

Process Capability Analysis Plot for a Product with Bilateral Specifications

Análisis de capacidad gráfico para un producto con especificaciones
bilaterales

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Abstract

A multi-process performance analysis plot based on process capability indices has been developed to analyse the manufacturing performance for multiple processes, which conveys critical information regarding the departure of the process mean from the target value and process variability, and provides a guideline for capability improvement. Existing performance analysis plots usually assume normal distributions and symmetric tolerances. In this paper, we present a graphical method to evaluate the process capability of a product composed of independent multiple processes with nominal-the-best specifications and symmetric or asymmetric tolerances. The proposed plot aims to get information about the location and spread of the studied characteristics which can follow any continuous distribution. An example showing the effect of processes with asymmetric tolerances on the capability of the product is considered.

Key words: Process capability indices, Process improvement, Statistical process control.

Resumen

El análisis gráfico de la capacidad de un proceso multivariado suministra información crítica acerca de la distancia que existe entre la media y su valor objetivo, para cada uno de los procesos; así como la variabilidad de cada uno de ellos. Esta información provee luces para mejorar la capacidad de todos los procesos que hacen forma del producto. En los gráficos existentes para analizar la capacidad de un producto, se asume que cada uno de los procesos que conforman el producto son independientes, tienen distribución normal y presentan tolerancia simétrica. En este artículo, proponemos

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un método gráfico para evaluar la capacidad de un producto conformado por procesos independientes, bajo cualquier distribución continua para tolerancias simétricas o asimétricas. El gráfico propuesto suministra información acerca de la localización y dispersión de cada una de las características que conforman el producto. Se presenta un ejemplo que ilustra el efecto de la asimetría sobre la capacidad de los procesos.

Palabras clave: control estadístico de procesos, índices de capacidad de procesos, mejoramiento de procesos.

1. Introduction

In the quality improvement processes, to establish if the products meet some preset specifications is important. Process capability indices (PCIs) have been widely used to measure whether the product quality meets the preset specifications.

In process capability studies there are two characteristics, –the process location, usually the process mean, in relation to its target value and the process dispersion–. Some process capability indices combine these two characteristics. The larger the value of a process capability index the more capable the process. A low value of a process capability index indicates that a process is non-capable, but it does not give us information if this situation is due to the variability, the deviation from target or both. See Deleryd & Vännman (1999). This drawback is overcome by graphical displays, which allow getting information about the location and spread of the studied characteristics.

Vännman (1997b) introduced a process capability plot, called the (δ, γ) -plot. The (δ, γ) -plot was defined for a normally distributed process output, bilateral specifications and symmetric tolerances. This is, the target value T is located at the mid-point of the specification interval. Chen, Huang & Li (2001) developed a chart to analyse the capability of a whole product, called PCAC. The PCAC evaluates simultaneously independent characteristics, with bilateral or unilateral specifications, symmetric or asymmetric specifications, but each process must have normal distribution. Huang & Chen (2003) developed a process capability chart for a multi-process product with bilateral specifications, under normal distributions. Chen, Hsu & Wu (2006) proposed a multi-process capability chart to evaluate non-normal processes capability, with symmetric tolerances. Chen, Huang & Hung (2008) took the PCAC of Chen et al. (2001) and proposed the $PCAC/C_{pm}$ chart. It can evaluate the capability of the product employing the univariate indices C_{pu} , C_{pl} and C_{pm} . $PCAC/C_{pm}$ is adequate for processes with normal distributions and symmetric tolerances.

In this paper we propose the $(\delta, \gamma, \varepsilon)$ -plot to evaluate the capability of a product with n independent characteristics with bilateral specifications. The proposed plot can be applied to normal or non-normal distributions and symmetric or asymmetric tolerances.

The remainder of this paper is organized as follows: in Section 2 some capability indices are presented. Then in Section 3 a process capability index for a multi-

process product with bilateral specifications is proposed. Section 4 describes the $(\delta, \gamma, \varepsilon)$ -plot. The practical advantages of the $(\delta, \gamma, \varepsilon)$ -plot are shown through an example in Section 5. Finally, some conclusions are given.

