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A Multivariate Analysis Approach to Forecasts Combination. Application to Foreign Exchange (FX) Markets

Una aproximación a la combinación de pronósticos basada en técnicas de análisis multivariante

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Abstract

Forecasting is characterized by the availability of a lot of methods and the fact that technological and economic forecast horizons are increasingly more different from each other. Combining forecasts is an adequate methodology for handling the above scenario, which is conceptually suitable for the application of several methods of multivariate analysis. This paper reviews some main problems in combining forecasts efficiently from the multivariate analysis view. In particular, a methodology to produce combined forecasts with a large number of forecasts is proposed. The usefulness of such a methodology is assessed in exchange rates forecasting. Further research is suggested for finance as well as for other practical contexts such as energy markets.

***Key words*:** Combining forecasting, Factor analysis, Forecasting methodology, Principal components analysis, Time series.

Resumen

El cálculo de pronósticos se caracteriza por la disponibilidad de muchos métodos y porque los horizontes de los pronósticos o las predicciones (económicas, tecnológicas, etc.) son cada vez más diferentes. Combinar pronósticos es una metodología adecuada para manejar el escenario anterior, el cual es conceptualmente adecuado para la aplicación de varios métodos de análisis multivariante. Este artículo revisa algunos problemas principales al combinar pronósticos de manera eficiente, empleando el marco del análisis multivariante. En concreto, se propone una metodología para generar pronósticos combinados con un gran número de pronósticos y se analiza una aplicación al mercado de divisas. Se valora la utilidad de esta metodología en finanzas y varios contextos prácticos, abriéndose posibilidades futuras de investigación a otros contextos aplicados, como los mercados de energía.

***Palabras clave*:** análisis de componentes principales, análisis factorial, combinar pronósticos, metodología para pronósticos, series temporales.

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1. Introduction

The 21^{st} century is providing diversity and multiple options for every problem and situation in the real world. Obvious examples are the fast transference in know-how from the academic to the non-academic world, improved knowledge in the scientific community, easy and flexible software, or increased availability due to new technologies (see, as an example of the above facts, the film *The Social Network*).

Forecasting is now characterised by the availability of a large number of methods, an updated review of which is available from several sources: Armstrong (2001), Diebold (2001), Elliott, Granger & Timmerman (2006), Makridakis, Wheelwright & Hyndman (1998).

Furthermore, forecast horizons for companies and institutions are increasingly different and yet interrelated. Decision-makers now require related predictions for the immediate and short run, or for the medium or long run, or even for the three standard horizons: short, medium and long ones. For example, Sánchez-Ubeda & Berzosa (2007) provide an application to forecasting industrial end-use natural gas consumption in Spain, where the company Enagás requires forecasts in a medium-term horizon (1–3 years) with a very high resolution (days). Other works covering this problem are Kang (2003), Andrawis, Atiya & El-Shishiny (2011) and references cited there.

Combining forecasts is generally regarded as an appropriate methodology for handling the above scenario, having reached maturity and recognition over the last forty years (Bates & Granger 1969, Bunn 1989, Clemen 1989, de Menezes, Bunn & Taylor 2000, Armstrong 2001, Timmerman 2006). In economics, a recent application is devoted to forecasting US employment growth (Rapach & Strauss 2008). On the other hand, Kapetanios, Labhard & Price (2008) report about that forecasts combination generally leads to a reduction in forecast error.

Such a setting is conceptually suitable for the application of *multivariate analysis methods* (MAM in the following). In the last fifteen years, these techniques have benefited from big improvements with regard to easiness of use, the overcoming of limitations and the recognition of enlarged usefulness (Hair, Anderson, Tatham & Black 1998). Hence, an adaptation of MAM to new technologies such as databases, Internet and others is an emerging area.

Given this scenario, this article proposes the employment of several methods of classical multivariate analysis to obtain combined forecasts. A recent paper (Poncela, Rodríguez, Sánchez-Mangas & Senra 2011) proposes the use of dimension reduction techniques in forecasts combination. However, our approach is different because it considers several forecast horizons, analyzes the possibilities in forecasts combination of the key set of multivariate methods and suggests different fields of applications such as financial markets and energy markets.

The paper is organized as follows. Section 2 presents the foreign exchange market and how forecasting is key in this market. Section 3 is devoted to the analysis of one forecast combination framework and highlights the multivariate analysis methods approach. Section 4 reviews one of the problems in combining forecasts efficiently and proposes a solution to this problem using principal component analysis. In addition, one application for EURUSD forecasting is presented. Section 5 outlines a new methodology for producing combined forecasts. In Section 6, the usefulness of such a methodology is considered when its practical implications in several contexts are analyzed. Finally, some conclusions from this research are provided.

2. Forecasting in the Foreign Exchange Market

According to Wikipedia, the **foreign exchange market** (**forex**, **FX**, or **currency market**) is a worldwide decentralized over-the-counter financial market for the trading of currencies. Financial centers around the world function as anchors of trading between a wide range of different types of around-the-clock buyers and sellers, with the exception of weekends. The foreign exchange market determines the relative values of different currencies. The primary purpose of foreign exchange is to assist international trade and investment, by allowing businesses to convert one currency to another currency. For example, it permits a US business to import British goods and pay Pound Sterling, even though its income is in US dollars. It also enables investors to invest in foreign exchange, and facilitates the carry trade, in which investors borrow low-yielding currencies and lend (invest in) high-yielding currencies, and which (it has been claimed) may lead to loss of competitiveness in some countries.

The foreign exchange market is highly competitive. In the second semester of 2010 media around the world devoted a lot of news to a possible new currency war. But this is not a new problem. For example, Posen (2004) analyzed the issue of avoiding a currency war.

With respect to the size and importance of the foreign exchange market, Eun & Sabherwal (2002) stated the following: "The market for foreign exchange is the largest financial market in the world. According to the Bank for International Settlements (2001) the average worldwide daily trading in trading foreign exchange markets is estimated to be US$1.2 trillion". More recently, King & Rime (2010) have analyzed the so called "$4 trillion question" giving explanations to the FX growth since 2007.

Subsequent to Eun & Sabherwal (2002), and in response to the increasing importance of this market, there have been a number of studies on forecasting exchange rates. The substance of many of these studies is the evaluation of the forecasting performance of one or more exchange rate models. These evaluations suggest that the best predictor of future spot rate is the current spot rate, i.e., the random-walk or "no-change" model, the most commonly used benchmark. This conclusion has also been obtained by Kilian & Taylor (2003).

There are a lot of websites producing forecasts for different time horizons. Some of them include forecasts for 3 (3M), 6 month (6M) and 12 (12M) month periods. For example, `http://www-2.danskebank.com/danskemarketsresearch`

Some of these sites include forecasts for a week, a month and a quarter. For example, the following website includes these forecasts by 16 experts.

`http://www.fxstreet.com/technical/forex-forecasts/experts-forecast-currencies-poll/2010/12/17/03/`

Table 1 shows an example of the information given in the above website.

TABLE 1: Elaborated from the information given in `www.fxstreet.com`

EUR/USD	**1 week**	**1 month**	**1 quarter**	**1 week**	**1 month**	**1 quarter**
		Levels			Trend	
Expert 1	1.36	1.38	1.32	Bullish	Bullish	Bearish
Expert 2	1.315	1.291	1.274	Bullish	Bullish	Bearish
...	...	...	...	Sideways	Bullish	Sideways
...	...	...	...	...	...	...
Expert 16	1.32	1.30	1.26	Very Bullish	Bearish	Bearish

King & Rime (2010) explain why electronic trading, based on electronic execution methods, is transforming FX markets. In particular, high-frequency trading (HFT) is one algorithmic trading strategy that profits from incremental price movements with frequent, small trades executed in milliseconds for investment horizons of typically less than one day. They state that HFT emerged with the advent of the 21^{st} century, becoming an important source of FX growth from 2004.

