, pp. 1898-1899).

An instance in the case of disjunction is (9).

(9) "Pat is in Rio or she is in Brazil" (Johnson-Laird, 2010, p. 206).

The possibilities of (9) do not match the possibilities in (7). Rio is a Brazilian city. Therefore, the second possibility in (7), that is, p & ¬q, cannot be the case.

A similar example for the conditional is (10).

(10) "If she played a musical instrument then she didn't play a flute" (Johnson-Laird, 2010, p. 201).

Beyond the fact that the consequent is negated in (10), a model in which she does not play a musical instrument and she plays a flute, that is, a model such as the second one in (8) (i.e., ¬p & q) would not be possible. What would be possible is the missing model in (10), that is, the model corresponding to the false case of the conditional if materially interpreted (i.e., p & ¬q).

More points make the theory of mental models different from classical logic. However, the account in this section is enough to develop the next sections. The account shows that, in particular cases, both the action of system 1 and modulation could have an influence on the arguments that will be presented below.

III. The introduction of disjunction

Inference (1) is a basic rule in propositional calculus to introduce disjunctions (e.g., Deaño, 1999). However, the theory of mental models has experimentally shown that people tend not to accept it (Orenes & Johnson-Laird, 2012). The reason is simple within the theory.

In (1), the premise establishes that p is true. But conjunction of possibilities (7), which is that corresponding to the formula derived in (1), includes a possibility incompatible with the premise. That possibility is the third one in (7), which expresses that p is false (see Orenes & Johnson-Laird, 2012).

An example built resorting to thematic content can illustrate this. If (11) is the premise in a deduction,

(11) Their last name is Smith

Propositional logic enables to deduce (12) from (11).

(12) Their last name is Smith or Archer

Disjunction (12) can be understood as exclusive. For this reason, the first possibility in (7), that is, p & q, can be eliminated. Nevertheless, the problem is the last possibility in (7), that is, ¬p & q. This is because the premise provides that it is true that their last name is Smith (i.e., as in (1), that p is true).

Theories such as the mental logic theory resolve this problem. They propose that there is a logic leading the human mind. But all the deductions that are valid in classical logic are not necessarily correct in that mental logic. One of those deductions the mental logic theory rejects is (1) (e.g., Braine & O'Brien, 1998a).

IV. The introduction of the conditional

The situation is not very different in the case of (2). (2) is also a basic rule in propositional logic (e.g., Deaño, 1999). Nonetheless, based on experimental results, the theory of mental models has claimed that people tend to deem it as unacceptable, too (Orenes & Johnson-Laird, 2012). Again, from the perspective of the theory, it is easy to understand the reasons.

In (2), the premise indicates that q is true. However, conjunction of possibilities (8), which is the conjunction that can be attributed to the conditional, has a possibility inconsistent with the premise. The possibility is the last one in (8). In that possibility, q is not the case (see Orenes & Johnson-Laird, 2012).

Resorting to thematic content again, an example can be that of (13) and (14). If (13) is a premise,

(13) They will go to the cinema

Classical propositional logic allows deriving (14).

(14) If they are from this town, then they will go to the cinema

Following (8), between the possibilities that can be assigned to (14), one of them is the scenario in which they are not from this town and they do not go to the cinema (¬p & ¬q). This last possibility cannot be accepted. In it, premise 'they will go to the cinema' is false.

This problem can also be removed from the perspective of theories such as the mental logic theory. These theories limit the use of (2). Pragmatics is important in the mental logic theory. So, inferences such as (2) are only correct when they pragmatically make sense (e.g., Braine & O'Brien, 1998b). In this way, the theory does not enable, given a premise, to build a conditional from it introducing an antecedent with any content (which is what propositional calculus admits). One might interpret that the conditional can only be introduced if the consequent has not been inferred and the assumption of the antecedent allow deducing the consequent (e.g., Braine & O'Brien, 1998b).

V. The conditional and the possibility of the antecedent and the consequent

The theory of mental models permits (3) because, given a conditional such as the premise in (3), its possibilities are those in (8). The first possibility in (8), that is, p & q, reveals that p is possible. The first and second possibilities in (8), that is, p & q and ¬p & q, indicate that q is possible. Therefore, both p and q are possible (e.g., Espino et al., 2020).

Thus, from the theory of mental models, if (14) is true, it is possible that they are from this town, and it is possible that they go to the cinema. This is not the case in usual normal modal logics. In them, (14) can be true even if it is impossible that they are from this town or they go to the cinema (for explanations such as this one, see, e.g., Espino et al., 2020).

VI. Disjunction and the possibility of its disjuncts

The account for (4) is akin to that of (3). Given a disjunction such as the premise in (4), the possibilities that can be deployed are those in (7). The first and second possibilities in (7), that is, p & q and p & ¬q, establish that p is possible. The first and third possibilities in (7), that is, p & q and ¬p & q, provide that q is possible (e.g., Khemlani et al., 2017).

Therefore, according to the theory of mental models, if (12) is true, it is possible that their last name is both Smith and Archer. But usual normal modal logics do not enable this either. In these logics, (12) can be true when it is impossible that their last name is Smith (it is enough that their last name is Archer). Likewise, (12) can also be true when it is impossible that their last name is Archer (that their last name is Smith suffices) (for explanations such as this one, see, e.g., Khemlani et al., 2017).

VII. The relation between the antecedent and the consequent following Chrysippus of Soli

Perhaps there is a way to introduce axioms linked to (3) and (4) in a hypothetical axiomatic system working in a manner similar to the theory of mental models (i.e., to the manner the human mind works following the theory of mental models). To do that, it may be enough to resort to the interpretation of the conditional Chrysippus of Soli proposes.