2. Process Capability Indices

Vännman (1995) defined a family of capability indices, depending on two non-negative parameters, u and v , as

$$C_p(u, v) = \frac{d - u |\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}$$

where μ is the process mean, σ the process standard deviation, T is the target value, $d = \frac{USL - LSL}{2}$ is half of the length of the specification interval, where USL and LSL are the upper and lower specification limits respectively, and $m = \frac{USL + LSL}{2}$ is the mid-point of the specification interval. By setting u and v equal to 0 or 1, we obtain the four basic indices, this is, $C_p(0, 0) = C_p$, $C_p(1, 0) = C_{pk}$, $C_p(0, 1) = C_{pm}$ and $C_p(1, 1) = C_{pmk}$. The indices $C_p(u, v)$ are appropriate for normal processes and symmetric tolerances, see Pearn, Wu & Wang (2005), but they have been shown to be inappropriate otherwise.

Pearn & Chen (1997) considered the generalization of $C_p(u, v)$, called $C_{Np}(u, v)$, which can be applied to processes with arbitrary distributions. $C_{Np}(u, v)$ is defined as

$$C_{Np}(u, v) = \frac{d - u |M - m|}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + v(M - T)^2}}$$

where M is the process median, P_α is the $(100\alpha)th$ percentile of the distribution and $\frac{P_{99.865} - P_{0.135}}{6}$ is the process spread. If the process follows the normal distribution, then $C_{Np}(u, v)$ reduces to $C_p(u, v)$.

Both, $C_p(u, v)$ and $C_{Np}(u, v)$ are appropriate for processes with symmetric tolerances. To handle asymmetric tolerances Vännman (1997a) proposed the index $C_{pa}(u, v)$, defined as

$$C_{pa}(u, v) = \frac{d - |\mu - m| - u |\mu - T|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}$$

Pearn et al. (2005) proposed the index $C''_{Np}(u, v)$, which is expressed as

$$C''_{Np}(u, v) = \frac{d^* - uA^*}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + vA^2}} \quad (1)$$

where $A = \max\left\{\frac{d(M-T)}{d_u}, \frac{d(T-M)}{d_l}\right\}$, $A^* = \max\left\{\frac{d^*(M-T)}{d_u}, \frac{d^*(T-M)}{d_l}\right\}$, $d_u = USL - T$, $d_l = T - LSL$, $d^* = \min(d_u, d_l)$. This index can be applied under non-normal distributions with symmetric or asymmetric tolerances. By letting $u = 0$ or 1 and $v = 0$ or 1, we denote $C''_{Np}(0, 0) = C''_{Np}$, $C''_{Np}(1, 0) = C''_{Npk}$, $C''_{Np}(0, 1) = C''_{Npm}$, and $C''_{Np}(1, 1) = C''_{Npmk}$. If $T = m$, then $C''_{Np}(u, v)$ reduces to $C_{Np}(u, v)$.

3. Process Capability Index for an Entire Product

Since a product usually holds many quality characteristics, Chen et al. (2001) defined an integrated process capability index, C_T , for independent characteristics, each of them following a normal distribution. Chen et al. (2006) modified the index C_T , to allow non-normal distributions. They assume that the product has t_n characteristics with bilateral specifications evaluated by N_{pn} , $t_u + t_l$ characteristics with unilateral specifications, where t_u is the number of smaller-the-better characteristics evaluated by N_{pu} and t_l is the number of larger-the-better characteristics evaluated by N_{pl} . The new index, denoted by N_T is defined as

$$N_T = \frac{1}{3} \Phi^{-1} \left\{ \frac{\left[\left(\prod_{i \in S} \prod_{j=1}^{n_i} [2\Phi(3N_{pij}) - 1] \right) + 1 \right]}{2} \right\}$$

where Φ^{-1} is the inverse cumulative distribution function of the standard normal distribution, $S = \{u, l, n\}$ denotes the type of process, smaller-the-better, larger-the-better and nominal-the-best respectively, and

$$N_{pij} = \begin{cases} \frac{USL - M}{(P_{99.865} - P_{0.135})/2}, & i = u, j = 1, 2, \dots, t_u \\ \frac{M - LSL}{(P_{99.865} - P_{0.135})/2}, & i = l, j = 1, 2, \dots, t_l \\ \frac{\min(USL - M, M - LSL)}{3\sqrt{\left[\frac{(P_{99.865} - P_{0.135})}{6}\right]^2 + (M - T)^2}}, & i = n, j = 1, 2, \dots, t_n \end{cases}$$

N_T can be calculated for unilateral and bilateral specifications. However, for bilateral specifications, the processes must have symmetric tolerances.

