**Revista Colombiana de Matemáticas**
Volumen 41(2007)2, páginas 397-406

# On the propagation of acceleration waves in thermoelastic micropolar medias

## Sobre la propagación de ondas aceleradas en medios micropolares termoelásticos

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ABSTRACT. The conditions for propagation of accelerating waves in a general nonlinear thermoelastic micropolar media are established. Deformation of micropolar media is described by the time-varying displacement vector $\boldsymbol{r}(t)$ and tensor of microrotation $\boldsymbol{r}(t)$ at each point. We call a surface $S(t)$ an accelerating wave (or a singular surface for a solution of the dynamic problem for the medium) if the points are points of continuity of both $\boldsymbol{r}(t)$ and $\mathbf{H}(t)$ and their first spatial and time derivatives while the second spatial and time derivatives (acceleration) of $\boldsymbol{r}(t)$ and $\mathbf{H}(t)$ have jumps on $S(t)$ (meaning that their one-sided limits at $S(t)$ differ). So $S(t)$ carries jumps in the acceleration fields as it propagat es through the body. In the thermomechanics of a micropolar continuum, similar propagating surfaces of singularities can exist for the fields of temperature, heat flux, etc.

We establish the kinematic and dynamic compatibility relations for the singular surface $S(t)$ in a nonlinear micropolar thermoelastic medium. An analog of Fresnel–Hadamard–Duhem theorem and an expression for the acoustic tensor are derived.

*Key words and phrases.* Acceleration waves, micropolar continuum, Cosserat continuum, nonlinear elasticity.

[a]The work was supported by the Russian Science Support Foundation and the Russian Foundation of Basic Research under grants 07-01-00525, 07-08-13589-ofi_c and by Universidad Nacional de Colombia, project of investigation No. DIB 8003061.

*2000 Mathematics Subject Classification.* 74A35, 74B20, 74J40.

Resumen. Se establecen las condiciones de propagación de ondas aceleradas en un medio no lineal micropolar termoelástico. Las deformaciones del medio micropolar son descritas por las variaciones temporales del vector de desplazamiento $\boldsymbol{r}(t)$ y del tensor de microrotación $\boldsymbol{r}(t)$ en cada punto. Llamamos una superficie $S(t)$ a una onda acelerada (o superficie singular para la solución del problema dinámico del medio) si los puntos son puntos de continuidad de $\boldsymbol{r}(t)$, $\mathbf{H}(t)$ y sus primeras derivas espaciales y temporales, mientras que las segundas derivadas espaciales y temporales tienen saltos en $S(t)$. Entonces $S(t)$ transporta los saltos en los campos acelerados cuando se propagan en el cuerpo. En la termomecánica de un continuo micropolar, superficies de propagación similares pueden existir para los campos de temperatura y de flujo de calor. Establecemos las relaciones de compatibilidad cinética y dinámica para las superficies singulares en un medio micropolar termoelástico no lineal. Un análogo del teorema Fresnel–Hadamard–Duhem y una expresión para el tensor acústico son establecidos.

*Palabras y frases clave.* Ondas de aceleración, continuo micropolar, continuo de Cosserat, elasticidad no-lineal.

## 1. **Introduction**

Propagation of nonlinear waves in solids is a complex process. Here analytic solutions are quite rare. However, the problem of propagation of acceleration waves is one of the exceptional cases. An acceleration wave (or wave of weak discontinuity) is a solution to the equations of motion of the medium that displays discontinuities in the second derivatives on certain surfaces that will be called singular. The existence of such solutions for nonlinear elastic and thermoelastic bodies and media with memory was proved in [20, 18, 19].

Acceleration waves in more complex media have been considered in a number of papers, e.g., [1, 10, 15, 16]. A generalization is presented in [12], where acceleration waves in elastic and viscoelastic micropolar media are considered. The relation between the existence of acceleration waves and the condition of strong ellipticity of the equilibrium equations is established in [3]. For micropolar shells, acceleration waves are studied in [4, 5].

