Revista Colombiana de Estadística
July 2015, Volume 38, Issue 2, pp. 413 to 429
DOI: http://dx.doi.org/10.15446/rce.v38n2.51669

Design of SkSP-R Variables Sampling Plans

Diseño de planes de muestreo SkSP-R

Muhammad Aslam[1,a], Saminathan Balamurali[2,b], Chi-Hyuck Jun[3,c], Batool Hussain[4,d]

[1]Department of Statistics, Faculty of Sciences, King Abdulaziz University, Saudi Arabia

[2]Departments of Mathematics institution, Krishnankoil, India

[3]Department of Industrial and Management Engineering, Pohang University of Science and Technology, Republic of Korea

[4]Department of Statistics, Kinnaird College for Women, Pakistan

Abstract

In this paper, we present the designing of the skip-lot sampling plan including the re-inspection called SkSP-R .The plan parameters of the proposed plan are determined through a nonlinear optimization problem by minimizing the average sample number satisfying both the producer's risk and the consumer's risks. The proposed plan is shown to perform better than the existing sampling plans in terms of the average sample number. The application of the proposed plan is explained with the help of illustrative examples.

***Key words*:** Acceptable Quality Level, Acceptance Sampling, Average Sample Number, Limiting Quality Level, Quality Control.

Resumen

En este artículo, se presenta el diseño de un plan de muestreo de lotes incluyendo reinspección llamado SkSP-R. Los parámetros del plan propuesto se determinan a través de un problema de optimización no lineal que minimiza el número de muestras promedio óptimo que satisface el riesgo del productor a un nivel de calidad aceptable y el riesgo del consumidor a un nivel de calidad límite. El plan propuesto se desempeña mejor que otros planes de muestreo existentes en términos del número de muestras promedio. Se presenta una aplicación del plan propuesto con la ayuda de tabulados.

***Palabras clave*:** características de calidad medibles, control de calidad, muestreo Skip-lot, nivel de calidad aceptable, nivel de calidad límite, muestreo de aceptación.

[a]Associate Professor. E-mail: aslam_ravian@hotmail.com
[b]Professor. E-mail: sbmurali@rediffmail.com
[c]Professor. E-mail: chjun@postech.ac.kr
[d]Ph.D. Student. E-mail: bhjafri@gmail.com

1. Introduction

Acceptance sampling is an important tool of statistical quality control. This tool is used to enhance the quality of the product through the inspection from the raw stage to the final stage. Without the proper inspection or testing the product may cause the bad reputation of the company in the global market. Good products sent to the market after the inspection increase the demand and alternately increase the profit of the company. Therefore, sampling plans have received the attention of the industrial engineers. Various sampling plans have been widely used in many industries including the electronic industry (Deros, Peng, Ab Rahman, Ismail & Sulong 2008), medical industry Fu, Tsai, Lin & Wei (2004) and construction industry Gharaibeh, Liu & Wani (2012).

Acceptance sampling plans are basically divided into two major categories namely; attribute sampling plans and variables sampling plans. The attribute sampling plans are used when the quality characteristic is just classified as good or bad. The variable sampling plans are used when the quality characteristic of interest can be measurable on numerical scale. The attribute sampling plans are easy to apply, however the variable sampling plans are generally more informative than the attribute sampling plans. Collani (1990) in one of his articles, criticized the variable sampling plans and, at the same time, Seidel (1997) proved that variable sampling plans are more optimal than the attribute sampling plans.

The single sampling plan (SSP) is one of the widely used sampling plans in the industries for the inspection of the finished products. This sampling plan is easy to apply and the industrial engineers can reach a decision quickly using this sampling plan. But, there are some other sampling schemes which are considered more efficient than the single sampling plan. As the cost of inspection is directly proportional to the sample size required for the acceptance or rejection decision, a large sample size incurs a large cost for the inspection which is not favorable for the producer and consumer. Therefore, some other sampling schemes such as double sampling, multiple sampling, sequential sampling and skip-lot sampling plans have been developed in order to save the cost and time of the inspection.

The skip-lot sampling plan (SkSP) is one of the sampling schemes widely used in the industry for the inspection purpose. The main advantage of the SkSP scheme is to provide the inspection of the product at a low cost (Hsu 1980). This scheme is shown to be more efficient than the single sampling plan Aslam, Wu, Azam & Jun (2013) in terms of the minimum average sample number. Dodge (1943, 1955) originally developed the skip-lot sampling procedure and designated it as SkSP-1 plan. Later on, Perry (1970, 1970) discussed the applications of the SkSP-2 scheme. More details about the procedure and applications of SkSP schemes can be seen in Bennett & Callejas (1980), MIL-STD 105D (1963), Okada (1967), Stephens (1979), Bennett & Callejas (1980), Cox (1980), Parker & Kessler (1981), Carr (1982), Schilling (1982), Liebesman & Saperstein (1983), Reetz (1984) , ANSI/ASQC Standard A2-1987 (1987), Liebesman (1987), Vijayaraghavan (1994), Besterfield (2004), Taylor (2005), Duffuaa, Turki & Kolus (2009), Aslam, Balamurali, Jun & Ahmad (2010), Balamurali & Jun (2011), Balamurali & Subramani

(2012), Wu, Aslam & Jun (2012), Aslam, Balamurali, Jun & Ahmad (2013) and Cao & Subramaniam (2013).

A re-inspection procedure can be used when the experimenters need to inspect the product again if they cannot make a decision on the basis of the original inspection. Govindaraju & Ganesalingam (1997) originally developed the sampling plan for resubmitted lots for the application of inspection of attribute quality characteristics. Recently, Aslam, Balamurali, Jun & Ahmad (2013) and Wu et al. (2012) proposed variable sampling plans using a process capability index for the inspection of resubmitted lots. By incorporating the idea of the re-inspection concept of Govindaraju & Ganesalingam (1997), Balamurali, Aslam & Jun (2014) introduced a new skip-lot sampling system designated as SkSP-R for attributing quality characteristics.

