https://doi.org/10.15446/recolma.v56n2.108379

Revista Colombiana de Matemáticas
Volumen 56(2022)2, páginas 189-211

On the Fischer matrices of a group of shape $2_+^{1+2n}{:}G$

Sobre las matrices de Fischer de un grupo de la forma $2_+^{1+2n}{:}G$

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Abstract. In this paper, the Fischer matrices of the maximal subgroup $\overline{G} = 2_+^{1+8}{:}(U_4(2){:}2)$ of $U_6(2){:}2$ will be derived from the Fischer matrices of the quotient group $Q = \frac{\overline{G}}{Z(2_+^{1+8})} \cong 2^8{:}(U_4(2){:}2)$, where $Z(2_+^{1+8})$ denotes the center of the extra-special 2-group 2_+^{1+8}. Using this approach, the Fischer matrices and associated ordinary character table of $\overline{G}$ are computed in an elegantly simple manner. This approach can be used to compute the ordinary character table of any split extension group of the form $2_+^{1+2n}{:}G$, $n \in \mathbb{N}$, provided the ordinary irreducible characters of 2_+^{1+2n} extend to ordinary irreducible characters of its inertia subgroups in $2_+^{1+2n}{:}G$ and also that the Fischer matrices $M(g_i)$ of the quotient group $\frac{2_+^{1+2n}{:}G}{Z(2_+^{1+2n})} \cong 2^{2n}{:}G$ are known for each class representative g_i in G.

Key words and phrases. split extension, extra-special p-group, irreducible projective characters, Schur multiplier, inertia factor groups, Fischer matrices.

2020 Mathematics Subject Classification. Primary: 20C15; Secondary: 20C40.

Resumen. En este artículo, las matrices de Fischer del subgrupo maximal $\overline{G} = 2^{1+8}+{:}(U_4(2){:}2)$ de $U_6(2){:}2$ serán derivadas a partir de las matrices de Fischer del grupo cociente $Q = \frac{\overline{G}}{Z(2^{1+8}+)} \cong 2^8{:}(U_4(2){:}2)$, donde $Z(2^{1+8}+)$ denota el centro del grupo 2-extra especial $2+^{1+8}$. Usando este enfoque, las matrices de Fischer y la tabla de caracteres asociadas de $\overline{G}$ son calculados de una manera elegante y simple. Este enfoque se puede utilizar para calcular la tabla de caracteres de cualquier extensión escindida de la forma $2_+^{1+2n}{:}G$, $n \in \mathbb{N}$, siempre y cuando los caracteres irreducibles ordinarios de 2_+^{1+2n} se extiendan a caracteres irreducibles ordinarios de sus subgrupos de inercia en $2_+^{1+2n}{:}G$

y también que las matrices de Fischer $M(g_i)$ del grupo cociente $\frac{2_+^{1+2n}:G}{Z(2_+^{1+2n})} \cong 2^{2n}:G$ sean conocidas para cada representante de clase g_i en G.

Palabras y frases clave. extensión escindida, p-grupo extra especial, caracteres proyectivos irreducibles, multiplicador de Schur, inertia factor groups, matrices de Fischer.

1. Introduction

The maximal subgroup $\overline{G} = 2_+^{1+8}{:}(U_4(2){:}2)$ (see [4]) of the automorphism group $U_6(2){:}2$ of the unitary simple group $U_6(2)$ is a split extension of the extra-special 2-group $N = 2_+^{1+8}$ by $G = U_4(2){:}2$. The center $Z(N) \cong 2$ is isomorphic to the cyclic group of order 2 and $N_1 = \frac{N}{Z(N)} \cong 2^8$ can be considered as an eight-dimensional $U_4(2){:}2$-module over the finite field $GF(2)$. In fact, up to isomorphism 2^8 afforded the unique representation of $U_4(2){:}2$ of degree eight over $GF(2)$ (see [9]).