As we can see generating accurate forecasts for the next 24 hours, or the next 60 minutes or the next 60 seconds, is a real challenge in FX markets.

3. A Multivariate Methods Approach for Combining Forecasts

In what follows, it will be assumed that we are interested in forecasting a magnitude, denoted by Y, for a large number of forecast horizons, denoted by n, from the information about it in different periods of time $1, 2, \dots$ until the present moment (denoted by t). With this objective in mind, the forecaster has several forecasts obtained using a number of forecasting methods, denoted by p. In the case of the application provided in Sánchez-Ubeda & Berzosa (2007), currently used by Enagás, Y is the daily demand for industrial end-use natural gas consumption and n is $365 \times 3 = 1095$.

TABLE 2: Structure of the information in the problem.

	$M1$	$M2$	...	...	Mp
$H1$	$f_{11}(t)$	$f_{12}(t)$	...	...	$f_{1p}(t)$
$H2$	$f_{21}(t)$	$f_{22}(t)$	...	...	$f_{2p}(t)$
...	...	...	...	...	...
...	...	...	...	...	...
Hn	$f_{n1}(t)$	$f_{n2}(t)$	...	...	$f_{np}(t)$

Hence, we will assume that we have p unbiased forecasts of the magnitude Y obtained by p forecasting methods (or experts or a combination of methods and experts) $(M1, M2, \ldots, Mp)$ for n forecast horizons $(H1, H2, \ldots, Hn)$. The notation $f_{ij}(t)$ stands for the forecast obtained in the ith forecast horizon by the jth method or expert with $i = 1, \ldots, n$ and $j = 1, \ldots, p$. All this information can be shown in a matrix structure (Table 2), denoted by $\mathbf{F}(t)$. Table 1 is a particular case of Table 2 transposing rows and columns.

In the above setting, the application of classical multivariate analysis methods (see, for example, Johnson & Wichern 2002, Rencher 2002, and Hair et al. 1998), where the variables are the forecast methods and the units are the forecast horizons, is appropriate.

From this information, MAM have been developed for the study of different problems in data analysis. In our case, the forecasts are obtained using several methods in different forecast horizons. One of the problems studied is the reduction of the dimensions of the problem (forecasting methods), the goal of this reduction being to facilitate the assimilation process of the relationships discovered by the analyst.

Another problem is the process of searching for dependence relationships which would facilitate the understanding of the behavior of some forecasting methods by taking into account the values that other methods show. Finally, an interesting problem is the analysis of possible interdependence relationships, which will permit the detection of subtle relationships between the forecast horizons.

An initial classification of MAM applied to the structure of Tables 1 and 2, which can be seen as a particular case of the general structure for MAM, is summarized in Table 3, which collects the proposed use of MAM in forecasting. It concerns the forecasting of horizons, forecast methods, and the tasks of analyzing dependence and interdependence between methods, or the problem of reducing the number of methods, or that of involving the grouping or classifying of forecast horizons.

In the following, the potential of MAM in the five problems in Table 3 will be briefly considered.

Problem 1. In our forecasting scenario the clearest task would be the reduction of the p unbiased forecasts to a smaller number of forecasts (ideally one or two) for every forecast horizon considered. With this goal in mind, methods such as Principal Component Analysis (PCA) or Factor Analysis (FA), linear or non-linear, or even Cluster Analysis applied to forecast methods, could deserve our attention. In a general context, the clustering approach in prediction has a long tradition (see, for example, Kusiak & Li 2010 for a recent application in wind power forecasting).

Problem 2. Granger & Ramanathan (1984) suggest that conventional forecast combining methods could be considered within the regression framework. Hence, the alternative methods for handling this problem in Table 2 must be explored. For example, MANOVA could be used to analyze if there are significant differences between a group of forecasts obtained using the same methodology (ARIMA, Neural Nets, Expert Opinions, ...) in different groups of time horizons such as short, medium or long ones. Depending on the conclusion reached, the decision of the forecaster as to the best way to combine would be different.

CCA could be used to determine the kind of relationship (linear or non-linear) between groups of forecasts to decide on the possibility of reducing the number of forecasts to be considered in the problem.

Problem 3. Depending on the different forecasting methods used, the usefulness of knowing the profile of the forecast horizons with respect to accuracy is beyond doubt. As many authors have said, there are no methods or methodologies that outperform the rest for every problem or every time horizon. It seems that DA and LR can help in future research into these questions. In particular, approaches in FOREX pose new challenges in International Finance.

Problem 4. Identifying groups of similar forecast horizons with respect to a series of forecasting methods or methodologies would make the creation of different types of combination for every cluster of forecast horizons possible.

Problem 5. This is one of the most complex problems in MAM and the same occurs in Table 3. Here, Partial Least Squares could help in solving the problem, commented on in the above paragraph, of deciding which methods should be included in clustering forecast horizons.

4. A Solution from Multivariate Analysis Methods to the Problem of Efficient Combined Forecasts

According to several authors Bunn (1988), Clemen & Winkler (1986), Winkler & Makridakis (1983), in addition to the notation introduced in Section 2, if we have p unbiased forecasts of the same variable Y for the same time horizon (let this be i), denoted by $f_{i1}(t), \dots, f_{ip}(t)$, then the composite forecast, denoted by $c_i(t)$, based upon the $p \times 1$ vector of the linear weights, $\mathbf{w}_i(t)$, will adopt the expression

$$c_i(t) = \mathbf{w}_i'(t)\,\mathbf{f}_i(t) \tag{1}$$

and will be optimum in the sense of having a minimum forecast error variance if $\mathbf{w}_i(t)$ is determined according to

$$\mathbf{w}_i(t) = \frac{\mathbf{S}_i^{-\mathbf{1}}(t)\,\mathbf{e}_i(t)}{\mathbf{e}_i'(t)\,\mathbf{S}_i^{-\mathbf{1}}(t)\,\mathbf{e}_i(t)} \tag{2}$$

where $\mathbf{e}_i(t) = (1, 1, \dots, 1)'$ is an $\mathbf{p} \times 1$ unit vector and $\mathbf{S}_i(t)$ is an $\mathbf{p} \times \mathbf{p}$ covariance matrix of forecast errors between the $\mathbf{p}$ forecasts for the ith forecast horizon.

According to a previous study (Clemen & Winkler 1986), if a normal or Gaussian is assumed for the forecasting errors, then the combined forecast $c_i(t)$ is a weighted average of the individual forecasts $f_{i1}(t), \dots, f_{ip}(t)$ with the vector of weights

$$\mathbf{w}_i'(t) = \frac{\mathbf{e}_i'(t)\left(\widehat{\sum}_i(t)\right)^{-\mathbf{1}}}{\mathbf{e}_i'(t)\left(\widehat{\sum}_i(t)\right)^{-\mathbf{1}}\mathbf{e}_i(t)} \tag{3}$$

Table 3: Brief summary of the proposed use of MAM in forecasting.