Here we generalize the analysis from [3] to the case of a thermoelastic micropolar medium. We establish a condition for existence of weakly discontinuous solutions to the equations of motion and heat transfer. We prove a theorem analogous to the Fresnel–Hadamard–Duhem theorem in the nonlinear mechanics of simple materials. Under natural assumptions on the form of the heat conductivity law, we demonstrate that the condition for existence of an acceleration wave is similar to that in an elastic medium without temperature strains.

In a micropolar medium (also called a Cosserat continuum), each particle has the six degrees of freedom associated with a rigid body. Hence rotational counteraction of the particles of the medium is taken into account. In the micropolar theory, besides ordinary stresses, couple stresses are introduced [2, 7, 6, 9, 14, 17]. The Cosserat model is used to describe granular, powder-like materials, soils, polycrystalline bodies, composite bodies and nanostructures, and magnetic liquids. It also has applications to the construction of nonlinear models for beams, plates, and shells.

It is worth mentioning the monographs [8, 13, 14], where wave processes in micropolar continua are studied.

## 2. General relations for Cosserat continuum

Let us recall the general relations for a micropolar medium.

The equations of motion for a micropolar continuum, which represent the balance of momentum and of moment of momentum for an arbitrary part of the body in the material frame, are [7, 6]

$$\begin{aligned} \nabla\cdot\mathbf{T}+\rho\boldsymbol{f} &= \rho\tfrac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}, \\ \nabla\cdot\mathbf{M}+(\mathbf{F}^T\cdot\mathbf{T})_\times+\rho\boldsymbol{m} &= \rho\gamma\tfrac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t}. \end{aligned} \tag{1}$$

Here $\mathbf{T}$ and $\mathbf{M}$ are the stress tensor and couple-stress tensor of Piola–Kirchoff type, $\mathbf{F}=\nabla\boldsymbol{r}$ is the deformation gradient, $\nabla$ is the nabla (Hamilton) operator in lagrangian coordinates, $\rho$ is the medium density in the reference configuration, $\boldsymbol{f}$ and $\boldsymbol{m}$ the vectors of mass forces and mass couples, respectively, $\rho\gamma$ is the scalar measure of rotational inertia of a particle, and $\boldsymbol{v}=\mathrm{d}\boldsymbol{r}/\,\mathrm{d}t$ is the velocity vector of a particle. The radius-vector $\boldsymbol{r}(t)$ describes the position of a particle of the medium at instant $t$, whereas the proper orthogonal tensor of microrotation $\mathbf{H}(t)$ defines its orientation. $\boldsymbol{\omega}=\dfrac{1}{2}\left(\dfrac{\mathrm{d}\mathbf{H}}{\mathrm{d}t}\cdot\mathbf{H}^T\right)_\times$ is the angular velocity of microrotation. $\mathrm{d}/\,\mathrm{d}t$ denotes the material derivative with respect to $t$. The symbol $\mathbf{T}_\times$ stands for the vector invariant of a second-rank tensor $\mathbf{T}$ [11]. In particular, for a dyad $\boldsymbol{a}\otimes\boldsymbol{b}$ we have $(\boldsymbol{a}\otimes\boldsymbol{b})_\times=\boldsymbol{a}\times\boldsymbol{b}$, where $\times$ is the cross product.

Let us introduce an orthonormal trihedron with directors $\boldsymbol{D}_k$ $(k=1,2,3)$ that describe the orientation of body particles in the material frame, and a similar trihedron $\boldsymbol{d}_k$ that defines the orientation of particles in the actual configuration. Then the microrotation tensor $\mathbf{H}$ is defined as $\mathbf{H}=\boldsymbol{D}_k\otimes\boldsymbol{d}_k$. The kinematics of a micropolar medium are depicted in Fig. 1. Here $\Omega$ and $\omega$ are the domains occupied by the body in the material and actual configurations respectively, $q^1,q^2,q^3$ are the Lagrange coordinates, and $\boldsymbol{r}_i=\partial\boldsymbol{r}/\partial q^i$ are the coordinate unit vectors.