By exploring the literature of acceptance sampling, we note that there is no development on SkSP-R plan available for the inspection of measurable quality characteristics. So, in this paper, we will focus on the development of the SkSP-R sampling plan for the variables inspection by assuming that the quality characteristic of interest follows a normal distribution with standard deviation a known or unknown. We will present the designing methodology, application and the efficiency of the proposed plan. We show that the proposed plan performs better than the existing sampling plan. The rest of the paper is organized as follows: the SkSP-R plan under variables inspection is proposed in Section 2, the designing methodology of the SkSP-R plan under variables inspection for the known standard deviation (sigma) case is given in Section 3, the designing methodology of the SkSP-R plan for the unknown standard deviation case is given in Section 4, a comparison of the SkSP-R plan under variables inspection with existing sampling plans is given in Section 5 and certain concluding remarks are given in the last section.

2. Execution of SkSP-R Plan

As pointed out earlier, Balamurali et al. (2014) developed a new system of skip-lot sampling plan designated as SkSP-R, which is based on the principles of both continuous sampling plans and the re-inspection scheme of Govindaraju & Ganesalingam (1997) for the quality inspection of the continuous flow of bulk products. The SkSP-R plan uses the concept of reference plan similar to the SkSP-2 plan of Perry (1970). In this paper, the SkSP-R plan uses the variables single sampling plan as the reference plan.

Suppose that the quality characteristic of interest has the upper specification limit U and follows a normal distribution with unknown mean μ and known standard deviation σ. The operating procedure of the SkSP-R plan with variables sampling plan as the reference plan is explained below.

1. Start with the normal inspection by applying the variables single sampling plan as the reference plan. During the normal inspection, lots are inspected one by one in order of being submitted.

2. From each lot submitted for inspection, take a random sample of size n_σ and measure the quality characteristics$(X_1, X_2, \ldots, X_{n_\sigma})$. Compute $V = \frac{(U-\bar{X})}{\sigma}$, where $\bar{X} = \frac{1}{n_\sigma}\sum_{i=1}^{n_\sigma} X_i$. Accept the lot if $v \geq k_\sigma$ and reject the lot if $v < k_\sigma$. (Note: In case of lower specification limit, the component v will be computed as $v = \frac{(\bar{X}-L)}{\sigma}$ and other calculations are the same).

3. When i consecutive lots are accepted based on the reference plan under normal inspection, discontinue the normal inspection and switch to the skipping inspection.

4. During the skipping inspection, inspect only a fraction f of lots selected at random by applying the variables single sampling plan as the reference plan. The skipping inspection is continued until a sampled lot is rejected.

5. When a lot is rejected after s consecutively sampled lots have been accepted, then go for re-inspection procedure for the immediate next lot as in step (5) given below.

6. During re-inspection procedure, perform the inspection using the reference plan. If the lot is accepted, then continue the skipping inspection. On non-acceptance of the lot, re-inspection is done for m times and the lot is rejected if it has not been accepted on (m-1)st resubmission.

7. If a lot is rejected on the re-inspection scheme, then we immediately revert to the normal inspection in Step (1).

8. Replace or correct all the non-conforming units found with conforming units in the rejected lots.

The proposed plan involves the reference plan and four parameters, namely f ($0<f<1$), the fraction of lots inspected in skipping inspection mode, i, the clearance number of normal inspection, s, the clearance number for re-inspection procedure and m, the number of time the lots are submitted for re-inspection. In general, i, s and m are positive integers. So, the plan is designated as SkSP-R (i, f, s, m). The operation of the proposed plan is depicted by a flow diagram as shown in Figure 1.

3. Known Sigma Variables SkSP-R Plan Design

Under variables sampling inspection, an item is classified as non-conforming if it exceeds the upper specification limit U. So, the fraction non-conforming in a lot based on normal distribution will be defined as

$$p = P\{X_i > U\} = 1 - \Phi\left(\frac{U-\mu}{\sigma}\right) \tag{1}$$

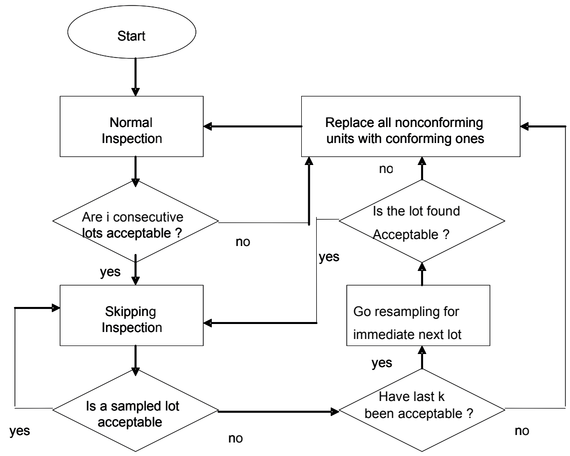

FIGURE 1: Operation of Proposed Skip lot Plan (Hussian et al., 2014).

In the case of lower specification limit L, the fraction non-conforming is determined as

$$p = P\{X_i < L\} = \Phi\left(\frac{\mu - L}{\sigma}\right)$$

where$\Phi(y)$is the normal cumulative distribution function and is given by

$$\Phi(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-z^2}{2}\right) dz \tag{2}$$

According to Balamurali et al. (2014), the operating characteristic (OC) function of the SkSP-R system, which gives the proportion of lots that are expected to be accepted for specified fraction non-conforming (product quality) p is given by

$$P_a(p) = \frac{fP + (1-f)P^i + fP^s(P^i - P)(1 - Q^m)}{f(1-P^i)\left[1 - P^s(1-Q^m)\right] + P^i(1 + fQP^s)} \tag{3}$$

where$P$is the probability of acceptance of the reference plan, i.e, the probability of accepting the lot under the variables single sampling plan with parameters (n_σ, k_σ) and $Q = 1 - P$. Here P is given by

$$P = \Phi(w)$$

Where $w = (v - k_\sigma)\sqrt{n_\sigma}$ and $v = \frac{U-\mu}{\sigma}$.

In general, any sampling plan can be designed based on two points on the OC curve approach. A well-designed sampling plan can significantly reduce the difference between the required and the actual existing quality of the products. The producer usually would focus on a specific level of product quality, called acceptable quality level (AQL), which would yield a high probability for accepting a lot. Alternatively, the consumer would also focus on another point at the other end of the OC curve, called limiting quality level (LQL). So, the producer wants the probability of acceptance at AQL to be larger than his confidence level $(1-\alpha)$ and the consumer desires that the lot acceptance probability at LQL should be less than his risk β. That is, the acceptance sampling plan must have its OC curve passing through those two designated points (AQL, $1-\alpha$) and (LQL, β). Generally the AQL is denoted by p_1 and the LQL is denoted by p_2.