We propose an integrated process capability index for a product with n normal or non-normal characteristics, bilateral specifications and symmetric or asymmetric tolerances. This index, denoted by C_{Np}^T , is defined as

$$C_{Np}^T = \frac{1}{3} \Phi^{-1} \left\{ \frac{\prod_{j=1}^n [2\Phi(3C_{Npmkj}^n) - 1] + 1}{2} \right\}$$

where $C_{Npmk}^n = C_{Np}^n(1, 1)$.

If the process capability for the entire product is greater or equal to a fixed value, let's say a , then

$$C_{Np}^T = \frac{1}{3} \Phi^{-1} \left\{ \frac{\prod_{j=1}^n [2\Phi(3C_{Npmkj}^n) - 1] + 1}{2} \right\} \geq a$$

Specifically, when the preset minimum values of process capabilities for individual characteristics are equal, we have

$$C_{Np}^T = \frac{1}{3} \Phi^{-1} \left\{ \frac{\left(\left[2\Phi \left(3C_{Npmk_j}^n \right) - 1 \right] \right)^n + 1}{2} \right\} \geq a$$

The critical value a_0 ($C_{Npmk_j}^n \geq a_0$) for individual process capability can be attained by solving the previous inequality when the integrated process yield exceeds a ($C_{Np}^T \geq a$), where

$$a_0 = \frac{1}{3} \Phi^{-1} \left[\frac{(2\Phi(3a) - 1)^{\frac{1}{n}} + 1}{2} \right] \quad (2)$$

4. The $(\delta, \gamma, \varepsilon)$ -plot for a Multi-Process Product

The index $C_{Np}^n(u, v)$ defined in (1) can be written as

$$C_{Np}^n(u, v) = \frac{1 - u|\delta|}{3\sqrt{\gamma^2 + v\varepsilon^2\delta^2}}$$

where

$$\delta = \begin{cases} \frac{M-T}{d_u} & M \geq T, \\ \frac{M-T}{d_l} & M \leq T \end{cases} \quad (3)$$

$$\gamma = \left(\frac{P_{99.865} - P_{0.135}}{6d^*} \right) \quad (4)$$

and

$$\varepsilon = \frac{d}{d^*} \quad (5)$$

δ measures the distance from the target value to the process median relative to d_u or d_l , γ measures how large the process spread is relative to d^* , and ε represents the process type of tolerance.

The $(\delta, \gamma, \varepsilon)$ -plot is a three dimensional contour plot which aims to visualize the capability of a product holding several quality characteristics. If the capability of the entire product is preset at a , $C_{Np}^T = a$, the minimum process capability index, a_0 , of each characteristic is calculated from equation (2).

The level curves are obtained solving the equation $C_{Np}^n(u, v) = a_0$ with respect to γ . This is by, setting

$$C_{Np}^n(u, v) = \frac{1 - u|\delta|}{3\sqrt{\gamma^2 + v\varepsilon^2\delta^2}} = a_0$$

we obtain

$$\gamma = \sqrt{\frac{(1 - u|\delta|)^2}{9a_0^2} - v\varepsilon^2\delta^2} \quad (6)$$

where $|\delta| \leq \frac{1}{u+3a_0\varepsilon\sqrt{v}}$.

The plot provides information instantly about the location, spread and tolerance of a multiprocess product, where each characteristic has bilateral specifications and follows a normal or non-normal distribution.

The region inside the contour plot represents capable processes, while the zone located outside the level plots corresponds to non-capable processes. The $(\delta, \gamma, \varepsilon)$ -plot depends on the values of u , v and a_0 . Figures 1a and 1b present the $(\delta, \gamma, \varepsilon)$ -plots for values of $C_{Np}''(0, 1)$ and $C_{Np}''(1, 1)$ equal to 1.0 and 2.0, according to the equation (6). These figures show us that the larger ε becomes the smaller the capable region is. For symmetric tolerances, that is $\varepsilon = 1$, we obtain the largest capable region. This region begins to reduce as the target value T is located far away from the mid-point of the specification interval.