Computing the table of marks within GAP it is noticed that there are 38 conjugacy classes of non-trivial subgroups of G having index less than 256. Hence G has 38 non-trivial subgroups G_i, where the degree of each of the permutation characters $\chi(G|G_i)$ of G acting on the classes of a subgroup G_i will be less than 256. Let $\chi(G|N_1)$ be the permutation character of G acting on the non-trivial classes of $N_1 = 2^8$. Then $\chi(G|N_1)$ will be the sum of some of these 38 permutation characters $\chi(G|G_i)$ such that for any non-trivial $g \in G$ it is required that $\chi(G|N_1)(g) = 2^k - 1$ for some $k \in \{0, 1, 2, 3, 4, 5, 6, 7\}$, i.e. the number $2^k - 1$ of elements of N_1 which is fixed by g. Using Exercise 4.2.4 in [11], it can be shown that for any element g in the conjugacy class $5A$ of G, we have that $\chi(G|N_1)(g) = 0$. Therefore, the only possibility for $\chi(G|N_1)$ will be the sum of two permutation characters $\chi(G|G_1)$ and $\chi(G|G_2)$ with degrees of 120 and 135, respectively. Hence G has three orbits on N_1 of lengths 1, 120 and 135. It is well known that $N_1 \cong V_8(2)$ (considered as a vector space of dimension 8 over $GF(2)$) and its dual space $N_1^*{:=}\mathrm{Hom}(N_1, \mathbb{C}^*)$ are isomorphic to each other. Since G has only one faithful irreducible eight-dimensional presentation over $GF(2)$ it follows that N_1 and N_1^* are also isomorphic as eight-dimensional modules for G over $GF(2)$. Moreover, N_1^* can be identified with set set $\mathrm{Irr}(N_1)$ and hence the action of G on the irreducible characters $\mathrm{Irr}(N_1)$ of N_1 will be the same as the action of G on N_1. Thus G has also three orbits of lengths 1, 120 and 135 on the 256 linear characters of N_1. Since the 256 linear characters of N come from N_1, G will also have three orbits on them with corresponding stabilizers H_1, H_2 and H_3 which have indices 1, 120 and 135 in $H_1 = G$. The last outstanding character of N is the unique faithful irreducible character θ_{257} of degree sixteen, which form on its own an orbit. Hence G has four orbits on the set $\mathrm{Irr}(N)$ and by checking the indices of the maximal subgroups of G in the ATLAS, the inertia factor groups corresponding to these orbits are identified as $H_1 = U_4(2){:}2$, $H_2 = 3_+^{1+2}{:}(2D_8)$, $H_3 = 2^4{:}S_4$ and $H_4 = U_4(2){:}2$.

Since G has 4 orbits on $\mathrm{Irr}(N)$ it follows that G will also has 4 orbits on the 257 conjugacy classes of elements of N. Hence under the action of G, N splits up into 4 conjugacy classes of $\overline{G}$. The first class contains the identity element, the second class the central element of order two, the third class $2^8 - 2^4 = 240$ elements of order 4 and the fourth class 270 elements of order two.

Having identified the inertia factor groups H_i, $i = 1, 2, 3, 4$, for the action of G on $\mathrm{Irr}(N)$ we proceed to use the technique of Fischer matrices (see [5] or [11]) to compute the ordinary irreducible characters of $\overline{G} = 2_+^{1+8}{:}(U_4(2){:}2)$. A summary of the method of Fischer matrices will be given in Section 2. In Section 3, the Fischer matrix $M(1A)$ of $\overline{G}$ corresponding to the identity class $1A$ of G will be computed and together with the decomposition of some ordinary irreducible characters of $U_6(2){:}2$ into the set $\mathrm{Irr}(N)$ it will be shown that the irreducible characters $\mathrm{Irr}(N)$ of N extend to ordinary irreducible characters of their inertia subgroups in $\overline{G}$. The quotient group $Q = \frac{\overline{G}}{Z(2_+^{1+8})} \cong 2^8{:}(U_4(2){:}2)$ is isomorphic to a subgroup $\overline{G}_1$ of $O_{10}^+(2)$ with shape $2^8{:}(U_4(2){:}2)$ (see [6]). The current author and others determined the Fischer matrices and ordinary character table of $\overline{G}_1$ in [6]. It will be discussed in Section 4 how each Fischer matrix $M(g)$ of $\overline{G}$ can be derived from the corresponding Fischer matrix $\widetilde{M}(g)$ of $\overline{G}_1$ by just adding a row and a column to $\widetilde{M}(g)$.