The OC function of the SkSP-R variables plan at the AQL ($=p_1$) and LQL ($=p_2$) satisfying the corresponding producer's risk α and consumer's risk β are respectively given as

$$P_a(p_1) = \frac{fP_1 + (1-f)P_1^i + fP_1^s(P_1^i - P_1)(1-Q_1^m)}{f(1-P_1^i)\left[1-P_1^s(1-Q_1^m)\right] + P_1^i(1+fQ_1P_1^s)} \geq 1-\alpha \tag{4}$$

and

$$P_a(p_2) = \frac{fP_2 + (1-f)P_2^i + fP_2^s(P_2^i - P_2)(1-Q_2^m)}{f(1-P_2^i)\left[1-P_2^s(1-Q_2^m)\right] + P_2^i(1+fQ_2P_2^s)} \leq \beta \tag{5}$$

where $P_1 = \Phi(w_1)$, which is the probability of acceptance of the reference plan at AQL, $Q = 1-\Phi(w_1)$, $P_2 = \Phi(w_2)$, which is the probability of acceptance of the reference plan at LQL and $Q_2 = 1-\Phi(w_2)$. Here w_1 is the value of w at $p = p_1$, w_2 is the value of w at $p = p_2$. That is,

$$w_1 = (v_1 - k_\sigma)\sqrt{n_\sigma} \quad \text{and} \quad w_2 = (v_2 - k_\sigma)\sqrt{n_\sigma} \tag{6}$$

wherev_1 is the value of v at AQL and v_2 is the value of v at LQL.

For given AQL or LQL, the values of i, f, s, m, k_σ and the sample size n_σ are determined by formulating a nonlinear optimization problem. Throughout this paper, we consider $s = i$ and $m = 2$ as suggested by Govindaraju & Ganesalingam (1997) in order to reduce the number of parameters. The average sample number (ASN), by definition, means the expected number of sampled units required for making a decision about the lot. It is also known that the ASN of the known sigma SkSP-R plan is given as (see Balamurali et al. 2014)

$$ASN(p) = \frac{nf + nfQP^{i+s} - nfP^s(1-P^i)(1-Q^m)}{f(1-P^i)\left[1-P^s(1-Q^m)\right] + P^i(1+fQP^s)} \tag{7}$$

The ASN at AQL and LQL respectively of the SkSP-R plan when s=i are given as

$$ASN(p_1) = \frac{nf + nfQ_1P_1^{2i} - nfP_1^i(1-P_1^i)(1-Q_1^m)}{f(1-P_1^i)\left[1-P_1^i(1-Q_1^m)\right] + P_1^i(1+fQ_1P_1^i)} \tag{8}$$

and

$$ASN(p_2) = \frac{nf + nfQ_2P_2^{2i} - nfP_2^i(1-P_2^i)(1-Q_2^m)}{f(1-P_2^i)\left[1-P_2^i(1-Q_2^m)\right] + P_2^i(1+fQ_2P_2^i)} \tag{9}$$

The ASN given above can be used as an objective function to be minimized in a nonlinear optimization problem since there are several choices for the objective function, it is considered here to minimize ASN at LQL given in (9) because it is larger than the ASN at AQL. Therefore, the problem will be reduced to the following nonlinear optimization problem.

$$\textit{Minimize } ASN(p_2) = \frac{nf + nfQ_2P_2^{i+s} - nfP_2^s(1-P_2^i)(1-Q_2^m)}{f(1-P_2^i)\left[1-P_2^s(1-Q_2^m)\right] + P_2^i(1+fQ_2P_2^s)}$$

Subject to

$$P_a(p_1) \geq 1 - \alpha$$

$$P_a(p_2) \leq \beta$$

$$n_\sigma > 1, \quad k_\sigma > 0, i \geq 1, m = 2, s = i \tag{10}$$

To solve the above problem finding of optimal parameters of (i, f, n_σ, k_σ), we use a grid search procedure. The parameters (i, f, n_σ, k_σ) for the known sigma plan are determined by six combinations of (α, β) namely (0.05, 0.1), (0.01,0.1),(0.01, 0.05), which are reported in Tables 1-3.

Table 1: Optimal parameters of variables SkSP-R plan for known standard deviation with $k = i$ and $m = 2$ with $\alpha = 0.05$ and $\beta = 0.10$.

		Optimal parameters					
p_1	p_2	n_σ	$k\sigma$	i	f	$P_a(p_1)$	ASN(p_2)
	0.002	68	3.04499	3	0.05	0.95018	67.229
	0.003	26	3.01499	3	0.05	0.95164	25.684
0.001	0.004	16	2.99499	3	0.05	0.95128	15.815
	0.005	12	2.97999	3	0.05	0.95141	11.881
	0.006	9	2.96499	3	0.05	0.95057	8.888
	0.006	797	2.55999	3	0.05	0.95933	786.746
	0.0075	146	2.54499	3	0.05	0.95011	144.205
0.005	0.008	108	2.53999	3	0.05	0.95009	106.686
	0.010	49	2.51998	3	0.05	0.95259	48.382
	0.012	30	2.50498	3	0.05	0.95223	29.626
	0.02	41	2.26498	3	0.05	0.95284	40.475
	0.03	15	2.22998	3	0.05	0.95022	14.807
0.01	0.04	10	2.26498	2	0.05	0.95069	9.512
	0.05	7	2.18498	3	0.05	0.95033	6.941
	0.06	5	2.15998	3	0.05	0.95006	4.936
	0.03	102	2.01499	3	0.05	0.95263	100.711
	0.04	34	1.98999	3	0.05	0.95000	33.654
0.02	0.05	18	1.96498	3	0.05	0.95068	17.778
	0.06	13	1.94998	3	0.05	0.95035	12.888
	0.07	10	1.98998	2	0.05	0.95155	9.512
	0.04	186	1.84999	3	0.05	0.95630	183.634
	0.05	54	1.82999		3 0.05	0.95026	53.335
0.03	0.06	29	1.80999	3	0.05	0.95135	28.669
	0.07	19	1.79499		3 0.05	0.95035	18.803
	0.08	14	1.77998		3 0.05	0.95079	13.863
	0.05	285	1.72499	3	0.05	0.95792	281.337
	0.06	81	1.70499	3	0.05	0.95522	79.957
0.04	0.07	40	1.68999	3	0.05	0.95169	39.493
	0.08	26	1.67499	3	0.05	0.95197	25.707
	0.09	18	1.65999	3	0.05	0.95182	17.771
	0.06	371	1.625 3	0.05	0.95161	0.95609	366.231
	0.07	110	1.605	3	0.05	0.95423	108.608
0.05	0.08	54	1.590	3	0.05	0.95019	53.330
	0.09	33	1.580	3	0.05	0.95159	32.623
	0.10	23	1.565	3	0.05	0.95018	22.715

Table 2: Optimal parameters of variables SkSP-R plan for known standard deviation with $k = i$ and $m = 2$ with $\alpha = 0.01$ and $\beta = 0.05$.