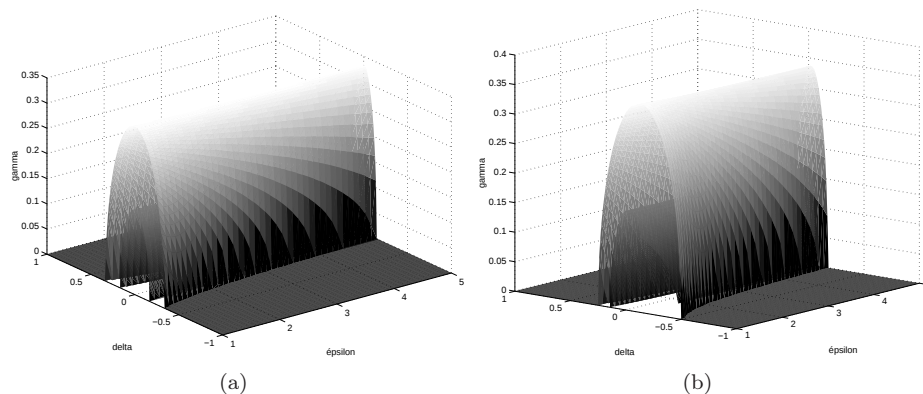
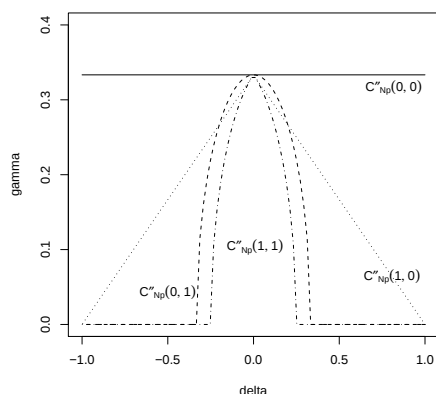


FIGURE 1: Example of a two Level curves for: (a) $C_{Np}''(0, 1) = 1.0, 2.0$ and (b) $C_{Np}''(1, 1) = 1.0, 2.0$; from top to bottom.

When each one of the characteristics measured in a product has symmetric tolerances, the $(\delta, \gamma, \varepsilon)$ -plot can be visualized in two dimensions as it is shown in Figure 2. Figure 2 coincides with the graphics of Deleryd & Vännman (1999), for the case of Gaussian processes.

Any characteristic has a point $(\delta, \gamma, \varepsilon)$ in a three-dimensional space. If the corresponding value of $(\delta, \gamma, \varepsilon)$ is inside the capability region then the characteristic meets the preset specifications. If the value is outside, we analyse the δ -axis to observe how far away the process median is relative to the target value, and the γ -axis to study the process spread. These analyses give us the key elements to be improved.

If the three-dimensional graphic is not easily visualized, we can plot two-dimensional graphics for each value of ε , so if we have r different values of ε , we construct r (δ, γ) -plots and analyse them. R code for implementing the proposed procedure is available from the first author upon request.

FIGURE 2: $(\delta, \gamma, \varepsilon)$ -plot for $C_{Np}''(u, v) = 1$, with $\varepsilon=1$, $u=0,1$ and $v=0,1$.

5. Example

We consider the data set presented by Huang & Chen (2003) and compare their results with those obtained by our graphical method. The application consists of 10 key characteristics of a Sea island microfiber product. Table 1 gives the specifications of these characteristics. To emphasize the cases with asymmetric tolerances we have slightly modified the target value of the 8th characteristic from 3.0000 to 3.0010.

First, we set a value for the integrated process capability, say $C_{Np}^T = 1.00$. Then, the corresponding minimum individual process capability for each quality characteristic is $a_0 = 1.214$, applying equation (2). By using equations (3), (4) and (5), the δ , γ , and ε coordinates for the 10 independent characteristics are calculated. See Table 1. The $(\delta, \gamma, \varepsilon)$ points are located on the graph as shown in Figure 3.

TABLE 1: Process capabilities for Sea-island micro-fiber TS2-135N.