For heat transfer, the dynamic equations (1) are supplemented by the heat conduction equation [7, 6]

$$\rho\theta\frac{\mathrm{d}\eta}{\mathrm{d}t}=-\nabla\cdot\boldsymbol{q}+\rho c, \tag{2}$$

where $\theta$ is the temperature, $\eta$ is the specific entropy, $\boldsymbol{q}$ is the heat flux of Piola–Kirchhoff type, and $c$ is the heat source.

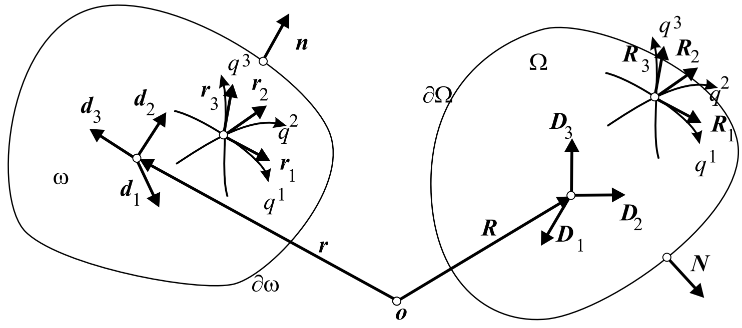


FIGURE 1. Kinematics of a micropolar medium

The constitutive equations for a Cosserat thermoelastic continuum can be written through the specific free energy

$$\psi = \psi(\mathbf{E}, \mathbf{K}, \theta)$$

as follows:

$$\mathbf{T} = \rho\psi,_{\mathbf{E}}\cdot\mathbf{H}, \quad \mathbf{M} = \rho\psi,_{\mathbf{K}}\cdot\mathbf{H}, \quad \eta = -\psi,\theta, \quad \boldsymbol{q} = \boldsymbol{q}(\mathbf{E}, \mathbf{K}, \theta, \nabla\theta), \tag{3}$$
$$\mathbf{E} = \mathbf{F}\cdot\mathbf{H}^T, \quad \mathbf{K}\times\mathbf{I} = -(\nabla\mathbf{H})\cdot\mathbf{H}^T, \quad \boldsymbol{q}\cdot\nabla\theta \le 0,$$

where $\mathbf{E}$ and $\mathbf{K}$ are the Cosserat deformation tensor and the wryness tensor (metric and bending strain measures) [3] and $\mathbf{I}$ is the unit tensor. From now on, we assume that $\psi$ is a twice continuously differentiable function and that the vector-function $\boldsymbol{q}$ is continuously differentiable. We use the following notation:

$$\psi,_{\mathbf{E}} = \frac{\partial\psi}{\partial\mathbf{E}}, \quad \psi,_{\mathbf{K}} = \frac{\partial\psi}{\partial\mathbf{K}}, \quad \psi,_{\theta} = \frac{\partial\psi}{\partial\theta},$$

etc.

## 3. Propagation of acceleration waves

Let us consider a motion of the medium during which a discontinuity of kinematic and dynamic quantities arises over a smooth surface $S(t)$. We shall call this surface *singular* (Fig. 2). We suppose the existence of unilateral limit values for the quantities at $S$; these, in general, may differ. The jump in $\boldsymbol{\Psi}$ at $S$ is denoted using the double square brackets: $[\![\boldsymbol{\Psi}]\!] = \boldsymbol{\Psi}^+ - \boldsymbol{\Psi}^-$.

Under the smoothness assumptions made above, let us derive conditions for existence of the acceleration wave — i.e., conditions under which weak discontinuous solutions may arise.

On a singular surface, the following balance equations must hold [7, 6]:

$$\rho V\,[\![\boldsymbol{v}]\!] = -\boldsymbol{N}\cdot[\![\mathbf{T}]\!], \quad \rho V\,[\![\boldsymbol{\omega}]\!] = -\boldsymbol{N}\cdot[\![\mathbf{M}]\!], \quad \rho\theta V\,[\![\eta]\!] = \boldsymbol{N}\cdot[\![\boldsymbol{q}]\!], \tag{4}$$

where $V$ is the intrinsic speed of propagation of $S$ in the direction $\boldsymbol{N}$ [20, 18, 19] and $\boldsymbol{N}$ is the unit normal to $S$.