		Optimal parameters					
p_1	p_2	n_σ	$k\sigma$	i	f	$P_a(p_1)$	ASN(p_2)
0.001	0.002	125	2.99999	3	0.05	0.99007	123.473
	0.003	48	2.94499	3	0.05	0.99002	47.428
	0.004	30	2.90499	3	0.05	0.99020	29.677
	0.005	22	2.87499	3	0.05	0.99009	21.786
	0.006	17	2.84499	3	0.05	0.99013	16.804
0.005	0.006	1518	2.54999	3	0.05	0.99000	1508.206
	0.0075	274	2.51499	3	0.05	0.99003	270.752
	0.008	201	2.49999	4	0.05	0.99032	200.645
	0.010	91	2.46999	3	0.05	0.99009	89.942
	0.012	55	2.43998	3	0.05	0.99004	54.309
0.01	0.02	75	2.20999	3	0.05	0.99004	74.043
	0.03	28	2.12499	4	0.05	0.99009	27.901
	0.04	17	2.07999	3	0.05	0.99023	16.788
	0.05	13	2.00449	3	0.05	0.99020	12.898
	0.06	10	2.00498	3	0.05	0.99024	9.913
0.02	0.03	187	1.97999	3	0.05	0.99006	184.651
	0.04	62	1.92499	3	0.05	0.99018	61.285
	0.05	34	1.87999	3	0.05	0.99017	33.605
	0.06	23	1.83999	3	0.05	0.99043	22.728
	0.07	17	1.80498	3	0.05	0.99044	16.787
0.03	0.04	331	1.82499	3	0.05	0.99022	326.735
	0.05	101	1.77999	3	0.05	0.99017	99.739
	0.06	54	1.73999	3	0.05	0.99064	53.338
	0.07	35	1.70999	3	0.05	0.99012	34.625
	0.08	25	1.66499	4	0.05	0.99043	24.958
0.04	0.05	506	1.705	3	0.05	0.99049	499.488
	0.06	151	1.665	3	0.05	0.99101	149.080
	0.07	75	1.625	4	0.05	0.99065	74.868
	0.08	47	1.595	4	0.05	0.99014	46.923
	0.09	34	1.575	3	0.05	0.99042	33.594
	0.06	709	1.605	4	0.05	0.99001	708.091
0.05	0.07	201	1.570	4	0.05	0.99000	200.740
	0.08	99	1.535	4	0.05	0.99077	98.826
	0.09	61	1.515	3	0.05	0.99021	60.251
	0.10	43	1.490	3	0.05	0.99023	42.489

Suppose that a quality characteristic has the upper specification limit U and the lower specification limit L and that an item having the quality characteristic beyond these limits is declared as nonconforming. The nominal-best quality characteristics usually have double specification limits. It is to be pointed out that in the case of double specification limits, the designing methodology is slightly different. However, a one sided case serves as a reasonable approximation. The sampling plans based on double specification limits have been investigated by many authors (see for example Lee, Aslam & Hun, 2012).

Example 1. For example, if $p_1 = 0.005$, $p_2 = 0.01$, $\alpha = 0.05$ and $\beta = 0.10$, Table 1 gives the optimal parameters as n_σ=49, $k_\sigma = 2.51998$, i=3 and f=0.05. Hence the optimal parameters of the SkSP-R plan for the specified requirements are i=3, f=0.05, s=3, m=2, n_σ=49 and $k_\sigma = 2.51998$. For this plan, the probability of acceptance at AQL is 0.95259 and ASN at LQL is 48.382.

TABLE 3: Optimal parameters of variables SkSP-R plan for known standard deviation with $k = i$ and $m = 2$ with $\alpha = 0.01$ and $\beta = 0.05$.

		Optimal parameters					
p_1	p_2	n_σ	$k\sigma$	i	f	$P_a(p_1)$	ASN(p_2)
0.001	0.002	160	3.00999	3	0.05	0.99021	159.669
	0.003	62	2.95999	3	0.05	0.99044	61.875
	0.004	38	2.92499	3	0.05	0.99029	37.929
	0.005	28	2.89999	3	0.05	0.99003	27.957
	0.006	22	2.87499	3	0.05	0.99009	21.966
0.005	0.006	1885	2.54999	5	0.05	0.99023	1884.989
	0.0075	362	2.51999	3	0.05	0.99118	361.25
	0.008	266	2.50999	4	0.05	0.99029	265.969
	0.010	118	2.47999	3	0.05	0.99076	117.76
	0.012	71	2.45499	3	0.05	0.99028	70.354
0.01	0.02	99	2.22499	3	0.05	0.99006	98.838
	0.03	37	2.15999	3	0.05	0.99014	36.937
	0.04	22	2.10999	3	0.05	0.99021	21.959
	0.05	16	2.07498	3	0.05	0.99000	15.976
	0.06	13	2.04498	3	0.05	0.99020	12.986
0.02	0.03	249	1.98999	3	0.05	0.99000	248.638
	0.04	79	1.93999	3	0.05	0.99012	78.852
	0.05	43	1.89999	3	0.05	0.99006	42.914
	0.06	29	1.86499	3	0.05	0.99024	28.941
	0.07	22	1.83499	3	0.05	0.99045	21.959
0.03	0.04	437	1.83	4	0.05	0.99000	436.953
	0.05	132	1.79	3	0.05	0.99081	131.728
	0.06	70	1.76	3	0.05	0.99012	69.894
	0.07	45	1.73	3	0.05	0.99014	44.927
	0.08	32	1.7	3	0.05	0.99038	31.934
0.04	0.05	654	1.71	3	0.05	0.99076	652.631
	0.06	192	1.675	3	0.05	0.99093	191.600
	0.07	97	1.645	3	0.05	0.99077	96.799
	0.08	60	1.62	3	0.05	0.99016	59.874
	0.09	43	1.595	3	0.05	0.99035	42.911
0.05	0.06	910	1.61	3	0.05	0.99100	908.099
	0.07	256	1.580	3	0.05	0.99071	255.472
	0.08	126	1.555	3	0.05	0.99009	125.764
	0.09	78	1.53	3	0.05	0.99021	77.843
	0.10	56	1.51	3	0.05	0.99009	55.911

4. Designing of Unknown Sigma SkSP-R Plan

Whenever the standard deviation is unknown, we should use the sample standard deviation S instead of σ. In this case, the operation of the reference plan is as follows.