Characteristic	LSL	T	USL	M	σ	δ	γ	ε	$C_{Np}''(1, 1)$	C_{pmk}''
1. Denier	3.3000	3.5000	3.7000	3.4900	0.0500	-0.0500	0.2500	1.0	1.2421	1.2421
2. Tenacity	4.5000	5.0000	5.5000	4.8000	0.1500	-0.4000	0.3000	1.0	0.4000	0.4000
3. Elongation	47.5000	50.0000	52.5000	50.7500	0.2035	0.3000	0.0814	1.0	0.7506	0.7506
4. OPU	1.4925	1.5000	1.5075	1.4992	0.0015	-0.1067	0.2000	1.0	1.3137	1.3137
5. Crumple number	10.0000	11.0000	12.0000	11.1245	0.0341	0.1245	0.0341	1.0	2.2608	2.2608
6. Crimple	11.7600	12.0000	12.2400	12.0150	0.0500	0.0625	0.2083	1.0	1.4367	1.4367
7. Rate of crimple elasticity	40.5000	45.0000	49.5000	46.5500	0.3575	0.3444	0.0794	1.0	0.6182	0.6182
8. Water	2.9850	3.0010	3.0090	2.9980	0.0002	-0.1875	0.0250	1.5	0.9592	1.4318
9. Hot-air shrinkage	4.9000	5.0000	5.0500	4.9800	0.0090	-0.2000	0.1800	1.5	0.7622	0.9911
10. Length	47.9400	51.0000	54.0600	50.6250	0.3500	-0.1225	0.1144	1.0	1.7448	1.7448

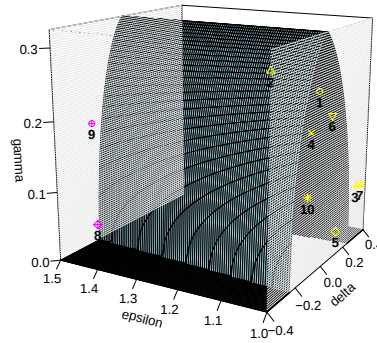


FIGURE 3: $(\delta, \gamma, \varepsilon)$ -plot for the quality status of Sea island micro-fiber TS2-135N.

Since there are two different values of ε , 1 and 1.5, as it can be seen in Table 1, we can construct a (δ, γ) -plot for each ε . See Figures 4 and 5.

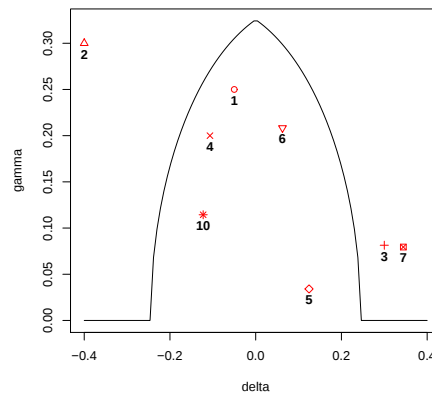


FIGURE 4: (δ, γ) -plot for the quality status of Sea island micro-fiber TS2-135N when $\varepsilon=1$.

From Figures 4 and 5 we observe that five characteristics (2, 3, 7, 8 and 9) are not within the contour line. Actions must be taken to improve the product quality. The process centers of characteristics 3, 7, 8 and 9 should be shifted toward the target to improve process capability. The spread of the distribution of characteristic 2 must be reduced and its mean moved closer to the target.

In column 9 of Table 1 we show the values of the index C_{pmk}'' used by Huang & Chen (2003) and in column 8 we show the values of index C_{Npmk}'' used in our proposal. Comparing columns 8 and 9 we see that both indices are equal when the tolerance is symmetric, but in presence of asymmetric tolerance the value of the index C_{Npmk}'' is less than C_{pmk}'' . This situation makes that the capability region used in our proposal for characteristics 8 and 9 be smaller than the capability

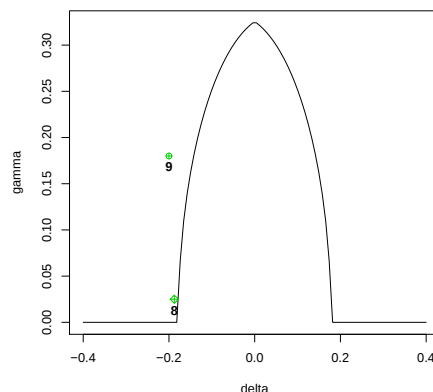


FIGURE 5: (δ, γ) -plot for the quality status of Sea island micro-fiber TS2-135N when $\varepsilon=1.5$.

region used for the others characteristics. See Figures 4 and 5. Besides, we see that characteristic 8 is not capable ($C''_{Npmk} = 0.9592 < a_0$). In the methodology of Huang & Chen (2003), the capability region is the same for all characteristics and in particular the 8th characteristic is capable ($C''_{pmk} = 1.4318 > a_0$). When the tolerance is symmetric, the capability region is the same for the two methods, therefore there is not difference in the analysis for characteristics 2, 3 and 7.