Note that $\overline{G}$ is the pre-image of the maximal subgroup $U_4(2){:}2$ of index 28 in $Sp_6(2)$ under the natural epimorphism modulo $N = 2_+^{1+8}$. Hence an isomorphic copy of $\overline{G}$ sits maximally inside the maximal subgroup $\overline{G}_2 = 2_+^{1+8}{:}Sp_6(2)$ of Co_2 (see [4]). In Section 4, the fusion map of the conjugacy classes of $U_4(2){:}2$ into $Sp_6(2)$ together with the permutation character of $\overline{G}_2$ on $\overline{G}$ and, if necessary, some of the ordinary irreducible characters of small degrees of $\overline{G}_2$ are restricted to the set $\mathrm{Irr}(\overline{G})$, to compute the orders of the elements of the conjugacy classes of $\overline{G}$ associated with each Fischer matrix $M(g)$ of $\overline{G}$. Note that the sizes of the centralizers of the classes of $\overline{G}$ coming from a coset Ng are easily determined by using the column orthogonality relation (see Section 2) of a Fischer matrix $M(g)$. Having obtained the conjugacy classes and Fischer matrices of $\overline{G}$ from each conjugacy class $[g]$ of G and together with the ordinary character tables of the inertia factor groups H_i, the ordinary character table of $\overline{G}$ (see Section 5) is constructed following the outline of the method discussed in Section 2. Using the algebra computer system GAP [8], the power maps of the elements of $\overline{G}$ are computed from the ordinary character table of $\overline{G}$ which was constructed in Section 5. Finally, the power maps of $\overline{G}$ and $U_6(2){:}2$ together with some restricted ordinary irreducible characters of $U_6(2){:}2$ to $\overline{G}$ are used to compute the fusion map of $\overline{G}$ into $U_6(2){:}2$.

Computations are carried out with the aid of the computer algebra systems MAGMA [3] and GAP and the notation of ATLAS is mostly followed. For an update on recent developments around Fischer matrices, interested readers are referred to the papers [17], [1], [2], [14], [15], [16] and [18].

The method used in this paper, to construct the Fischer matrices and ordinary character table of $\overline{G}$, works for any finite split extension of the form $\overline{S} = 2^{1+2n}{:}G_1$, $n \geq 1$, provided the ordinary irreducible characters of the extra-special 2-group 2^{1+2n} (of type "+" or type "-") extend to ordinary irreducible characters of their inertia subgroups $\overline{H_i}$ in $\overline{S}$. Furthermore, the Fischer matrices of the quotient group $\frac{\overline{S}}{Z(2^{1+2n})} \cong 2^{2n}{:}G$ are also known. In fact, this method can be extended to any extension group of the shape $\overline{E} = p^{1+2n}.G_1$, p a prime, if such a group $\overline{E}$ exists.

2. Theory of Fischer Matrices

Since the ordinary character table of $\overline{G} = 2_+^{1+8}{:}(U_4(2){:}2)$ will be constructed by the technique of Fischer matrices, a brief overview of this method is given as found in [20].

Let $\overline{G} = N.G$ be an extension of N by G, where N is normal in $\overline{G}$ and $\overline{G}/N \cong G$. Denote the set of all irreducible characters of a finite group G_1 by $\mathrm{Irr}(G_1)$. Also, define $\overline{H} = \left\{x \in \overline{G} | \theta^x = \theta\right\} = I_{\overline{G}}(\theta)$ as the inertia group of $\theta \in \mathrm{Irr}(N)$ in $\overline{G}$ then N is normal in $\overline{H}$. Let $\overline{g} \in \overline{G}$ be a lifting of $g \in G$ under the natural homomorphism $\overline{G} \longrightarrow G$ and $[g]$ be a conjugacy class of elements with representative g. Let $X(g) = \{x_1, x_2, \cdots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\overline{G}$ from the coset $N\overline{g}$ whose images under the natural homomorphism $\overline{G} \longrightarrow G$ are in $[g]$ and we take $x_1 = \bar{g}$. Now let $\theta_1 = 1_N, \theta_2, \cdots, \theta_t$ be representatives of the orbits of $\overline{G}$ on $\mathrm{Irr}(N)$ such that for $1 \leq i \leq t$, we have $\overline{H_i}$ with corresponding inertia factors H_i. By Gallagher [10] we obtain