Step 1: From each submitted lot, take a random sample of size $n_S$and measure the quality characteristics$\left(X_1, X_2, \dots, X_{n_S}\right)$

Step 2: Compute$v = \frac{(U - \bar{X})}{S}$, where $\bar{X} = \frac{1}{n_S}\sum_{i=1}^{n_S} X_i$ and $S = \sqrt{\frac{\sum (X_i - \overline{X})^2}{n_S - 1}}$.

Accept the lot if $v \geq k_S$ and reject the lot if $v < k_S$.

The operation of SkSP-R plan for unknown sigma case is exactly the same as in the known sigma case, but the only difference is that the reference plan will be operated as mentioned above.

Thus, the unknown sigma variable SkSP-R plan has the parameters namely, i, s, m along with the sample size n_S, and the acceptable criterion k_S. The OC function for the unknown sigma case is different from the known sigma case. It is known that $\overline{X} \pm k_S S$is approximately normally distributed with mean $\mu \pm k_S E(S)$ and variance $\frac{\sigma^2}{n_S} + k_S Var(S)$ (see Duncan 1986, Balamurali & Jun 2006). That is,

$$\overline{X} + k_S S \sim N\left(\mu + k_S\sigma, \frac{\sigma^2}{n_S} + k_S^2 \frac{\sigma^2}{2n_S}\right)$$

Therefore, the probability of accepting a lot is given by

$$P\left\{\overline{X} \leq U - k_S S\,|p\right\} = P\left\{\overline{X} + k_S S \leq U\,|p\right\}$$

$$= \Phi\left(\frac{U - k_S\sigma - \mu}{(\sigma/\sqrt{n_S})\sqrt{1 + \frac{k_S^2}{2}}}\right) = \Phi\left((v - k_S)\sqrt{\frac{n_S}{1 + \frac{k_S^2}{2}}}\right)$$

If we let, $w_S = \left((v - k_S)\sqrt{\frac{n_S}{1+\frac{k_S^2}{2}}}\right)$, then the probability of accepting a lot is considered as $\Phi(w_S)$.

Hence the lot acceptance probability for the sigma unknown case of SkSP-R should satisfy the following two inequalities at AQL and LQL:

$$P_a(p_1) = \frac{fP_1 + (1-f)P_1^i + fP_1^s(P_1^i - P_1)(1 - Q_1^m)}{f(1 - P_1^i)\left[1 - P_1^s(1 - Q_1^m)\right] + P_1^i(1 + fQ_1P_1^s)} \geq 1 - \alpha \quad (11)$$

and

$$P_a(p_2) = \frac{fP_2 + (1-f)P_2^i + fP_2^s(P_2^i - P_2)(1 - Q_2^m)}{f(1 - P_2^i)\left[1 - P_2^s(1 - Q_2^m)\right] + P_2^i(1 + fQ_2P_2^s)} \leq \beta \quad (12)$$

where $P_1 = \Phi(w_{1S})$,$Q = 1 - \Phi(w_{1S})$, $P_2 = \Phi(w_{2S})$and $Q_2 = 1 - \Phi(w_{2S})$. Here w_{1S} is the value of w at p=p_1, w_{2S} is the value of w at p=p_2. That is,

$$w_{1S} = (v_1 - k_S)\sqrt{n_S} \quad \text{and} \quad w_{2S} = (v_2 - k_S)\sqrt{n_S} \quad (13)$$

wherev_1 is the value of v at AQL and v_2 is the value of v at LQL. In this case, the nonlinear optimization problem becomes

$$\textit{Minimize } ASN(p_2) = \frac{nf + nfQ_2P_2^{i+s} - nfP_2^k(1-P_2^i)(1-Q_2^m)}{f(1-P_2^i)\left[1-P_2^s(1-Q_2^m)\right] + P_2^i(1+fQ_2P_2^s)}$$

$$\textit{Subject to } P_a(p_1) \geq 1 - \alpha$$

$$P_a(p_2) \leq \beta$$

$$n_S > 1, \;\; k_S > 0, i \geq 1, m = 2, s = i \quad (14)$$

We may determine the parameters of the unknown sigma SkSP-R plan by solving the nonlinear problem given in (14). For given AQL or LQL, the values of i, f, s, m, k_S and the sample size n_S are determined by using a search procedure. The parameters (i, f, s, m, n_S, k_S) for the unknown sigma plan are determined for six combinations of (α, β) namely (0.05, 0.1), (0.01, 0.1), (0.01, 0.05), which are reported in Tables 4-6.

TABLE 4: Optimal parameters of variables SkSP-R Plan for unknown standard deviation with $k = i$ and $m = 2$ with $\alpha = 0.05$ and $\beta = 0.10$.