6. Conclusions

Process capability plots have been proved to provide more information than process capability indices. In this paper, we have proposed a three-dimensional process capability graphic, called the $(\delta, \gamma, \varepsilon)$ -plot. The $(\delta, \gamma, \varepsilon)$ -plot allows simultaneous monitoring of several independent characteristics of a product with bilateral specifications and symmetric or asymmetric tolerances, which follow normal or non-normal distributions. The plot defines contour lines, which define the capability regions, and provides information about the location and spread of the characteristics. This information, provided to the practitioners, generates more efficient actions for improving the product quality. An issue, for a future research, is to extend this method to several correlated characteristics.

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Appendix

A1. Introducción

En los procesos de mejoramiento de calidad es importante determinar si los procesos se ajustan a unas especificaciones previamente establecidas. Los índices de capacidad de procesos, PCI por su sigla en inglés, miden este ajuste. Algunos de ellos combinan dos características básicas en los estudios de capacidad de procesos: la localización del proceso con relación a su valor objetivo y la dispersión del proceso. Un valor bajo del índice indica que el proceso no es capaz, pero desafortunadamente no expresa si esta información se debe a la variabilidad del proceso o que su localización está alejada del valor objetivo. Véase Deleryd & Vännman (1999). Esta deficiencia es superada mediante el empleo de métodos gráficos.

Vännman (1997b) introdujo un gráfico para representar la capacidad de un proceso, denominado el gráfico (δ, γ) . Este gráfico requiere que el proceso tenga distribución normal, con especificaciones bilaterales y tolerancias simétricas (el valor objetivo se localiza en el punto medio del intervalo de especificación). Chen et al. (2001) desarrollaron un gráfico para analizar la capacidad de un producto, denominado PCAC. Este gráfico evalúa simultáneamente características independientes con especificaciones bilaterales o unilaterales, pero se requiere que la distribución de cada característica sea normal. Huang & Chen (2003) propusieron un gráfico de capacidad de procesos, bajo cualquier distribución pero sujeto a especificaciones bilaterales. Chen et al. (2006) plantearon un gráfico multiproceso para evaluar la capacidad de procesos no normales, pero con tolerancias simétricas. Chen et al. (2008) tomaron el PCAC planteado por Chen et al. (2001) y proponen el gráfico $PCAC/C'_{pm}$, que puede evaluar la capacidad de un producto empleando los índices univariados básicos C'_{pu} , C'_{pl} and C'_{pm} . El $PCAC/C'_{pm}$ es adecuado para procesos con distribuciones normales y tolerancias simétricas.

En este artículo se propone el gráfico $(\delta, \gamma, \varepsilon)$ para evaluar la capacidad de un producto conformado por n características independientes cuyas especificaciones son bilaterales. Este gráfico no requiere que cada proceso tenga distribución normal ni que las tolerancias sean simétricas.

El resto del artículo está organizado como sigue: en la sección A2 algunos índices de capacidad son presentados. Posteriormente, en la sección A3 un índice para la medir capacidad de un producto con especificaciones bilaterales es propuesto. En la sección A4 través de un ejemplo se presentan las ventajas del gráfico $(\delta, \gamma, \varepsilon)$. Este apéndice no incluye la descripción del gráfico $(\delta, \gamma, \varepsilon)$ ni las conclusiones del artículo.

A2. Índices de capacidad de proceso

Vännman (1995) definió una familia de índices de capacidad, dependiendo de dos parámetros no negativos, u and v , como:

$$C_p(u, v) = \frac{d - u |\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}$$

donde μ es la media del proceso, σ la desviación estándar del proceso, T el valor objetivo, $d = \frac{USL - LSL}{2}$ la mitad de la longitud del intervalo de especificación, USL y LSL los límites de especificación superior e inferior, respectivamente; y $m = \frac{USL + LSL}{2}$ es el punto medio del intervalo de especificación. Cuando u y v toman valores de 0 o 1 se obtienen los cuatro índices básicos: $C_p(0, 0) = C_p$, $C_p(1, 0) = C_{pk}$, $C_p(0, 1) = C_{pm}$ y $C_p(1, 1) = C_{pmk}$. Los índices $C_p(u, v)$ son apropiados para procesos con distribución normal y tolerancias simétricas. Véase Pearn et al. (2005).