$$\mathrm{Irr}(\overline{G}) = \bigcup_{i=1}^{t} \{(\psi_i\overline{\beta})^{\overline{G}} | \beta \in \mathrm{IrrProj}(H_i), \text{ with factor set } \alpha_i^{-1}\}$$

where ψ_i is a projective character of $\overline{H_i}$ with factor set $\overline{\alpha}_i$ such that $\psi_i \downarrow_N = \theta_i$. Note that $\overline{\beta}$ is a lifting for β into $\overline{H_i}$ and α_i is obtained from $\overline{\alpha_i}$. We have that $\overline{H_1} = \overline{G}$ and $H_1 = G$. Choose $y_1, y_2, .., y_r$ to be representatives of the α_i^{-1}-regular conjugacy classes of elements of H_i that fuse to $[g]$ in G. We define

$$R(g) = \{(i, y_k) \,|\, 1 \leq i \leq t, H_i \cap [g] \neq \varnothing, 1 \leq k \leq r\}$$

and we note that y_k runs over representatives of the α_i^{-1}-regular conjugacy classes of elements of H_i which fuse into $[g]$ in G. We define $y_{l_k} \in \overline{H_i}$ such that y_{l_k} ranges over all representatives of the conjugacy classes of elements of $\overline{H}_i$ which map to y_k under the homomorphism $\overline{H_i} \longrightarrow H_i$ whose kernel is N.

Lemma 2.1. With notation as above,

$$(\psi_i\overline{\beta})^{\overline{G}}(x_j) = \sum_{y_k:(i,y_k)\in R(g)} \beta(y_k) \sum_{l}{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|} \psi_i(y_{l_k})$$

Proof. See [20] ☑

Then the Fischer matrix $M(g) = \left(a^{j}_{(i,y_k)}\right)$ is defined as

$$\left(a^{j}_{(i,y_k)}\right) = \left(\sum_{l}{}^{'} \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|}\psi_i(y_{l_k})\right),$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$ and where $\sum_l'$ is the summation over all l for which $y_{l_k} \sim x_j$ in $\overline{G}$. So, we can write Lemma 2.1 as

$$(\psi_i\overline{\beta})^{\overline{G}}(x_j) = \sum_{y_k:(i,y_k)\in R(g)} a^{j}_{(i,y_k)}\beta(y_k).$$

The Fischer $M(g)$ (see Figure 1) is partitioned row-wise into blocks, where each block corresponds to an inertia group $\overline{H}_i$. We write $|C_{\overline{G}}(x_j)|$, for each $x_j \in X(g)$, at the top of the columns of $M(g)$ and at the bottom we write $m_j \in \mathbb{N}$, where we define $m_j = [C_{\overline{g}}{:}C_{\overline{G}}(x_j)]{=}|N|\frac{|C_G(g)|}{|C_{\overline{G}}(x_j)|}$ and $C_{\overline{g}} = \{x \in \overline{G} | x(N\overline{g}) = (N\overline{g})x\}$. On the left of each row we write $|C_{H_i}(y_k)|$, where the α_i^{-1}-regular class $[y_k]$ fuses into the class $[g]$ of G. Then in general we can write $M(g)$ with corresponding weights for rows and columns as follows, where blocks $M_i(g)$ corresponding to the inertia groups $\overline{H_i}$ are separated by horizontal lines.