Problem	MAM	Comments
1. To reduce the original information to a smaller set of new forecasting methods (factors)	- Factor Analysis (FA) - Principal Component Analysis (PCA)	Factors usually are determined by principal component analysis or by a common factor analysis.
2. To determine the existence of dependence relationships between two groups of forecasting methods	- Regression - Analysis of variance (ANOVA) - Canonical Correlation Analysis (CCA) - Conjoint Analysis (CoA)	The differences between methods are due to the number and kind of forecasting methods that are included in every group of forecasting methods.
3. To assign forecast horizons to groups characterized by a series of forecasting methods	- Discriminant Analysis (DA) - Logistic Regression (LR)	The first method is more general than the second one, however the latter raises more flexible hypotheses.
4. To make or to identify groups of forecast horizons similar between them	- Cluster Analysis (ClA) - Multidimensional Scaling (MS) - Correspondence Analysis (CA) - CHAID Analysis	The first method provides groups of forecast horizons with maximum internal homogeneity and maximum heterogeneity between them. The second one facilitates the discovery of the number of dimensions where the forecasts horizons (units) are similar. While, the third one allows accommodating different type of data and non-linear relationships. The last one proposes groups according to a dependent variable or forecasting method, making easy the classification of new forecast horizons in the corresponding groups.
5. Searching for crossed relationships of multiple dependence and the representation of non-observed concepts in such relationships	- Structural Equation Modelling (SEM) - Partial Least Squares (PLS)	SEM makes it possible to analyse several relationships at the same time. It is an extension of several MAM like FA or Regression. PLS facilitates the discovery of the number of dimensions where the forecast horizons are similar.

depending on $\widehat{\sum}_i(t)$. Thus, the estimate $\widehat{\sum}_i(t)$ which can be determined from past data on estimation errors, prior information or some combination thereof plays an important role in the determination of the combined forecast.

4.1. Combined Forecasts with Principal Component Analysis

Principal Component Analysis (PCA), initially introduced by Pearson in 1901 and independently by Hotelling in 1933, is one of the oldest MAM. This technique, applied to our problem, will involve describing or explaining the structure of variation in a multivariate data set (with evidence of interdependence between the variables, see Table 2 in Section 3) in terms of a set of uncorrelated variables where every one is a specific linear combination of the original forecasting methods. That is, a linear combination of forecasts can be obtained as an alternative to usual ways of proceeding.

In the following, we denote by $\mathbf{f}(t)$ the vector of the p forecast methods with the available information until t, in such a way that if the jth method or column of Figure 1 is designed by $f_j(t)$, it follows that

$$\mathbf{f}(t) = (f_1(t), f_2(t), \ldots, f_p(t))$$

Furthermore, the population covariance matrix associated with the $\mathbf{F}(t)$ matrix in Table 2 will be denoted by $\mathbf{S}(t)$.

In this situation, the first component extracted will be a linear combination of the p forecast methods, designed by $C1(t)$, such that

$$C1(t) = \mathbf{a}_1' \cdot \mathbf{f}(t) \tag{4}$$

whose sample variance is the biggest for all vector of coefficients $\mathbf{a}_1$. Consequently, the sample variance can be increased without limit and a restriction to these coefficients must be considered. That the sum of squares will be one, i.e.,

$$\mathbf{a}_1' \cdot \mathbf{a}_1 = 1 \tag{5}$$

Similarly, the second component denoted by $C2(t)$ will be

$$C2(t) = \mathbf{a}_2' \cdot \mathbf{f}(t)$$

with maximum variance restricted to the two conditions $\mathbf{a}_2' \cdot \mathbf{a}_2 = 1$ and $\mathbf{a}_2 \cdot \mathbf{a}_1 = 0$.

Extending the above reasoning to the ith component, it will be verified that it is a linear combination of the p original variables, denoted by $Cj(t)$, such that

$$Cj(t) = \mathbf{a}_j' \cdot \mathbf{f}(t) \tag{6}$$

with maximum variance restricted to the following conditions

$$\mathbf{a}_j' \cdot \mathbf{a}_j = 1 \quad \text{and} \quad \mathbf{a}_j' \cdot \mathbf{a}_i = 0 \quad (i < j) \tag{7}$$

Thus, the problem of finding the principal components (PC) in a matrix like that of Table 2 consists in obtaining the vectors of coefficients $\mathbf{a}_j$, for $j = 1, 2, \ldots$; with a limit in p. We can show (Johnson & Wichern 2002) that this problem is equivalent to that of obtaining the eigenvectors of the correlation matrix between the forecast methods. The associated eigenvalues will be $\lambda_1 > \cdots > \lambda_p$ and the total variance of the p PC will be equal to the variance of the original forecasts. Moreover, the trace (sum of the diagonal elements) of the covariance matrix, $tr(\mathbf{S}(t))$, will verify

$$\sum_{i=1}^{p} \lambda_i = tr(\mathbf{S}(t)) \tag{8}$$

Hence, the jth PC accounts for a proportion of variance

$$\frac{\lambda_i}{tr(\mathbf{S}(t))} \tag{9}$$

of the total variance in the original forecast methods.

The usefulness of PCA lies in that the components, the different combination of the forecast methods in our setting, employ the variance in a decreasing proportion. Thus, the important issue for obtaining an appropriate reduction of the forecasts consists of answering the following question: How many components should be extracted?

There are a series of formal techniques (see, for example, Basilevsky 2004), but we will restrict ourselves to informal rules used in practical contexts Gorsuch (1983). They are the following.

1. The inclusion of a sufficient number of components to explain some large relative percentage of the total variance. The quotes between 70 and 90 percent are usually suggested. However, lower values arise when p or n increase.

2. The exclusion of those components whose eigenvalues are less than the average.

3. Plotting the eigenvalues λ_i(in decreasing order) versus the index i, called *scree plot*, to select the number of components needed to extract the value of i corresponding to an elbow in the curve. That is, the point where big eigenvalues soften and small eigenvalues begin.

The above development is based on terms of eigenvectors and eigenvalues of the covariance matrix $\mathbf{S}(t)$. However, these components are usually extracted from the correlation matrix (that associated to $\mathbf{F}(t)$ in our setting), which will be denoted by $\mathbf{R}(t)$.

The reasons for the last statement are not difficult to understand if we imagine a multivariate data set where the variables represent very different approaches to the problem. In such a case, the structure of the PC obtained from the covariance matrix will depend on the choice, essentially arbitrary, of the units of measurement. Moreover, if there are great differences between the variances of the forecasting methods, those whose variances are biggest will tend to dominate the first CP.

Extracting PC by the eigenvalues of $\mathbf{R}(t)$ will exclude the above difficulty. This procedure is equivalent to calculating the PC of the original methods after standardizing for having unit variance.

However, in the rest of the situations, the PC must be extracted from the covariance matrix, because this is closer to the spirit and substance of the PCA, especially if the components will be employed in further computations, which is very common when forecasting with several forecasts.

With respect to the PC extracted from $\mathbf{R}(t)$ and $\mathbf{S}(t)$, the following can be stated.

1. Eigenvectors, eigenvalues and coefficients from the PC extracted from $\mathbf{S}(t)$ will differ from the PC extracted from $\mathbf{R}(t)$.

2. The percentage of variance that components of $\mathbf{R}(t)$ account for will differ from the percentage that account for the extracted components from $\mathbf{S}(t)$.

3. The extracted components from a matrix $\mathbf{R}(t)$ are not unique or exclusive to that matrix.

Finally, several measuring to indicate the adequacy of the data matrix $\mathbf{F}(t)$ to PCA have been developed. One of the most simple is the Kaiser-Meyer-Olkin or KMO test. According to Hair et al. (1998), this test assesses the adequacy of using PCA on data. Values greater than 0.7 suggest that it is appropriate to apply PCA to the considered matrix. Moreover, measures to elucidate the question of whether a forecasting method must be included in the problem of the reduction of the information are available. The most commonly used is MSA (measure of sample adequacy) and recommendations about this amount have been given Rencher (2002).

4.2. One Basic Application to FX markets

Several cases have been run to evaluate the above approach. To show a particular case, the daily EURUSD close rate from January 1999 to December 2010 has been considered. This series will be denoted by $Y(t)$. To analyze this series, the series of differences should be obtained. That is, $\Delta Y(t) = Y(t) - Y(t-1)$.

Plots of these series are shown in Figures 1 and 2.