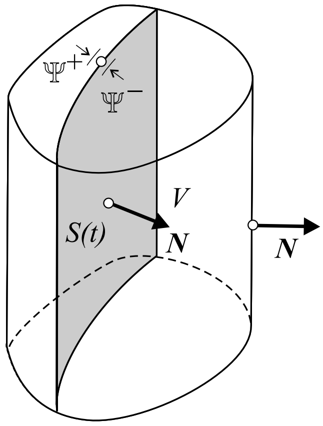


FIGURE 2. Singular surface

*An acceleration wave* (or *weak discontinuity wave*, or *singular surface of second order*) is a traveling singular surface at which the second spatial and time derivatives of the position vector $\boldsymbol{r}$ and micro-rotation tensor $\mathbf{H}$ have jumps, while the functions themselves and their first derivatives are continuous. So on $S$ we have

$$[\![\mathbf{F}]\!] = \mathbf{0}, \quad [\![\nabla\, \mathbf{H}]\!] = \mathbf{0}, \quad [\![\boldsymbol{v}]\!] = \mathbf{0}, \quad [\![\boldsymbol{\omega}]\!] = \mathbf{0}. \tag{5}$$

Let us also suppose that the temperature field and its derivatives are continuous in a neighborhood of $S$:

$$[\![\theta]\!] = \mathbf{0}, \quad [\![\nabla\, \theta]\!] = \mathbf{0}, \quad \left[\!\!\left[\frac{\mathrm{d}\theta}{\mathrm{d}t}\right]\!\!\right] = \mathbf{0}. \tag{6}$$

Hence we are considering homothermal acceleration waves.

For brevity we write

$$\dot{(...)} \equiv \frac{\mathrm{d}}{\mathrm{d}t}(...).$$

3.1. **Transformation of the dynamic equations.** First we consider the jump relations that follow from the motion equations.

Equations (5) and (6) imply continuity of the measures of deformation $\mathbf{E}$ and $\mathbf{K}$ in some neighborhood of $S$:

$$[\![\mathbf{E}]\!] = \mathbf{0}, \quad [\![\mathbf{K}]\!] = \mathbf{0}.$$

Hence, in view of the constitutive equations (3), we establish continuity of the tensors $\mathbf{T}$ and $\mathbf{M}$, of the entropy density $\eta$, and of the heat flow vector:

$$[\![\mathbf{T}]\!] = \mathbf{0}, \quad [\![\mathbf{M}]\!] = \mathbf{0}, \quad [\![\eta]\!] = \mathbf{0}, \quad [\![\boldsymbol{q}]\!] = \mathbf{0}.$$

It follows immediately that the balance equations (4) hold identically on $S$.

An application of Maxwell's theorem [20, 18, 19] to the continuous fields of the velocities $\boldsymbol{v}$ and $\boldsymbol{\omega}$, the stresses $\mathbf{T}$, and the couples $\mathbf{M}$ yields a system of equations relating the jumps in the derivatives with respect to the spatial variables and time [3]:

$$\begin{aligned} [\![\dot{\boldsymbol{v}}]\!] &= -V\boldsymbol{a}, & [\![\nabla \boldsymbol{v}]\!] &= \boldsymbol{N}\otimes\boldsymbol{a}, \\ [\![\dot{\boldsymbol{\omega}}]\!] &= -V\boldsymbol{b}, & [\![\nabla \boldsymbol{\omega}]\!] &= \boldsymbol{N}\otimes\boldsymbol{b}, \\ V[\![\nabla\cdot\mathbf{T}]\!] &= -\boldsymbol{N}\cdot\left[\!\!\left[\dot{\mathbf{T}}\right]\!\!\right], & V[\![\nabla\cdot\mathbf{M}]\!] &= -\boldsymbol{N}\cdot\left[\!\!\left[\dot{\mathbf{M}}\right]\!\!\right], \end{aligned} \tag{7}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are the vectorial amplitudes of the jumps in the linear and angular accelerations.