$$M(g) = \begin{array}{c} \\ |C_G(g)| \\ |C_{H_2}(y_1)| \\ |C_{H_2}(y_2)| \\ \vdots \\ |C_{H_i}(y_1)| \\ |C_{H_i}(y_2)| \\ \vdots \\ |C_{H_t}(y_1)| \\ |C_{H_t}(y_2)| \\ \vdots \\ \\ \end{array} \begin{array}{cccc} |C_{\overline{G}}(x_1)| & |C_{\overline{G}}(x_2)| & \cdots & |C_{\overline{G}}(x_{c(g)})| \\ a^{1}_{(1,g)} & a^{2}_{(1,g)} & \cdots & a^{c(g)}_{(1,g)} \\ \hline a^{1}_{(2,y_1)} & a^{2}_{(2,y_1)} & \cdots & a^{c(g)}_{(2,y_1)} \\ a^{1}_{(2,y_2)} & a^{2}_{(2,y_2)} & \cdots & a^{c(g)}_{(2,y_2)} \\ \vdots & \vdots & \vdots & \vdots \\ \hline a^{1}_{(i,y_1)} & a^{2}_{(i,y_1)} & \cdots & a^{c(g)}_{(i,y_1)} \\ a^{1}_{(i,y_2)} & a^{2}_{(i,y_2)} & \cdots & a^{c(g)}_{(i,y_2)} \\ \vdots & \vdots & \vdots & \vdots \\ \hline a^{1}_{(t,y_1)} & a^{2}_{(t,y_1)} & \cdots & a^{c(g)}_{(t,y_1)} \\ a^{1}_{(t,y_2)} & a^{2}_{(t,y_2)} & \cdots & a^{c(g)}_{(t,y_2)} \\ \vdots & \vdots & \vdots & \vdots \\ \hline m_1 & m_2 & \cdots & m_{c(g)} \end{array}$$

FIGURE 1. The Fischer Matrix $M(g)$

In practice we will never compute the y_{l_k} or the ordinary irreducible character tables of the inertia subgroups $\overline{H}_i$. The reason for this is that the ordinary irreducible characters of the $\overline{H}_i$ are in general much larger and more complicated to compute than the one for $\overline{G}$. Instead of using the above formal definition of a Fischer matrix $M(g)$, the arithmetical properties of $M(g)$ below are used to compute the entries of $M(g)$ (see [12]).

(a) $a^j_{(1,g)} = 1$ for all $j = \{1, 2, .., c(g)\}$.

(b) $|X(g)| = |R(g)|$.

(c) $\sum_{j=1}^{c(g)} m_j \, a^j_{(i,y_k)} \, \overline{a^j_{(i',y'_k)}} = \delta_{(i,y_k),(i',y'_k)} \frac{|C_G(g)|}{|C_{H_i}(y_k)|} |N|$.

(d) $\sum_{(i,y_k)\in R(g)} a^j_{(i,y_k)} \, \overline{a^{j'}_{(i,y_k)}} \, |C_{H_i}(y_k)| = \delta_{jj'} \, |C_{\overline{G}}(x_j)|$.

(e) $M(g)$ is square and nonsingular.

If N is elementary abelian, then we obtain the following additional properties of $M(g)$:

(f) $a^1_{(i,y_k)} = \frac{|C_G(g)|}{|C_{H_i}(y_k)|}$.

(g) $|a^1_{(i,y_k)}| \geq |a^j_{(i,y_k)}|$.

(h) $a^j_{(i,y_k)} \equiv a^1_{(i,y_k)} (\mathrm{mod}\ p)$, if $|N| = p^n$, for p a prime and $n \in \mathbb{N}$

The matrix $M(g)$ is square, where the number of rows is equal to the number of α_i^{-1}- regular classes of the inertia factors H_i's, $1 \leq i \leq t$, which fuse into $[g]$ in G and the number of columns is equal to the number $c(g)$ of conjugacy classes of $\overline{G}$ which is obtained from the coset $N\overline{g}$. Then the partial character table of $\overline{G}$ on the classes $\{x_1, x_2, \cdots, x_{c(g)}\}$ is given by

$$\begin{bmatrix} C_1(g)\, M_1(g) \\ C_2(g)\, M_2(g) \\ \vdots \\ C_t(g)\, M_t(g) \end{bmatrix}$$

where the Fischer matrix $M(g)$ (see Figure 1) is divided into blocks $M_i(g)$ with each block corresponding to an inertia group $\overline{H}_i$ and $C_i(g)$ is the partial character table of H_i with factor set α_i^{-1} consisting of the columns corresponding to the α_i^{-1}-regular classes that fuse into $[g]$ in G. We obtain the characters of $\overline{G}$ by multiplying the relevant columns of the projective characters of H_i with factor set α_i^{-1} by the rows of $M(g)$. We can also observe that

$$|\mathrm{Irr}(\overline{G})| = \sum_{i=1}^{t} |\mathrm{IrrProj}(H_i, \alpha_i^{-1})|.$$