A simple and visual analysis of Figures 1 and 2 allows us to conclude that the EURUSD series has the following features:

1. Soft linear trend, with several long periods of decreasing trend and other periods of increasing trend. In consequence, several turning points can be detected.

2. Due to sharp decreasing or increasing, presence or absence of structural breaks should be considered.

3. The differentiated series is not a white noise. Therefore, the EURUSD series is not a random walk.

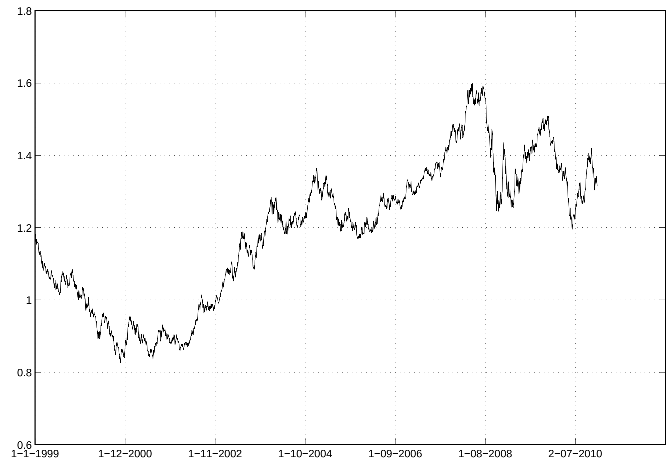

Figure 1: Plot of daily EURUSD.

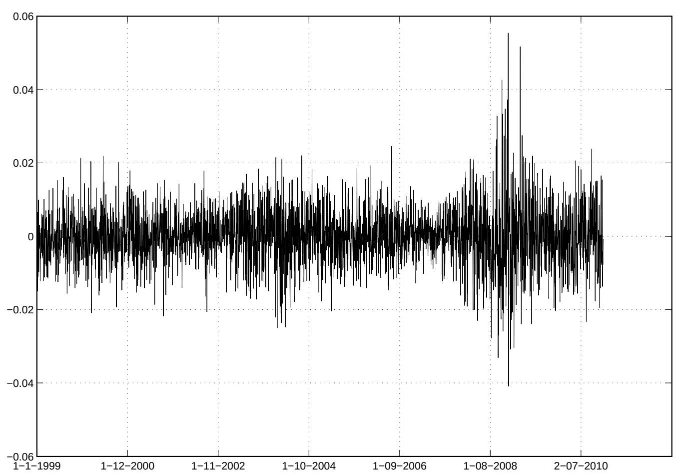

Figure 2: Plot of the series EURUSD differenced.

These facts lead to conclude that the EURUSD series is predictable, but not for all years (e.g., the first half of 2007 in the differenced series is close to a white noise). In addition, several methods should be considered and information from the past which is not longer valid could be removed.

Figure 3 shows the boxplots of both datasets, $Y(t)$ and $\Delta Y(t)$.

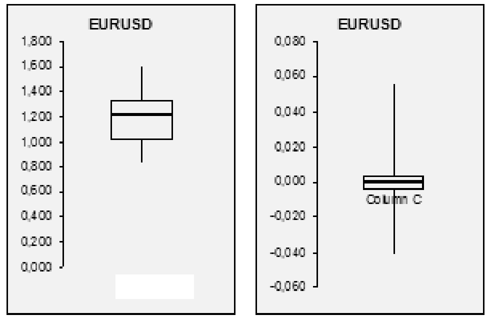

Figure 3: Boxplots of series 1 and series 1 differentiated.

In the reminder of this article, the results will be shown using the STATGRAPHICS CENTURION software. There are, however, a variety of software packages and we do not wish to give the impression that we endorse one over any other.

The forecasting methods employed are very basic, but usual in FOREX, to give more relevance to the application proposed itself instead of using sophisticated or complicated methods. They came from the exponential smoothing approach, polynomial trend and the ARIMA methodology, using the Forecast method procedure in STATGRAPHICS in an automatic or a manual way. Forecasts were obtained using six methods, see Table 4 for details, and a matrix like that in Table 2 was available. ARIMA models are usually considered in FOREX markets (see, e.g., Dunis & Williams 2005).

Table 4: Accuracy measures with the six methods employed.

Model	#	RMSE	MAE	MAPE	AIC
Random walk	0	0.00767929	0.0056745	0.479297	−9.73846
ARIMA (3,0,2) with const.	1	0.00765786	0.0056721	0.478967	−9.74405
ARIMA (1,0,0)	2	0.00767834	0.00567289	0.47914	−9.7387
Holt, alpha = 0.9966 and beta = 0.0075	3	0.00769148	**0.00566384**	**0.478415**	−9.734
ARIMA (6,0.5) with const.	4	**0.00765561**	0.00567007	0.478764	**−9.74463**
ARIMA (4,1,6)	5	0.007668	0.00567807	0.479561	−9.7414
Quadratic trend $-6.29 + 0.00058\ t$ $-1.0399\text{E-}8\ t^2$	6	0.104693	0.0849887	7.52837	−4.51345

As a result of extracting the first PC of the five first methods considered, and using the covariance matrix associated to the forecasting matrix, Figure 4 is obtained. Using only a PC, we will retain practically all the information in the five forecasting methods considered.

The main conclusion obtained from Table 5 is that combining by means of the proposed approach in this paper is the same as combining by means of the simple average. That is, the following important property can be stated: combining with the simple average theoretically will produce the maximum variance between linear combinations of the considered forecasting methods. Timmerman (2006) concludes that the equal weights average is a benchmark that has proved surprisingly difficult to beat. An interesting research issue is to connect both results from a theoretical point of view.

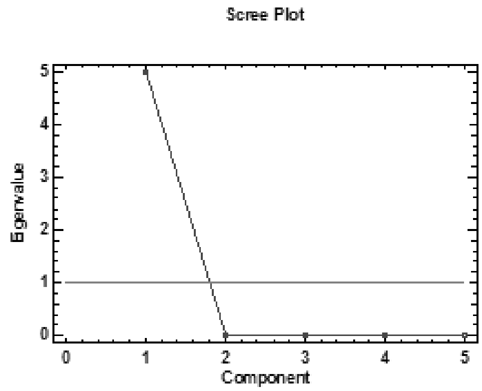

Figure 4: Scree plot for 5 forecasting methods with the EURUSD series.

Table 5: Matrix of weights for the combination of the five forecasting methods with the EURUSD series.

	Component
	1
FORECASTS_M1	0.447214
FORECASTS_M2	0.447214
FORECASTS_M3	0.447213
FORECASTS_M4	0.447214
FORECASTS_M5	0.447214

Another use of PCA is in the detection of forecasting methods with null or low influence on the structure of variability in the forecasting matrix, where PCA will permit us to discard those methods in the combination. For example, Table 6 shows that situation with the sixth method used to obtain the PC.

5. A New Methodology to Produce Combined Forecasts for Several Forecast Horizons

As has been mentioned in the introduction to this paper, forecasters of the 21^{st} century will have at their disposal a large number of forecasts that they may want to combine. See, for example, the 'suite of statistical forecasting models' (the 'Suite') of The Bank of England in Kapetanios et al. (2008).

Table 6: Matrix of weights for the combination of the six forecasting methods with the EURUSD series.

	Component
	1
FORECASTS_M1	0.415608
FORECASTS_M2	0.415609
FORECASTS_M3	0.4156
FORECASTS_M4	0.415608
FORECASTS_M5	0.415608
FORECASTS_M6	0.369264

In 1989, Granger reviewed some alternatives to original methods of forecasting, stating that the latter were too complicated for using with a large number of forecasts available for combination.