Pearn & Chen (1997) consideran la generalización de $C_p(u, v)$, denominada $C_{Np}(u, v)$, la que puede ser aplicada a procesos con distribución arbitraria. $C_{Np}(u, v)$ es definido como,

$$C_{Np}(u, v) = \frac{d - u |M - m|}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + v(M - T)^2}}$$

donde M es la media del procesos, P_α es el (100α) percentil de la distribución y la fracción $\frac{P_{99.865} - P_{0.135}}{6}$ constituye la dispersión del proceso. Si el proceso sigue una distribución normal, entonces $C_{Np}(u, v)$ se reduce a $C_p(u, v)$. Ambos, $C_p(u, v)$ y $C_{Np}(u, v)$ son apropiados para procesos con tolerancia simétrica. Para trabajar con tolerancias asimétricas, Vännman (1997a) planteó el índice $C_{pa}(u, v)$, definido como:

$$C_{pa}(u, v) = \frac{d - |\mu - m| - u |\mu - T|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}$$

Pearn et al. (2005) proponen el índice $C''_{Np}(u, v)$, que se expresa como:

$$C''_{Np}(u, v) = \frac{d^* - uA^*}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + vA^2}}$$

donde $A = \max\left\{\frac{d(M-T)}{d_u}, \frac{d(T-M)}{d_l}\right\}$, $A^* = \max\left\{\frac{d^*(M-T)}{d_u}, \frac{d^*(T-M)}{d_l}\right\}$, $d_u = USL - T$, $d_l = T - LSL$, $d^* = \min(d_u, d_l)$. Este índice puede ser aplicado a procesos con distribuciones no normal y con tolerancia simétrica o asimétrica. Cuando u o v toman valores de 0 o 1, este índice puede ser denotados como $C''_{Np}(0, 0) = C''_{Np}$, $C''_{Np}(1, 0) = C''_{Npk}$, $C''_{Np}(0, 1) = C''_{Npm}$, y $C''_{Np}(1, 1) = C''_{Npmk}$. Si $T = m$, then $C''_{Np}(u, v)$ se reduce a $C_{Np}(u, v)$.

A3. Índice de capacidad de procesos para todo un producto

Puesto que un producto usualmente contiene varias características, Chen et al. (2001) definen un índice de capacidad integrado, C_T , para características independientes, cada una de las cuales sigue una distribución normal. Chen et al. (2006) modifican el índice C_T para permitir procesos con distribuciones diferentes a la normal. Ellos asumieron que el producto tiene t_n características con especificaciones bilaterales evaluadas por el índice N_{pn} , $t_u + t_l$ características con especificaciones unilaterales, donde t_u es el número de especificaciones tales que cuanto más pequeña sea es mejor la característica evaluadas por el índice N_{pu} , y t_l es el número de características tales que cuanto mayor sea el valor que tomen mejor es la característica. Estas son evaluadas por el índice N_{pl} . El nuevo índice es denotado por N_T y es definido como

$$N_T = \frac{1}{3} \Phi^{-1} \left\{ \frac{\left[\left(\prod_{i \in S} \prod_{j=1}^{n_i} [2\Phi(3N_{pij}) - 1] \right) + 1 \right]}{2} \right\}$$

donde Φ^{-1} es la función de distribución acumulada inversa de una distribución normal estándar, $S = \{u, l, n\}$ denota el tipo de proceso, cuanto más pequeño mejor, cuanto más grande mejor y procesos con especificaciones bilaterales, respectivamente, y

$$N_{pij} = \begin{cases} \frac{USL - M}{(P_{99,865} - P_{0,135})/2}, & i = u, j = 1, 2, \dots, t_u \\ \frac{M - LSL}{(P_{99,865} - P_{0,135})/2}, & i = l, j = 1, 2, \dots, t_l \\ \frac{\min(USL - M, M - LSL)}{3\sqrt{\left[\frac{(P_{99,865} - P_{0,135})}{6}\right]^2 + (M - T)^2}}, & i = n, j = 1, 2, \dots, t_n \end{cases}$$

N_T puede ser calculado para especificaciones unilaterales y bilaterales. Sin embargo, para especificaciones bilaterales, el proceso debe tener tolerancia simétrica.