When the mass forces and couples are continuous, the relations

$$[\![\nabla\cdot\mathbf{T}]\!] = \rho\,[\![\dot{\boldsymbol{v}}]\!], \quad [\![\nabla\cdot\mathbf{M}]\!] = \rho\gamma\,[\![\dot{\boldsymbol{\omega}}]\!]$$

follow from the dynamic equations (1).

By differentiating the constitutive equations (3) and using (7), we can transform these to a form that contains only the vectorial amplitudes $\boldsymbol{a}$ and $\boldsymbol{b}$:

$$\begin{aligned} \boldsymbol{N}\cdot\psi,_{\mathbf{EE}}\cdot\cdot\,(\mathbf{H}\cdot\boldsymbol{a}\otimes\boldsymbol{N}) + \boldsymbol{N}\cdot\psi,_{\mathbf{EK}}\cdot\cdot\,(\mathbf{H}\cdot\boldsymbol{b}\otimes\boldsymbol{n}) &= V^2\boldsymbol{a}\cdot\mathbf{H}^T, \\ \boldsymbol{N}\cdot\psi,_{\mathbf{KE}}\cdot\cdot\,(\mathbf{H}\cdot\boldsymbol{a}\otimes\boldsymbol{N}) + \boldsymbol{n}\cdot\psi,_{\mathbf{KK}}\cdot\cdot\,(\mathbf{H}\cdot\boldsymbol{b}\otimes\boldsymbol{N}) &= \gamma V^2\boldsymbol{b}\cdot\mathbf{H}^T. \end{aligned}$$

Using matrix notation for tensors and vectors defined in a 6-dimensional space, we may rewrite these relations in the more compact form

$$\mathbb{A}(\boldsymbol{N})\cdot\boldsymbol{\xi} = V^2\mathbb{B}\cdot\boldsymbol{\xi}, \tag{8}$$

where

$$\boldsymbol{\xi} = (\boldsymbol{a}', \boldsymbol{b}') \in \mathbb{R}^6,\ \boldsymbol{a}' = \boldsymbol{a}\cdot\mathbf{H}^T,\ \boldsymbol{b}' = \boldsymbol{b}\cdot\mathbf{H}^T,$$

$$\mathbb{A}(\boldsymbol{N}) \equiv \begin{bmatrix} \psi,_{\mathbf{EE}}\{\boldsymbol{N}\} & \psi,_{\mathbf{EK}}\{\boldsymbol{N}\} \\ \psi,_{\mathbf{KE}}\{\boldsymbol{N}\} & \psi,_{\mathbf{KK}}\{\boldsymbol{N}\} \end{bmatrix},$$

$$\mathbb{B} \equiv \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \gamma\mathbf{I} \end{bmatrix}.$$

We have denoted

$$\mathbf{G}\{\boldsymbol{N}\} \equiv G_{klmn}N_kN_m\boldsymbol{i}_l\otimes\boldsymbol{i}_n$$

for an arbitrary 4th rank tensor $\mathbf{G}$ and an arbitrary vector $\boldsymbol{N}$, which are represented in a Cartesian basis $\boldsymbol{i}_k$ $(k = 1, 2, 3)$.

$\mathbb{A}(\boldsymbol{N})$ is analogous to the acoustical tensor for the micropolar medium. Symmetry of $\mathbb{A}(\boldsymbol{N})$ follows from the existence of the free energy function $\psi$. This provides that the squared velocity of propagation for an acceleration wave in an elastic micropolar medium is real-valued. The requirement that $\mathbb{A}(\boldsymbol{N})$ be positive definite is necessary for existence of an acceleration wave. It coincides with the condition of strong ellipticity of the equilibrium equations for an elastic micropolar medium [3].