		Optimal parameters					
p_1	p_2	n_σ	$k\sigma$	i	f	$P_a(p_1)$	ASN(p_2)
0.001	0.002	381	3.04499	3	0.05	0.95003	376.585
	0.003	142	3.01499	3	0.05	0.95125	140.174
	0.004	86	2.99499	3	0.05	0.95076	84.927
	0.005	62	2.97999	3	0.05	0.95008	61.254
	0.006	49	2.96499	3	0.05	0.95081	48.412
0.005	0.006	3406	2.55999	3	0.05	0.95929	3362.096
	0.0075	617	2.54499	3	0.05	0.95003	609.335
	0.008	455	2.53999	3	0.05	0.95007	449.403
	0.010	204	2.51998	3	0.05	0.95251	201.403
	0.012	124	2.50499	3	0.05	0.95219	122.448
0.01	0.02	146	2.26499	3	0.05	0.95281	144.123
	0.03	53	2.22999	3	0.05	0.95056	52.352
	0.04	31	2.19999	3	0.05	0.95121	30.603
	0.05	22	2.17999	3	0.05	0.95028	21.737
	0.06	17	2.15998	3	0.05	0.95057	16.798
0.02	0.03	308	2.01499	3	0.05	0.95263	304.057
	0.04	99	1.98499	3	0.05	0.95337	97.731
	0.05	53	1.96499	3	0.05	0.95079	52.351
	0.06	35	1.94498	3	0.05	0.95093	34.563
	0.07	26	1.92999	3	0.05	0.95019	25.695
0.03	0.04	503	1.84999	3	0.05	0.95622	496.547
	0.05	144	1.82999	3	0.05	0.95018	142.208
	0.06	75	1.80999	3	0.05	0.95135	74.079
	0.07	49	1.79499	3	0.05	0.95003	48.466
	0.08	35	1.77499	3	0.05	0.95251	34.577
0.04	0.05	709	1.725	3	0.05	0.95791	699.889
	0.06	199	1.705	3	0.05	0.95525	196.449
	0.07	97	1.690	3	0.05	0.95166	95.765
	0.08	61	1.675	3	0.05	0.95136	60.248
	0.09	43	1.660	3	0.05	0.95193	42.463
0.05	0.06	861	1.625	3	0.05	0.95162	849.940
	0.07	251	1.605	3	0.05	0.95602	247.792
	0.08	121	1.590	3	0.05	0.95397	119.441
	0.09	74	1.580	3	0.05	0.95012	73.147
	0.10	51	1.565	3	0.05	0.95151	50.361

Example 2. For example, i f$p_1 = 0.005$, $p_2 = 0.01$, $\alpha = 0.05$ and $\beta = 0.10$, Table 4 gives the optimal parameters as $n_S = 204$, $k_S = 2.51998$, $i = 3$ and $f = 0.05$. Hence the optimal parameters of the SkSP-R plan for the specified requirements are $i = 3$, $f = 0.05$, $s = 3$, $m = 2$, $n_S = 204$, $k_S = 2.51998$. For this plan, the probability of acceptance at AQL is 0.95251 and ASN at LQL is 201.403.

5. Comparison

In this section we compare the variables SkSP-R plan with the variables single sampling plan. For this purpose we provide Table 7 which gives the ASN values at LQL of both sampling plans with $\alpha = 5\%$ and $\beta = 10\%$ for various combinations of AQL and LQL. For the comparison, we have considered both known and unknown standard deviation sampling plans.

TABLE 5: Optimal parameters of variables SkSP-R plan for unknown standard deviation with $k = i$ and $m = 2$ with $\alpha = 0.01$ and $\beta = 0.10$.

		Optimal parameters					
p_1	p_2	n_σ	$k\sigma$	i	f	$P_a(p_1)$	ASN(p_2)
0.001	0.002	683	2.99499	4	0.05	0.99001	681.889
	0.003	253	2.93499	4	0.05	0.99024	252.556
	0.004	153	2.88999	4	0.05	0.99065	152.73
	0.005	109	2.86999	3	0.05	0.99024	107.639
	0.006	84	2.82999	4	0.05	0.99011	83.862
0.005	0.006	6444	2.54499	3	0.05	0.99216	6442.787
	0.0075	1138	2.51499	3	0.05	0.99000	1124.397
	0.008	829	2.49999	4	0.05	0.99032	827.533
	0.010	366	2.46999	3	0.05	0.99001	361.632
	0.012	218	2.43999	3	0.05	0.99001	215.227
0.01	0.02	25	2.20999	3	0.05	0.99004	254.701
	0.03	92	2.12499	4	0.05	0.99019	91.844
	0.04	53	2.06499	4	0.05	0.99033	52.907
	0.05	37	2.03499	3	0.05	0.99013	36.529
	0.06	30	2.00499	3	0.05	0.99020	29.736
0.02	0.03	551	1.97999	3	0.05	0.99001	543.963
	0.04	175	1.92499	3	0.05	0.99006	172.902
	0.05	92	1.86999	4	0.05	0.99003	91.849
	0.06	61	1.83999	3	0.05	0.99026	60.238
	0.07	44	1.79499	4	0.05	0.99000	43.932
0.03	0.04	883	1.825	3	0.05	0.99023	871.663
	0.05	259	1.78	3	0.05	0.99009	255.677
	0.06	133	1.735	4	0.05	0.99001	132.798
	0.07	85	1.70	4	0.05	0.99011	84.879
	0.08	59	1.665	4	0.05	0.99028	58.896
0.04	0.05	1241	1.705	3	0.05	0.99049	1225.006
	0.06	358	1.66	4	0.05	0.99120	357.364
	0.07	174	1.625	4	0.05	0.99065	173.693
	0.08	106	1.595	4	0.05	0.99004	105.820
	0.09	74	1.565	4	0.05	0.99023	73.871
0.05	0.06	1621	1.605	4	0.05	0.99000	1618.914
	0.07	449	1.570	4	0.05	0.99001	448.421
	0.08	216	1.535	4	0.05	0.99079	215.624
	0.09	130	1.515	3	0.05	0.99012	128.357
	0.10	89	1.490	3	0.05	0.99002	87.864

From this table, it is clearly understood that the ASN of variables SkSP-R plan is considerably smaller as compared to the variables single sampling plan for any combinations of AQL and LQL. For example, if $p_1 = 0.01$ and $p_2 = 0.03$, Table 7 gives the ASN of the variables single sampling plan and variables SkSP-R plan as 44 and 14.807 for the known sigma case. It indicates that the variables SkSP-R plan achieves a reduction of over 66% in ASN compared to the ASN of the s, the ASN values are obtained from Table 7 as 137 and 52.352 respectively for the variables single sampling plan and the variables SkSP-R plan under the unknown sigma case. By comparing these values, we conclude that the variables SkSP-R plan achieves over a 61% reduction in ASN over the variables single sampling plan. However it is to be pointed out that the SkSP-R plan does not offer the same protection as the variables single sampling plan except under the stationary conditions of the underlying Markov chain requiring a higher number of lots of the same quality to achieve conditions. Under periods of changing quality, like the onset of a problem, the protection offered by SkSP-R plan is considerably lesser

TABLE 6: Optimal Parameters of Variables SkSP-R Plan for Unknown Standard Deviation with $k = i$ and $m = 2$ with $\alpha = 0.01$ and β=0.05.