Se propone un índice de capacidad integrado para un producto con características normales o no normales, especificaciones bilaterales y tolerancias simétricas o asimétricas. Este índice, denotado por C_{Np}^T , es definido como

$$C_{Np}^T = \frac{1}{3} \Phi^{-1} \left\{ \frac{\prod_{j=1}^n [2\Phi(3C_{Npmkj}'' - 1)] + 1}{2} \right\}$$

donde $C_{Npmk}'' = C_{Np}''(1, 1)$.

Si el índice de capacidad para todo el producto es mayor o igual a valor fijo, digamos a , entonces

$$C_{Np}^T = \frac{1}{3} \Phi^{-1} \left\{ \frac{\prod_{j=1}^n \left[2\Phi \left(3C_{Npmk_j}'' \right) - 1 \right] + 1}{2} \right\} \geq a$$

Específicamente, cuando los valores mínimos preestablecidos para las capacidades de proceso para cada característica individual son iguales, se tiene:

$$C_{Np}^T = \frac{1}{3} \Phi^{-1} \left\{ \frac{\left(\left[2\Phi \left(3C_{Npmk_j}'' \right) - 1 \right] \right)^n + 1}{2} \right\} \geq a$$

El valor crítico a_0 ($C_{Npmk_j}'' \geq a_0$) para la capacidad individual de procesos puede ser obtenido solucionando la desigualdad previa cuando la capacidad integrada del producto excede a ($C_{Np}^T \geq a$), donde

$$a_0 = \frac{1}{3} \Phi^{-1} \left[\frac{(2\Phi(3a) - 1)^{\frac{1}{n}} + 1}{2} \right]$$

A4. Ejemplo

Se consideran los datos presentados por Huang & Chen (2003) y comparamos sus resultados con los obtenidos en la propuesta. La aplicación consta de 10 características fundamentales de una microfibras de Sea island. La tabla 1 presenta las especificaciones de estas características. Para enfatizar el caso con tolerancia asimétrica, se ha modificado ligeramente el valor objetivo de la característica 8 de 3.0000 a 3.0010.

Primero, se establece un valor para la capacidad integrada del proceso, digamos $C_{Np}^T = 1,00$. Entonces, la mínima capacidad de proceso individual para cada característica es $a_0 = 1,214$, aplicando la ecuación (2). Usando las ecuaciones (3), (4) y (5), se calculan las coordenadas para las 10 características independientes (tabla 1). Los puntos (δ , γ , ε) son ubicados en el gráfico como se muestra en la figura 3.

Dado que hay dos valores diferentes de ε , 1 y 1.5, como se puede observar en la tabla 1, se construye un gráfico (δ , γ) para cada valor de ε , ver figuras 4 and 5.

A partir de las figuras 4 y 5 se observa que cinco características (2, 3, 7, 8 y 9) no están dentro de la línea de contorno; por consiguiente deben emprenderse acciones para mejorar la calidad del producto. Así, las medianas de las características 2, 3, 7, 8 y 9 deben desplazarse hacia el valor objetivo, y debe reducirse la dispersión de la característica 2.

En la columna 9 de la tabla 1 se muestran los valores del índice C_{pmk}'' usado por Huang & Chen (2003); en la columna 8 se muestran los valores del índice C_{Npmk}''

empleado en esta propuesta. Comparando las columnas 8 y 9 se observa que ambos índices son iguales cuando la tolerancia es simétrica, pero en presencia de tolerancia asimétrica el valor del índice C_{Npmk}^n es menor que el valor del índice C_{pmk}^n . Esta situación hace que la región de capacidad usada en nuestra propuesta para las características 8 y 9 sea más pequeña que la región de capacidad usada para las otras características (figuras 4 y 5). Además, se observa que la característica 8 no es capaz ($C_{Npmk}^n = 0,9592 < a_0$). En la metodología de Huang & Chen (2003), la región de capacidad es igual para todas las características; en particular la característica 8 es capaz ($C_{pmk}^n = 1,4318 > a_0$). Cuando la tolerancia es simétrica, la región de capacidad es igual para los dos métodos; por consiguiente, no hay diferencia en el análisis para las características 2, 3 y 7.