3.2. **Transformation of the heat conductivity equation.** Now let us consider the influence of the temperature field on the existence of acceleration waves, and derive some relations for the jumps.

Applying Maxwell's theorem to the field of $\boldsymbol{q}$ and to the temperature gradient, we get

$$V\,[\![\nabla\boldsymbol{\cdot}\,\boldsymbol{q}]\!] = -\boldsymbol{N}\cdot[\![\dot{\boldsymbol{q}}]\!]\,, \quad [\![\nabla\,\nabla\,\theta]\!] = \boldsymbol{N}\otimes\boldsymbol{g}, \quad \left[\!\!\left[\dot{(\nabla\,\theta)}\right]\!\!\right] = -V\boldsymbol{g}, \tag{9}$$

where $\boldsymbol{g}$ is the vector amplitude of the jump in the second gradient of the temperature. Similar to (7), from (2) it follows that

$$[\![\nabla\boldsymbol{\cdot}\,\boldsymbol{q}]\!] = -\rho\theta\,[\![\dot{\eta}]\!]\,. \tag{10}$$

From (9) and (10) we have

$$\boldsymbol{N}\cdot[\![\dot{\boldsymbol{q}}]\!] = \rho\theta\,[\![\dot{\eta}]\!]\,. \tag{11}$$

Let us restrict our consideration to the constitutive equation for $\boldsymbol{q}$ by Fourier's law

$$\boldsymbol{q} = -\mathbf{k}(\theta)\cdot\nabla\,\theta, \quad \boldsymbol{h}\cdot\mathbf{k}(\theta)\boldsymbol{h} > 0, \quad \forall\boldsymbol{h}\neq\mathbf{0}, \tag{12}$$

where $\mathbf{k}$ is a positive definite thermoconductivity tensor.

Differentiating $(3)_3$ and (12) with respect to $t$ and using (11), we get

$$\boldsymbol{N}\cdot\boldsymbol{k}(\theta)\cdot\boldsymbol{g} = \rho\theta\left(\boldsymbol{N}\cdot\psi,_{\theta\mathbf{E}}\cdot\boldsymbol{a}' + \boldsymbol{N}\cdot\psi,_{\theta\mathbf{K}}\cdot\boldsymbol{b}'\right). \tag{13}$$

Now, using matrix notation, we can rewrite (8) and (13):

$$\mathbb{A}_\theta(\boldsymbol{N})\boldsymbol{\cdot}\boldsymbol{\zeta} = V^2\mathbb{B}_\theta\boldsymbol{\cdot}\boldsymbol{\zeta}, \tag{14}$$

where $\zeta = (\boldsymbol{a}', \boldsymbol{b}', \boldsymbol{g}) \in \mathbb{R}^9$, $\mathbb{A}_\theta$, and $\mathbb{B}_\theta$ are matrices with tensor components

$$\mathbb{A}_\theta(\boldsymbol{N}) \equiv \begin{bmatrix} \psi,_{\mathbf{EE}}\{\boldsymbol{N}\} & \psi,_{\mathbf{EK}}\{\boldsymbol{N}\} & \mathbf{0} \\ \psi,_{\mathbf{KE}}\{\boldsymbol{N}\} & \psi,_{\mathbf{KK}}\{\boldsymbol{N}\} & \mathbf{0} \\ -\rho\theta\boldsymbol{N}\cdot\psi,_{\theta\mathbf{E}} & -\rho\theta\boldsymbol{N}\cdot\psi,_{\theta\mathbf{E}} & \boldsymbol{N}\cdot\mathbf{k}(\theta) \end{bmatrix},$$

$$\mathbb{B}_\theta \equiv \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \gamma\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Thus, given Fourier's law for the heat flow vector, we reduce the problem of propagation of an acceleration wave in a thermoelastic micropolar medium to the spectrum problem (14). Namely, an acceleration wave exists only if (14) has nontrivial solutions and the eigenvalues for the problem (14) are real and positive.