In this section, a methodology for the production of combined forecasts is proposed, for a context in which a set of forecasting methods are available for several forecast horizons. In the sequel, it will be assumed that we are interested in forecasting a magnitude Y for a large number of forecast horizons, denoted by n, using the information available up to the present moment (designed by t). With this objective in mind, the forecaster has obtained several forecasts using a number of forecasting methods, denoted by p. All this information is shown in the matrix structure of Table 2, introduced in Section 3.

In some MAM it is customary to consider that $p << n$, but situations where $p >> n$ are frequent and important analyses can be conducted. In forecasting, every situation for all possible values of p and n must be considered.

Case $\boldsymbol{p \geq n}$ (there are at least as many forecasting methods as forecast horizons)

In this situation, experience in the use of MAM indicates that it would be appropriate to use procedures for the reduction of the dimension of the problem as PCA or FA do. One possibility would be to cluster the forecasting methods and then to explore a way of constructing combined forecasts from the centroids of the clusters.

Case $\boldsymbol{p < n}$ (there are fewer forecasting methods than forecast horizons)

Here, we will assume that the ratio is less than five to one (no more than five forecast horizons for every forecasting method). Hence, the use of techniques for reducing the dimension of the problem as PCA or FA would not be appropriate. However, these techniques could be used in the way that is analysed in the following case.

Case $\boldsymbol{p << n}$ (there are far more forecast horizons than forecasting methods)

In this situation, the practice of MAM suggests the use of techniques (such as PCA or FA) for reducing the dimension of the problem. The literature on these methods is much too vast to adequately cite here; but there are excellent general developments (Johnson & Wichern 2002, Rencher 2002, Hair et al. 1998) and a thorough coverage of these techniques (Gorsuch 1983, Basilevsky 2004).

In our context, factor analysis would be concerned with the identification of an underlying structure within a set of forecasting methods. Hence, we will try to simplify the complicated interrelations among a large number of forecasting methods, in a search of certain unobservable common factors which are not related to each other. In consequence, the goal of this technique is the reduction of forecasting methods and the ascertaining of the relevant characteristics of the original set of methods by the construction of a smaller number of new common factors or latent variables with a minimum loss of information.

The adaptation of the general model of factor analysis (Johnson & Wichern 2002, Jobson 1992, Basilevsky 2004) to the forecasting context developed in Section 3 will involve handling three sets of variables: a set of p forecasting methods $FM_1, \dots, FM_p$ with mean vector μ and covariance matrix Σ; a set of m unobserved variables or combined forecasts called common factors $C_1, \dots, C_m$ where m $\leq$ p; and a set of p unique but unobserved factors $e_1, \dots, e_p$. The model will be

$$FM_1 - \mu_1 = l_{11}C_1 + l_{12}C_2 + \cdots + l_{1m}C_m + e_1$$

$$FM_2 - \mu_2 = l_{21}C_1 + l_{22}C_2 + \cdots + l_{2m}C_m + e_2$$

$$\circ$$

$$\circ$$

$$FM_p - \mu_p = l_{p1}C_1 + l_{p2}C_2 + \cdots + l_{pm}C_m + e_p$$

or, in matrix notation,

$$\underset{(p\times 1)}{\mathbf{FM}} - \mu = \underset{(\mathbf{p}\times\mathbf{m})}{\mathbf{L}}\underset{(m\times 1)}{C} + \underset{(\mathbf{p}\times\mathbf{1})}{\mathbf{e}}$$

The coefficient l_{ij} accompanying C_j in the linear combination describing $FM_i - \mu_i$ is called the loading of the ith forecasting method on the jth common factor, so the matrix $\mathbf{L}$ is the matrix of factor loadings or the factor pattern matrix.

In this model,

$$E(\mathbf{C}) = \mathbf{0},\ Cov(\mathbf{C}) = \mathbf{I}$$

$$E(\mathbf{e}) = \mathbf{0},\ Cov(\mathbf{e}) = \Psi$$

$$E(\mathbf{e} \cdot C) = \mathbf{0}$$

Then, $\Sigma = \mathbf{LL'} + \Psi$. Hence,

$$Var(FM_i) = \sum_{j=1}^{m} l_{ij}^2 + \Psi_i; \quad i = 1, \dots, p$$

$$Cov(FM_i, FM_k) = \sum_{j=1}^{m} l_{ij} l_{kj}$$

Hence, the variance of the ith forecasting method can be decomposed into two parts: the first part $\sum_{j=1}^{m} l_{ij}^2$ denotes the portion of $Var\left(FM_i\right)$ related to the common factors and is called the ith communality; and the second term Ψ_i is the unique variance, which is not related to the common factors.

The FA Methodology applied to the Forecasting Data Matrix

From the observations on the above p forecasting methods obtained for n forecast horizons, which are denoted by $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$; and with the application of the linear model discussed previously, the FA method is designed to find a small number of combined forecasts (factors) that represent the data adequately. Noting the sample covariance matrix by $\mathbf{S}$, we have an estimation of the unknown population covariance matrix Σ. If the off-diagonal elements of $\mathbf{S}$ are small or those of the sample correlation matrix $\mathbf{R}$ essentially zero, the forecasting methods are not related and factor analysis will not be appropriate. In another case, a factor model can be established and the first problem is to estimate the matrices $\mathbf{L}$ and Ψ.

Methods giving solutions to this problem are principal component analysis (PCA), principal factors (PF), maximum likelihood factoring, alpha factoring, unweighted least square factoring, image factoring and so on. The most commonly used are PCA and PF. PCA is commonly employed as a preliminary extraction technique, followed by one or more of the other procedures, perhaps varying the number of factors, the communality estimates, and the rotational methods in each run Tabachnik & Fidell (1996). The basic difference between PCA and PF is that the former focuses on variance and the latter on covariance (communality).

The goal of PCA in the proposed forecasting context would be to extract maximum variance from the forecasts matrix $\mathbf{F}(t)$ with a few orthogonal combined forecasts, while the aim of PF would be to reproduce the correlation matrix $\mathbf{R}(t)$ between forecasting methods with a few orthogonal factors that will act as combined forecasts. Thus, the proposed methodology consists of the following steps.

1. The reduction of the dimensions of the forecasting matrix $\mathbf{F}(t)$ data to a few factors or PC (very probably dependents).

2. The repetition of '1' in order to extract one or two factors, called in the context of FA *higher-order factors*, and the taking of these factors of the combined forecasts as the final combination of forecasts. That is, new combinations are created based on the different combined forecasts.

There is a practical case of extracting higher-order factors, in a different setting to that of this paper (Maté & Calderón 2000).

6. Some Practical Contexts and Application Issues

In this section, three practical forecasting contexts will be considered. The usefulness of the proposed methodology for these contexts and for some patterns of time series will be analyzed.

Energy Markets

Forecasting is a very important task for people involved in the management of energy markets, even when there is a stable economic setting. Energy producers urgently need automated and accurate forecasts so that they can anticipate demand, prices and other market variables.

All over the world, the entire sector is currently undergoing an important restructuring process towards deregulation and competition. The emergence and development of this new context implies that the management of energy producing companies can no longer be based on administrative and centralized procedures: their profit now depends directly on their own managerial decisions. Decision-making processes become more complex and risky due to the higher uncertainty which results from the new operational framework.

It has been pointed out in previous research that a careful calculation of expected demand and supply can lead to contracts that enhance the profitability of companies. Hence, more research for very short run forecasts will be required (Ramanathan, Engle, Granger, Vahid-Araghi & Brace 1997). In this article, the development of a number of models to produce very short run forecasts of hourly system loads was considered. The required forecast horizons vary from 16 to 40 hours from Monday to Thursday, and from 16 to 88 hours on Friday. However, the availability of forecasts for all week, 168 hours, is normal in energy companies. For example, Hahn, Meyer-Nieberg & Pickl (2009), is a recent survey of mathematical methods and ideas which have been used for electricity load forecasting.