Chrysippus did not comprehend the conditional as Philo of Megara did: Chrysippus' interpretation is not the material interpretation classical logic offers (e.g., O'Toole & Jennings, 2004). In Chrysippus' view, a connection between the antecedent and the consequent was necessary (e.g., Barnes et al., 2008). This led to a 'connexive logic' (e.g., Lenzen, 2019), which "… claimed a fight between the antecedent and the negation or denial of the second clause" (López-Astorga, 2021a, p. 37). Several ancient writers, for example, Cicero, Diogenes Laërtius, or Sextus Empiricus, seem to attribute this view to Chrysippus of Soli (see, e.g., Gould, 1970; López-Astorga, 2021a; O'Toole & Jennings, 2004).

A formula capturing Chrysippus' idea has been given (Lenzen, 2019). That is formula (15).

(15) $(p \Rightarrow q) \leftrightarrow \neg\Diamond(p \wedge \neg q)$

Where '⇒' represents the conditional relation as understood by Chrysippus of Soli and '↔' stands for biconditional relation.

Formula (15) is formula (24) in López-Astorga (2021a), which is already classical in modal logic. Formula (15) can help construct axioms corresponding to (3) and (4) for a hypothetical axiomatic system respecting main theses of the theory of mental models.

VIII. Two axioms based on deductions of possibilities not admitted in usual normal modal logics

If the relation between the premise and the conclusion in (3) is deemed as a conditional relation consistent with Chrysippus' view, (3) can be expressed as (16).

(16) $(p \rightarrow q) \Rightarrow (\Diamond p \wedge \Diamond q)$

Formulae (15) and (16) lead to (17).

(17) $\neg\Diamond[(p \rightarrow q) \wedge \neg(\Diamond p \wedge \Diamond q)]$

If (17) holds, (18) holds, in general, in normal modal logics.

(18) $N\neg[(p \rightarrow q) \wedge \neg(\Diamond p \wedge \Diamond q)]$

Where 'N' denotes the modal operator of necessity.

Classical propositional logic enables to transform (18) into (19).

(19) $N[\neg(p \rightarrow q) \vee (\Diamond p \wedge \Diamond q)]$

And, again, in classical propositional calculus, (19) is equivalent to (20).

(20) $N[(p \rightarrow q) \rightarrow (\Diamond p \wedge \Diamond q)]$

Formula (20) can be the axiom for (3). It establishes that the conditional that can be formed from (3) is necessary.

A similar process can be thought for (4). If the deduction relation shown in (4) is assumed as a conditional relation in accordance with Chrysippus' criterion between the premise and the conclusion, (4) can be presented as (21).

(21) $(p \vee q) \Rightarrow (\Diamond p \wedge \Diamond q)$

Formulae (15) and (21) allow deriving (22).

(22) $\neg\Diamond[(p \vee q) \wedge \neg(\Diamond p \wedge \Diamond q)]$

In general, in normal modal logics, (23) can be deduced from (22).

(23) $N\neg[(p \vee q) \wedge \neg(\Diamond p \wedge \Diamond q)]$

By classical propositional calculus, (23) can be transformed into (24).

(24) $N[\neg(p \vee q) \vee (\Diamond p \wedge \Diamond q)]$

That very calculus enables to infer (25) from (24).

(25) $N[(p \vee q) \rightarrow (\Diamond p \wedge \Diamond q)]$

Formula (25) could be the second axiom. It reveals that the conditional corresponding to the deduction in (4) is necessary. In this way, the two axioms to include in a hypothetical axiomatic system simulating the functioning of the human mind according to the theory of mental models are (20) and (25).

IX. Conclusions

Many characteristics differentiate the theory of mental models from standard logic. Therefore, if the aim is to build an axiomatic system considering most of the theses of the theory of mental models, those characteristics should be taken into account. This paper has dealt with some of them.

In classical propositional calculus, there are rules people often reject, or, at least, they do not apply. Two examples are those of (1) and (2). To overcome the difficulties associated to those rules is not hard. This has been done from the perspective of the mental logic theory. The latter theory has claimed that (1) is not a basic rule of the real logic leading the human mind. As far as (2) is concerned, that very theory restrains its use in some circumstances.

There are, at a minimum, two more problems in this regard. According to the theory of mental models, conditionals lead to assume that both their antecedents and their consequents are possible. On the other hand, following this last theory, disjunctions also lead to consider their disjuncts to be possible. Hence, an axiomatic system trying to be as consistent with the theory of mental models as possible should not forget these facts.

The key can be the way Chrysippus of Soli interpreted the conditional. Chrysippus requires a relation between the clauses of the conditional: the negation of the second clause cannot be compatible with the first one. If conditional formulae respecting Chrysippus' idea are built from the two last deductions mentioned, that allows coming to two necessary formulae. One of them provides that the conditional relation between, on the one hand, a conditional and, on the other hand, the conjunction of the possibility of its antecedent and the possibility of its consequent is necessary. The second formula indicates that the conditional relation between, on the one hand, a disjunction and, on the other hand, the conjunction of the possibility of one of its disjuncts and the possibility of its other disjunct is necessary, too. Those two formulae can be taken as axioms.

Those axioms would enable to keep moving forward in the construction of an axiomatic system coherent with the theory of mental models. The axioms would not be enough. More features of the theory of mental models distinguishing it from standard classical logic would have to be considered. However, they would have an advantage: if it is the case that the theory of mental models describes the way people infer conclusions, some derivations akin to those that the human mind actually makes could already be correct in the provisional system.

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