		Optimal parameters					
p_1	p_2	n_σ	$k\sigma$	i	f	$P_a(p_1)$	ASN(p_2)
0.001	0.002	883	3.00999	3	0.05	0.99018	881.158
	0.003	332	2.95999	3	0.05	0.99038	331.313
	0.004	197	2.92499	3	0.05	0.99009	196.594
	0.005	142	2.89499	3	0.05	0.99035	141.711
	0.006	111	2.86999	3	0.05	0.99044	110.770
0.005	0.006	8011	2.54999	5	0.05	0.99022	8010.952
	0.0075	1508	2.51999	3	0.05	0.99116	1504.841
	0.008	1101	2.50999	4	0.05	0.99025	1100.871
	0.010	478	2.47999	3	0.05	0.99069	476.998
	0.012	284	2.45499	3	0.05	0.99024	283.405
0.01	0.02	343	2.22499	3	0.05	0.99003	342.431
	0.03	122	2.15999	3	0.05	0.99002	121.781
	0.04	70	2.10999	3	0.05	0.99005	69.861
	0.05	49	2.06999	3	0.05	0.99015	48.906
	0.06	37	2.03499	3	0.05	0.99013	36.924
0.02	0.03	742	1.98999	3	0.05	0.99059	741.913
	0.04	226	1.93999	3	0.05	0.99004	225.559
	0.05	120	1.89999	3	0.05	0.99000	119.755
	0.06	79	1.86499	3	0.05	0.99018	78.836
	0.07	58	1.83499	3	0.05	57.882	0.99025
	0.04	1169	1.83	4	0.05	0.99000	1168.875
0.03	0.05	343	1.79	3	0.05	0.99079	342.287
	0.06	176	1.755	3	0.05	0.99089	175.635
	0.07	111	1.73	3	0.05	0.99000	110.807
	0.08	78	1.7	3	0.05	0.99035	77.838
0.04	0.05	1610	1.71	3	0.05	0.99076	1606.628
	0.06	461	1.675	3	0.05	0.99092	460.036
	0.07	228	1.645	3	0.05	0.99076	227.525
	0.08	139	1.61999		3 0.05	0.99018	138.711
	0.09	98	1.595	3	0.05	0.99038	97.801
0.05	0.06	2088	1.61	3	0.05	0.99099	2083.623
	0.07	574	1.580	3	0.05	0.99068	572.799
	0.08	277	1.555	3	0.05	0.99003	276.468
	0.09	168	1.53	3	0.05	0.99013	167.648
	0.10	119	1.51	3	0.05	0.99001	118.804

than represented by AQL and LQL. In contrast, the variables single sampling plan maintains the protection represented by the AQL and LQL under all transitive conditions of changing quality.

6. Conclusions

The SkSP-R sampling plan is designed for the variable data in this paper. The necessary measures of the proposed plan for known and unknown standard deviation of normal distribution have been derived. The proposed plan can be used in the industry when the quality of interest follows the normal distribution. The efficiency of the proposed plan over the existing plan is studied. The proposed plan performs better than the existing variables single sampling plan in terms of minimum ASN. The application of the proposed plan in the industry can reduce the inspection cost. The extensive tables have been developed for various

Table 7: ASN comparison of the proposed plan with variables single sampling plan with $\alpha = 0.05$ and $\beta = 0.10$.

p_1	p_2	ASN at (p_2)			
		Known Sigma		Unknown Sigma	
		SSP	SkSP-R	SSP	SkSP-R
0.001	0.002	191	67.229	67.229	376.585
	0.003	74	25.684	25.684	140.174
	0.004	45	15.815	15.815	84.927
	0.005	33	11.881	11.881	61.254
	0.006	26	8.888	8.888	48.412
0.005	0.006	-	786.746	786.746	3362.096
	0.0075	417	144.205	144.205	609.335
	0.008	-	106.686	106.686	449.403
	0.010	138	48.382	48.382	201.403
	0.012	85	29.626	29.626	122.448
0.01	0.02	116	40.475	40.475	144.123
	0.03	44	14.807	14.807	52.352
	0.04	26	9.512	9.512	30.603
	0.05	19	6.941	6.941	21.737
	0.06	15	4.936	4.936	16.798
0.02	0.03	287	100.711	100.711	304.057
	0.04	94	33.654	33.654	97.731
	0.05	52	17.778	17.778	52.351
	0.06	35	12.888	12.888	34.563
	0.07	26	9.512	9.512	25.695
0.03	0.04	506	183.634	183.634	496.547
	0.05	154	53.335	53.335	142.208
	0.06	81	28.669	28.669	74.079
	0.07	53	18.803	18.803	48.466
	0.08	38	13.863	13.863	34.577
0.04	0.05	-	281.337	281.337	699.889
	0.06	224	79.957	79.957	196.449
	0.07	114	39.493	39.493	95.765
	0.08	72	25.707	25.707	60.248
	0.09	51	17.771	17.771	42.463
0.05	0.06	-	366.231	366.231	849.940
	0.07	300	108.608	108.608	247.792
	0.08	149	53.330	53.330	119.441
	0.09	93	32.623	32.623	73.147
	0.10	65	22.715	22.715	50.361

Note: (-) shows that plan parameters do not exist.

combinations of AQL and LQL and various producer and consumer risks are provided for this purpose. The proposed plan for non-normal distributions will be considered as future research. The current study only considers the case of constant process fraction non-conforming. The performance of the proposed plan should be evaluated for the case of shifted fraction non-conforming in a future study.

Acknowledgements

The authors are deeply thankful to the Editor and the three reviewers for their valuable suggestions to improve the quality of this manuscript.This article was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah. The author, Muhammad Aslam, therefore, acknowledge with thanks DSR technical and financial support.

[Received: February 2014 — Accepted: January 2015]

References

ANSI/ASQC Standard A2-1987 (1987), Terms, symbols, and definitions for acceptance sampling, American Society for Quality Control, Milwaukee.