The methodology proposed in our paper can be applied to price forecasts in energy companies. According to Zhao, Dong, Xu & Wong (2008), electricity price forecasting is a difficult yet essential task for market participants in a deregulated electricity market. Pindyck (1999) analyzes the influence of price forecasts on investment decisions and the choice of products to produce.

Stock Markets

Forecasting is a crucial task for brokers and people involved in the management of stocks. The planning of strategies might be improved with the availability of forecasts for returns, prices and volatility, in those periods where the day is divided to provide information about the evolution of a stock or groups of stocks. For example, if a quarter of an hour is taken as a unit of time division, forecasts for 32 horizons are required in an 8-hour trading day. Obviously, other divisions of time, like 5 minutes, could be used.

If, as has been suggested (Makridakis & Winkler 1983), forecasts for the 32 time horizons obtained by using four or five methods are available for the broker, the use of the proposed methodology might provide more accurate forecasts.

This approach opens a new alternative in this context because the standard practice in stock markets is to generate one-step-ahead forecasts. For example, with respect to the daily stock market data, the use of Artificial Neural Networks to combine time series forecasts of market volatility from USA, Canada, Japan and UK has been discussed (Donaldson & Kamstra 1996).

Sales

Issues concerning the design of forecasting methodologies have been looked at by other researchers (Winklhofer, Diamantopoulos & Witt 1996). One of them was the time horizon. Some conclusions were that the majority of firms prepared sales forecasts on a yearly basis, that the most popular short-term sales forecasting horizon was one month and that firms operating in highly competitive markets put more emphasis on short term than on long-term forecasts.

The above suggests that the proposed approach in this paper will not be useful for many companies that employ sales forecasts. However, the increasingly competitive context in which some firms develop their activity, in a situation with some parallelism with that of the energy or stock markets, can increase the need to consider the desirability of using more forecasting methods and a large number of forecast horizons. If adopted, these changes in approach could be handled with the methodology proposed in this paper.

Seasonal Series

Series with a seasonal component reproduce the behaviour of the magnitude every certain period of time. So we can assume that the forecast horizons act as units of study similar to the problems where multivariate analysis is usually applied.

Trending Series

Series with a trend component do not behave with the same pattern through several forecast horizons. So the forecast horizons perhaps do not show the same structure as the units in the usual multivariate analysis problems.

Cyclical Series

Series with a cyclical component must come back to a previous structure in the behaviour of the magnitude at regulars periods of time. So, as with seasonal series, we can assume that forecast horizons act as units of study similar to the problems where MAM are usually applied.

Automation

One of the key points for a successful implementation of the proposed methodology is its automation. For this reason, the issues of versatility, accuracy, speed and intelligibility must be strongly considered, for as long as a limited amount of resources (software, knowledge, data and so on) are available.

Forecasting Methods to Be Included

Several classic statistical forecasting models (exponential smoothing, ARIMA, GARCH, and so on) are widely used as part of forecasting systems, giving satisfactory results when applied in different fields. They provide many possibilities of modeling for one or several variables since they allow different behaviors and horizons be taken into account. Likewise, they have very intuitive use features and are very user-friendly, because the parameters they involve represent the underlying process in a simple way.

In addition, many of the variables included in the practical applications behave well in many cases or moments of their history, justifying the view that linear or classic forecasting models may be seen suitable for such contexts. However, the possibility of using artificial neural networks and other non-linear methods should be considered.

There are several references for a further study of general forecasting methods (Armstrong 2001, Diebold 2001, Elliott et al. 2006, Makridakis et al. 1998).

Integration with Rule-Based Forecasting and Expert Systems

A forecasting technique which integrates judgement and statistical extrapolation is the so-called rule-based forecasting technique. Rule-based forecasting automates some tasks associated with maintaining a complex body of knowledge, given in an accessible and modifiable form, to provide more accurate forecasts. There are several examples of rule-based forecast systems (Collopy & Armstrong 1992, Armstrong, Adya & Collopy 2001), and we think the proposed approach could provide a new way of looking at them.

The potential and benefits of using expert systems in forecasting tasks, both to formulate the model and to select the forecasting model, has been demonstrated (Arinze 1994, Armstrong et al. 2001, Collopy, Adya & Armstrong 2001). The use of intelligent decision-support systems based on neural network technology for model selection and forecasting has been explored (Venkatachalam & Sohl 1999).

The framework developed in this article might be incorporated in the future into expert systems as well as into intelligent forecasting systems.

7. Conclusions

Competitive settings, real-time econometrics and deregulated management frameworks will require an increasing number of forecast horizons, particularly in the globalised FX market.

Economists, engineers, forecasters and managers will have at their disposal a set of forecasts for the magnitudes of interest obtained by using several methods.

The aim of this paper has been to provide a starting point for the handling of the above scenario by showing the possibility of using multivariate analysis (especially principal components analysis and factor analysis) in combining forecasts. A conceptual framework has been developed and suggestions for using several multivariate analysis methods to solve the problem of how to effectively combine forecasts have been made. In addition, a new methodology for the construction of combined forecasts has been proposed.

In particular, it has been shown that the employment of PCA in the considered scenario of FX markets is very promising for:

1. Reinforcing the properties of easy combinations like that of simple average.
2. Selecting the methods to include in the combination.

Moreover, some practical contexts have been considered and the usefulness of the proposed methodology in those contexts and for some patterns of time series commented on.

It seems that multivariate analysis may be very useful in combining forecasts. However, much more research is required to evaluate the exciting possibilities offered by these methods in forecasting, in general, and in combining economic, financial or technological forecasts, in particular.

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Appendix

A1. Introducción

El siglo XXI está caracterizándose por ofrecer múltiples opciones para cada problema y situación del mundo real. Muchos hechos corroboran la afirmación anterior; por ejemplo una rápida transferencia de conocimientos del mundo académico al no académico, una comunidad científica con un conocimiento cada vez más amplio, programas de ordenador sencillos y flexibles y un incremento en cuanto a disponibilidad de nuevas tecnologías (véase, como ejemplo de lo anterior, el caso de la película *La red social*).

En nuestros días la predicción se caracteriza por la disponibilidad de un gran número de métodos. Se pueden consultar varias fuentes para actualizar el conocimiento de los mencionados métodos (por ejemplo, Armstrong 2001, Diebold 2001, Elliott et al. 2006, Makridakis et al. 1998).

Además, los horizontes de predicción para las empresas y las instituciones son cada vez más diferentes, aunque se encuentran relacionados. Quienes toman las decisiones necesitan predicciones relacionadas con el plazo inmediato, así como para el corto, el mediano y el largo plazo, o incluso para los tres horizontes estándar: corto, mediano y largo. Por ejemplo, Sánchez-Ubeda & Berzosa (2007) proponen una aplicación para pronosticar el consumo de gas natural de los usuarios finales en España, donde la empresa ENAGAS necesita predicciones para un horizonte de mediano plazo (1 a 3 años) con una resolución muy elevada (días). Otros trabajos que abordan dicho problema son Kang (2003), Andrawis et al. (2011) y las referencias citadas en ambos.

La combinación de pronósticos se considera una metodología apropiada para manejar el escenario anterior, habiendo alcanzado madurez y reconocimiento en los últimos 40 años (Bates & Granger 1969, Bunn 1988, Clemen 1989, de Menezes et al. 2000, Armstrong 2001, Timmerman 2006). En economía, una aplicación reciente se dedica a pronosticar el crecimiento del empleo en Estados Unidos (Rapach &

Strauss 2008). Además, Kapetanios et al. (2008) constatan que la combinación de pronósticos por lo general conduce a una reducción en el error de predicción.