3.3. **Existence of acceleration waves.** Let us consider the existence of positive solutions to the spectral problem (14).

Now $\mathbb{A}_\theta$ is not symmetric and $\mathbb{B}_\theta$ is not positive definite. However, there is a theorem which generalizes the Fresnel–Hadamard–Duhem theorem for an thermoelastic micropolar medium.

**Theorem 1.** *For any propagation directions defined by the vector $\boldsymbol{N}$, the homothermal acoustic numbers are real.*

*Proof.* The acoustic numbers are the squared speeds of propagation of acceleration waves. The spectral problem (14) splits into two problems, namely (8) and (13). $\mathbb{A}$ is symmetric as it is composed of the mixed derivatives of the free energy $\psi$. $\mathbb{B}$ is a symmetric matrix as well, and is positive definite. So the spectral problem (8) has only real-valued solutions. First solve (8). As $\mathbf{k}$ is a nonsingular tensor, from equation (13) we find a vector $\boldsymbol{g}$ that is uniquely defined.

Thus the theorem is proved under the assumptions of [3], which are supplemented by the requirement that the heat conductivity tensor $\mathbf{k}$ be positive definite. ☑

Similar to [3], we also have the following theorem.

**Theorem 2.** *The condition for existence of homothermal acceleration waves for all directions of propagations in a micropolar thermoelastic medium is equivalent to the condition of strong ellipticity of the equilibrium equations of the medium.*

*Proof.* Existence of acceleration waves means the spectral problem (14) has only positive eigenvalues for any $\boldsymbol{N}$, which means we must have $V^2 > 0$. As (14) is equivalent to (8) and (13), it is valid if and only if $\mathbb{A}$ is positive definite for any values of $\boldsymbol{N}$. Hence the inequality

$$\boldsymbol{\xi} \cdot \mathbb{A}(\boldsymbol{N}) \cdot \boldsymbol{\xi} > 0, \quad \forall \boldsymbol{N} : |\boldsymbol{N}| = 1, \quad \boldsymbol{\xi} \neq \mathbf{0} \tag{15}$$

must hold. This is an additional restriction imposed on the constitutive relations of a thermoelastic micropolar medium. As shown in [3], the inequality (15) is the strong ellipticity condition for the equilibrium equations for a micropolar medium. This completes the proof. ☑

**Remark.** Let us note that positive definiteness of $\mathbf{k}$ coincides with strong ellipticity of the steady-state thermoconductivity equation as well. Degeneration of one or both of $\mathbb{A}$ and $\mathbf{k}$ implies the possibility of existence of non-smooth solutions to the equilibrium equations or the steady-state thermoconductivity equation.

## 4. Conclusion

The conditions for existence of homothermal acceleration waves in a thermoelastic micropolar medium are established. It is shown that the conditions for existence of acceleration waves in a thermoelastic micropolar medium do not depend on the thermoconductivity defined by Fourier law. Hence the acceleration waves in a thermoelastic micropolar medium with Fourier's law of thermoconductivity propagate under the same conditions as in an elastic medium.

It is easy to see that the results of this paper extend to the case of a nonlinear thermoconductivity law having the form

$$\boldsymbol{q} = \boldsymbol{q}(\theta, \nabla\theta).$$

It is sufficient to require that

$$\boldsymbol{g} \cdot \boldsymbol{q}_{,\nabla\theta} \cdot \boldsymbol{g} < 0, \quad \text{for all} \quad \boldsymbol{g} \neq \mathbf{0}\,.$$

The case involving a more general form of the thermoconductivity law

$$\boldsymbol{q} = \boldsymbol{q}(\theta, \nabla\theta, \mathbf{E}, \mathbf{K})$$

requires special study, because the structure of its corresponding matrix $\mathbb{A}_\theta$ is more complex.

**Acknowledgements.** The authors are indebted to Michael J. Cloud (Lawrence Technological University) for assistance in the preparation of this paper.

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(Recibido en agosto de 2007. Aceptado en octubre de 2007)

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