Aslam, M., Balamurali, S., Jun, C.-H. & Ahmad, M. (2010), 'Optimal designing of a skip-lot sampling plan by two point method', *Pakistan Journal of Statistics* **26**(4), 585–592.

Aslam, M., Balamurali, S., Jun, C.-H. & Ahmad, M. (2013), 'Optimal design of skip lot group acceptance sampling plans for the weibull distribution and the generalized exponential distribution', *Quality Engineering* **5**(3), 237–246.

Aslam, M., Wu, C. W., Azam, M. & Jun, C.-H. (2013), 'Variable sampling inspection for resubmitted lots based on process capability index c_{pk} for normally distributed items', *Applied Mathematical Modeling* **37**, 667–675.

Balamurali, S., Aslam, M. & Jun, C.-H. (2014), 'A new system of product inspection based on skip-lot sampling plans including resampling', *The Scientific World Journal* pp. 1–6. ID 192412.

Balamurali, S. & Jun, C.-H. (2006), 'Repetitive group sampling procedure for variables inspection', *Journal of Applied Statistical Science* **33**(3), 327–338.

Balamurali, S. & Jun, C.-H. (2011), 'A new system of skip-lot sampling plans having a provision for reducing normal inspection', *Applied Stochastic Models in Business and Industry* **27**(3), 348–363.

Balamurali, S. & Subramani, J. (2012), 'Optimal designing of skip-lot sampling plan of type sksp-2 with double sampling plan as the reference plan', *Journal of Statistical Theory and Practice* **26**, 354–362.

Bennett, G. K. & Callejas, C. J. (1980), The economic design of skip-lot sampling plans, Transactions on Spring Annual Conference, pp. 349–355.

Besterfield, D. H. (2004), *Quality Control*, 7 edn, Pearson Prentice Hall.

Cao, Y. & Subramaniam, V. (2013), 'Improving the performance of manufacturing systems with continuous sampling plans', *IIE Transactions* **45**(6), 575–590.

Carr, W. E. (1982), 'Sampling plan adjustment for inspection error and skip-lot plan', *Journal of Quality Technology* **14**(1), 10–18.

Collani, E. V. (1990), 'A note on acceptance sampling for variables', *Metrika* **38**, 19–36.

Cox, D. C. (1980), 'Skip-lot sampling plans', *Quality* **21**(8), 26–27.

Deros, B. M., Peng, C. Y., Ab Rahman, M. N., Ismail, A. R. & Sulong, A. B. (2008), 'Assessing acceptance sampling application in electrical and electronic product', *Journal of Achievements in Materials and Manufacturing Engineering* **31**(2), 622–628.

Dodge, H. F. (1943), 'A sampling inspection plan for continuous production', *Annals of Mathematical Statistics* **14**(3), 264–279.

Dodge, H. F. (1955), 'Skip-lot sampling plan', *Industrial Quality Control* **11**(5), 3–5.

Duffuaa, S. O., Turki, U. M. & Kolus, A. A. (2009), 'Process-targeting model for a product with two dependent quality characteristics using acceptance sampling plans', *International Journal of Production Research* **47**(14), 4031–4046.

Duncan, A. J. (1986), *Quality Control and Industrial Statistics*, 5 edn, R. D. Irwin, Homewood, Illinois.

Fu, H., Tsai, H., Lin, C. & Wei, D. (2004), 'Application of a single sampling plan for auditing medical-claim payments made by taiwan national health insurance', *Health Policy* **70**(2), 185–95.

Gharaibeh, N., Liu, L. & Wani, S. (2012), 'Skip-lot acceptance sampling plans for highway construction and materials', *Journal of Construction Engineering Management* **138**(7), 791–796.

Govindaraju, K. & Ganesalingam, S. (1997), 'Sampling inspection for resubmitted lots', *Communications in Statistics- Simulation and Computation* **26**, 1163–1176.

Hsu, J. I. S. (1980), 'A cost model for skip-lot destructive testing', *IEEE Transactions on Reliability* **26**(1), 70–72.

Lee, S.-H., Aslam, M. & Hun, C.-H. (2012), 'Repetitive group sampling plans with double specification limits', *Pakistan Journal of Statistics* **28**(1), 41–57.

Liebesman, B. S. (1987), 'The development of an attribute skip-lot sampling standard', *Frontiers in Statistical Quality Control* **3**, 3–33.

Liebesman, B. S. & Saperstein, B. (1983), 'A proposed attribute skip-lot sampling program', *Journal of Quality Technology* **15**(3), 130–139.

MIL-STD 105D (1963), Sampling procedures and tables for inspection by attributes, Government Printing Office, Washington D.C.

Okada, H. (1967), 'Skip-lot sampling inspection plan combined with MIL-STD 105D', *Reports of Statistical Application Research* **14**(4), 13–20.

Parker, R. D. & Kessler, L. (1981), 'A modified skip-lot sampling plan', *Journal of Quality Technology* **13**(1), 31–35.

Perry, R. L. (1970), A System of Skip-lot Sampling Plans for Lot Inspection, PhD thesis, Rutgers-The State University.

Reetz, D. (1984), 'Optimal skip-lot sampling plans for Markov chains – theoretical foundation', *Frontiers in Statistical Quality Control* **2**, 25–36.

Schilling, E. G. (1982), *Acceptance Sampling in Quality Control*, American Society for Quality, Milwaukee.

Seidel, W. (1997), 'Is sampling by variables worse than sampling by attributes? A decision theoretic analysis and a new mixed strategy for inspecting individual lots', *Sankhya B* **59**, 96–107.

Stephens, K. S. (1979), How to perform continuous sampling (csp), *in* 'ASQC Basic References in Quality Control, Statistical Techniques', Vol. 2, American Society for Quality Control, Milwaukee.

Taylor, W. A. (2005), 'Selecting statistically valid sampling plans'. http://www.variation.com/ techlib/ as-7.html.

Vijayaraghavan, R. (1994), 'Construction and selection of skip-lot sampling inspection plans of type sksp-2 indexed by indifference quality level and maximum allowable percent defective', *Journal of Applied Statistics* **21**(5), 417–423.

Wu, C.-W., Aslam, M. & Jun, C.-H. (2012), 'Variables sampling inspection scheme for resubmitted lots based on the process capability index c_{pk}', *European Journal of Operational Research* **217**(3), 560–566.