Los escenarios anteriores son conceptualmente adecuados para la aplicación de los *métodos de análisis multivariante* (en lo sucesivo MAM). En los últimos quince años estas técnicas han experimentado de grandes mejoras en cuanto a la facilidad de uso, la superación de ciertas limitaciones y el reconocimiento de una utilidad mayor (Hair et al. 1998). En consecuencia, una adaptación de los MAM a las nuevas tecnologías como bases de datos, Internet, etc., es un área nueva.

Dado este escenario, este artículo propone el empleo de varios métodos de análisis multivariante clásico para obtener pronósticos combinados. Un artículo reciente (Poncela et al. 2011) propone el uso de técnicas de reducción de la dimensión en la combinación de pronósticos. Nuestro enfoque, sin embargo, es diferente porque considera distintos horizontes de predicción, analiza las posibilidades en la combinación de pronósticos del conjunto clave de métodos multivariantes y sugiere diferentes campos de aplicación, como los mercados financieros o los mercados de energía.

Esta versión reducida en español se organiza de la siguiente forma. La sección A2 se dedica al análisis de un marco de combinación de pronósticos y resalta la aproximación de los métodos del análisis multivariante. La sección A3 presenta la utilidad de la metodología propuesta en diferentes contextos prácticos. Por último, se muestran algunas conclusiones de la investigación.

A2. Combinación de pronósticos bajo el marco del análisis multivariante

En lo que sigue se supondrá que estamos interesados en pronosticar una magnitud, Y, para un número grande de horizontes de pronósticos, denotado por n, a partir de la información sobre la magnitud en diferentes periodos $1, 2, \dots$ hasta el momento (denotado por t). Con este objetivo en mente, el profesional de la predicción dispone de varios pronósticos obtenidos mediante un número de métodos de predicción, notado por p.

Por tanto, supondremos que tenemos p pronósticos centrados o no sesgados de la magnitud Y obtenidos mediante p métodos (o expertos o una combinación de métodos y expertos) $(M1, M2, \dots, Mp)$ para n horizontes de pronóstico $(H1, H2, \dots, Hn)$. La notación $f_{ij}(t)$ se refiere al pronóstico dado para el horizonte i-ésimo por el método o experto j-ésimo, con $i = 1, \dots, n$ y $j = 1, \dots, p$. Esta información se puede mostrar en una estructura de matriz (tabla 2), notada por

$$\mathrm{F}(t)$$

En el escenario anterior, resultará apropiada la aplicación de los métodos de análisis multivariante (Johnson & Wichern 2002, Rencher 2002, Hair et al. 1998), considerando variables los métodos de predicción y unidades los horizontes de pronóstico.

A partir de esta información, los MAM se han desarrollado para el estudio de diferentes problemas en el análisis de datos. En nuestro caso, los pronósticos se obtienen empleando varios métodos para diferentes horizontes de pronóstico. Uno de los problemas estudiado es la reducción de las dimensiones del problema (métodos de pronóstico), cuyo objetivo es facilitar el proceso de asimilación de las relaciones descubiertas por el analista.

Otro problema es el proceso de búsqueda de relaciones de dependencia que simplificaría la comprensión del comportamiento de algunos métodos de predicción, teniendo en cuenta los valores que muestren otros métodos. Por último, un problema interesante es el análisis de las posibles relaciones de interdependencia que permitirá detectar relaciones sutiles entre los horizontes de pronóstico.

Una clasificación inicial de los MAM, aplicada la estructura de la tabla 2 se resume en la tabla 3, que recoge el uso propuesto de los MAM en la predicción. Nos referimos a la predicción para diferentes horizontes, los métodos de pronóstico y las tareas de analizar dependencia e interdependencia entre métodos, o al problema de reducir el número de métodos, o al de agrupar o clasificar horizontes de pronóstico.

A continuación se considera brevemente el potencial de los MAM en los cinco problemas de la tabla 3.

Problema 1. En nuestro escenario de predicción, la tarea más evidente sería la reducción de los p pronósticos nosesgados a un número más pequeño de pronósticos (idealmente uno, o a lo sumo 2), para cada horizonte de pronóstico considerado. Con este objetivo en mente, métodos como el Análisis de Componentes Principales (PCA) o el Análisis Factorial (FA), lineal o no lineal, o incluso el Análisis de Conglomerados aplicado a los métodos de pronóstico, llamarían nuestra atención. En un contexto general, la aproximación de conglomerados en predicción tiene una larga tradición. Se puede consultar Kusiak & Li (2010) para una aplicación reciente en la predicción de la producción de energía eólica.

Problema 2. Granger & Ramanathan (1984) han sugerido que los métodos convencionales de combinar pronósticos podrían considerarse en el marco de la regresión. Por tanto, deberían explorarse los métodos alternativos para manejar este problema en la Tabla 1.

Por ejemplo, MANOVA se podría emplear para analizar si hay diferencias significativas entre un grupo de pronósticos obtenidos utilizando la misma metodología (ARIMA, redes neuronales, opiniones expertas, etc.) en diferentes grupos de horizontes de tiempo como corto, mediano o largo plazo. En función de la conclusión alcanzada, sería diferente la decisión del profesional de la predicción sobre la mejor forma de combinar.

CCA se podría utilizar para determinar el tipo de relación (lineal o no lineal) entre grupos de pronósticos a fin de decidir sobre la posibilidad de reducir el número de pronósticos que van a considerarse en el problema.

Problema 3. En función de los métodos de predicción utilizados, la utilidad de conocer el perfil de los horizontes de pronóstico con respecto a la exactitud está fuera de duda. Como han manifestado muchos autores, no hay ningún método ni metodología que supere o rebase al resto en cada problema considerado o en cada horizonte de tiempo. Por consiguiente, parece que el DA y la LR podrían ayudar en investigaciones futuras sobre estas cuestiones. En concreto, las aproximaciones en FOREX plantean nuevos retos en finanzas internacionales.

Problema 4. Identificar grupos de horizontes de pronóstico similares con respecto a una serie de métodos o metodologías de pronósticos haría posible la configuración de diferentes tipos de combinación para cada conglomerado de horizontes de pronósticos.

Problema 5. Este es uno de los problemas más complejos en los MAM. Eso sucede en la tabla 3. Aquí, los Mínimos Cuadrados Parciales (PLS) podrían ser una metodología útil al resolver el problema comentado de decidir qué métodos se deberían incluir al agrupar en conglomerados los horizontes de pronóstico.

A3. Algunos contextos prácticos y algunas cuestiones de aplicación

En esta sección se consideran tres contextos prácticos de predicción. Se analiza la utilidad de la metodología propuesta en dichos contextos, así como en algunos patrones de series temporales.

Mercados de energía

La predicción es una tarea muy importante para las personas que trabajan gestionando los mercados de energía, incluso cuando hay un escenario económico estable. Los productores de energía necesitan con urgencia pronósticos exactos que se encuentren automatizados, de manera que puedan anticipar la demanda, los precios y otras variables del mercado.

En la mayor parte de las zonas económicas más relevantes del mundo, el sector de la energía está acometiendo un importante proceso de reestructuración cuya meta es la competencia y la liberalización de cuestiones de regulación que impiden la libre competencia de los productores y proveedores de la energía. Todo esto implica que la gestión de las empresas productoras de energía ya no puede estar basada en procedimientos administrativos y centralizados; su beneficio ahora depende directamente de las decisiones gerenciales que se tomen. Los procesos de la toma de decisiones son cada vez más complejos y arriesgados, debido a la mayor incertidumbre que resulta del nuevo marco de operaciones en los mercados de energía del siglo XXI.

En investigaciones previas se ha resaltado que un cálculo cuidadoso de la demanda esperada y de la oferta en los mercados de energía puede conducir a contratos que incrementen los beneficios de las empresas. Por tanto, se va a requerir más investigación para pronósticos a muy corto plazo (Ramanathan et al. 1997). En este artículo, se ha considerado el desarrollo de una serie de modelos para producir pronósticos a muy corto plazo en sistemas de demanda horaria. La disponibilidad de pronósticos para toda la semana, 168 horas, es el estándar en las empresas de energía. Por ejemplo, Hahn et al. (2009) recogen información de una encuesta sobre métodos matemáticos e ideas que se han empleado para predecir la demanda de electricidad.

La metodología propuesta en este artículo se puede aplicar al cálculo de pronósticos de precios en las empresas de energía. Según Zhao et al. (2008), obtener estos pronósticos de precios constituye una tarea difícil pero esencial para los participantes en un mercado de electricidad desregulado. Pindyck (1999) analiza la influencia de los pronósticos de precios sobre las decisiones de inversión y la elección de los productos que se van a producir.

Mercados de acciones

El cálculo de pronósticos resulta una tarea crucial para los agentes de bolsa y las personas involucradas en la gestión de las acciones. Se podría mejorar la planificación de estrategias mediante la disponibilidad de pronósticos para retornos, precios y volatilidad, en algunos periodos del día. Por ejemplo, si un cuarto de hora se toma como unidad de división del tiempo, se van a necesitar pronósticos para 32 horizontes en un día de 8 horas. Obviamente, se utilizan con mucha frecuencia otras divisiones de tiempo; por ejemplo, 5 minutos. Por tanto, si como se ha sugerido (Makridakis & Winkler 1983) se obtuvieran pronósticos para los 32 horizontes de tiempo utilizando 4 o 5 métodos, el uso de la metodología propuesta podría proporcionar pronósticos más exactos a los agentes de bolsa.

Ventas

En algunas investigaciones (Winklhofer et al. 1996) se ha estudiado la cuestión del horizonte de tiempo en el ámbito de los pronósticos en ventas. Algunas conclusiones fueron que la mayoría de las empresas preparaba pronósticos de ventas sobre una base anual, que el horizonte más popular en los pronósticos de ventas a corto plazo era un mes y que las empresas que operan en mercados muy competitivos ponen más énfasis en el corto plazo que en el largo.

Esto sugiere que la aproximación propuesta en este artículo no será útil para muchas empresas que emplean pronósticos de ventas. Sin embargo, el contexto crecientemente competitivo donde algunas empresas desarrollan su actividad, en una situación con cierto paralelismo con la de los mercados de energía y la de los mercados de acciones, puede incrementar la necesidad de considerar interesante emplear más métodos de predicción y un gran número de horizontes de pronóstico. Una vez que estos cambios fueran adoptados se podrían manejar con la aproximación propuesta.

Series estacionales

Series con una componente estacional reproducen el comportamiento de la magnitud cada cierto periodo. En consecuencia, se puede suponer que los horizontes de pronóstico actúan como unidades de estudio, de manera similar a los problemas donde el análisis multivariante se aplica habitualmente.

Series con tendencia

Las series que muestran una componente de tendencia no se comportan con el mismo patrón a través de varios horizontes de pronóstico. Por consiguiente, en esta situación los horizontes de pronóstico quizás no muestran la misma estructura que las unidades en los problemas usuales del análisis multivariante.

Series cíclicas

Las series que presentan una componente de ciclo con sus correspondientes puntos de giro deben retornar a una estructura previa en el comportamiento de la magnitud. Por tanto, como ocurre con las series estacionales, se puede suponer que los horizontes de pronóstico actúan como unidades de estudio, de manera similar a los problemas donde los MAM se aplican habitualmente.

Automatización

Uno de los puntos clave para una implantación con éxito de la metodología propuesta es la automatización de la misma. Por esta razón, las cuestiones de versatilidad, exactitud, velocidad e inteligibilididad se deben considerar de manera rotunda, porque solo habrá disponible una cantidad limitada de recursos (software, conocimiento, datos, etc.).

Métodos de cálculo de pronósticos a incluir

Varias metodologías de pronóstico clásicas (alisado exponencial, ARIMA, GARCH, etc.) se emplean ampliamente como parte de los sistemas de predicción, dando resultados satisfactorios cuando se aplican en diferentes campos. Proporcionan muchas posibilidades de modelado para una o varias variables, puesto que permiten considerar horizontes y comportamientos diferentes. Además, presentan rasgos muy intuitivos y son muy favorables, ya que los parámetros que manejan representan al proceso de manera simple.

Además, muchas de las variables incluidas en las aplicaciones prácticas se comportan bien en muchos casos o momentos de su historia, justificando la opinión de que los modelos lineales o clásicos de cálculo de pronósticos se pueden ver adecuados para tales contextos. Sin embargo, debería considerarse la posibilidad de emplear Redes Neuronales Artificiales y otros métodos no lineales.

Hay varias referencias para un estudio detallado de los métodos generales de cálculo de pronósticos (Makridakis et al. 1998, Diebold 2001, Elliott et al. 2006).

Integración con predicción basada en reglas y sistemas expertos

Una técnica de pronóstico que integra opinión experta y extrapolación estadística se denomina técnica de pronóstico basada en reglas. La predicción basada en reglas automatiza algunas tareas asociadas al mantenimiento de un cuerpo complejo de conocimiento, dado en una forma accesible y modificable, a fin de proporcionar pronósticos más exactos. Existen varios ejemplos de sistemas de elaboración de pronósticos basados en reglas (Collopy & Armstrong 1992, Armstrong et al. 2001). El enfoque propuesto podría proporcionar una nueva perspectiva a los citados sistemas.

En distintos trabajos (Arinze 1994, Armstrong et al. 2001, Collopy et al. 2001) se ha mostrado el potencial y los beneficios de emplear sistemas expertos en las tareas de predicción, tanto para formular el modelo como para seleccionarlo. También, se ha explorado el uso de sistemas inteligentes de apoyo a la decisión basados en redes neuronales para la selección del modelo y el cálculo de pronósticos (Venkatachalam & Sohl 1999).

El marco desarrollado en este artículo se podría incorporar en el futuro tanto en sistemas expertos como en sistemas inteligentes de cálculo de pronósticos.

A4. Conclusiones

Entornos competitivos, econometría en tiempo real y marcos de gestión liberalizados van a requerir un número creciente de horizontes de pronósticos; particularmente en el mercado globalizado de divisas.

Economistas, ingenieros, gerentes y profesionales de la predicción tendrán a su disposición, para las magnitudes de interés, un conjunto de pronósticos obtenidos mediante el empleo de varios métodos.

El objetivo de este artículo es proporcionar un punto inicial para el manejo del escenario anterior, mostrando la posibilidad de utilizar análisis multivariante (especialmente análisis de componentes principales y análisis factorial) al combinar pronósticos. Se ha presentado un marco conceptual y se han dado sugerencias para utilizar métodos de análisis multivariante para resolver el problema de combinar pronósticos de forma eficaz. Además, se ha propuesto una nueva metodología para la construcción de pronósticos combinados.

En concreto, se ha mostrado que el empleo de PCA en el escenario considerado de los mercados de divisas es muy prometedor para lo siguiente.

1. Reafirmar ciertas propiedades que tienen las combinaciones sencillas de pronósticos, como el promedio simple.
2. Seleccionar los métodos que se van a incluirse en la combinación.

Asimismo, se han considerado algunos contextos prácticos y la utilidad de la metodología propuesta en estos para algunos patrones de series temporales.

Parece que el análisis multivariante puede ser muy útil a la hora de combinar pronósticos. Sin embargo, se requiere mucha más investigación a fin de evaluar las excitantes posibilidades que ofrecen estos métodos en la predicción, en general, y al combinar pronósticos económicos, financieros o tecnológicos, en particular.

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