3. On the type of characters of the inertia factors H_i

The group $\overline{G} = 2_{+}^{1+8}{:}(U_4(2){:}2)$ can be regarded as the 2-fold cover $Z(N)^{\cdot}(2^8{:}(U_4(2){:}2))$ for the group $\overline{G}_1 = 2^8{:}(U_4(2){:}2)$. Using Fischer-Clifford theory we noticed that $\overline{G}_1$ will have two orbits on $Z(N) = Z(\overline{G}) \cong 2$, with one orbit containing the identity element $1_{Z(N)}$ and the second orbit the central element z of order two. Hence the orbits will have both $\overline{G}_1$ as their respective set stabilizer. By Lemma 5.2 in [7], $\overline{G}_1$ will also have two orbits on $\mathrm{Irr}(Z(N))$ containing one element each and with corresponding inertia factors $H_1 = H_2 = \overline{G}_1$. To construct the ordinary character table of $2^{\cdot}(2^8{:}(U_4(2){:}2))$ via the technique of Fischer matrices, we will then require the ordinary character table of H_1 and an irreducible projective character table of H_2 with factor set α of order two. The ordinary character table of $\overline{G}_1 = 2^8{:}(U_4(2){:}2)$ was constructed in [6] using Fischer matrices and it was shown that $U_4(2){:}2$ acts irreducibly on its unique eight-dimensional module 2^8, where $U_4(2){:}2$ has three orbits of lenghts 1, 120 and 135 on $\mathrm{Irr}(2^8)$ with corresponding inertia factor groups $U_4(2){:}2$, $3_{+}^{1+2}{:}(2D_8)$ and $2^4{:}S_4$. Since $\overline{G}_1$ is a split extension and 2^8 elementary abelian, only the ordinary character tables of the inertia factor groups of 2^8 in $\overline{G}_1$ were used in construction the set $\mathrm{Irr}(\overline{G_1})$ as a consequence of Mackey's Theorem in [10]. Hence we will also use the ordinary character tables of the inertia factors $H_1 = U_4(2){:}2$, $H_2 = 3_{+}^{1+2}{:}(2D_8)$ and $H_3 = 2^4{:}S_4$ of $\mathrm{Irr}(2_{+}^{1+8})$ in $\overline{G}$ to construct the ordinary character table of $\overline{G} = 2_{+}^{1+8}{:}(U_4(2){:}2)$ using Fischer matrices. To determine which type of irreducible characters (ordinary or projective) will be used for $H_4 = U_4(2){:}2$, we will use the Fischer matrix $M(1_G)$ together with decompositions of some ordinary characters of small degrees of $U_6(2){:}2$ into the ordinary irreducible characters of N. We can add here that for the first $c(g)-1$ rows of each Fischer matrix $M(g)$ of size $c(g)$ the properties (f), (g) and (h) found in Section 2 are also applicable since 2^8 is elementary abelian.

Having obtained the inertia factors $H_1 = U_4(2){:}2$, $H_2 = 3_{+}^{1+2}{:}(2D_8)$, $H_3 = 2^4{:}S_4$ and $H_4 = U_4(2){:}2$ for the action of G on $\mathrm{Irr}(N)$, we can form the Fischer matrix $M(1A)$ corresponding to the identity coset $N1_G = N$ as below. Properties (a) and (f) (see Section 2) were used to find the entries for the first row and the first three entries of the first column of $M(1A)$.

$$
\begin{array}{cc}
 & \begin{array}{cccc} 26542080 & 26542080 & 110592 & 98304 \end{array} \\
M(1A) = \begin{array}{c} 51840 \\ 432 \\ 384 \\ 51840 \end{array} & \left(\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 120 & a & b & c \\ 135 & d & e & f \\ g & h & i & j \end{array} \right) \\
 & \begin{array}{cccc} 1 & 1 & 240 & 270 \end{array}
\end{array}
$$

The column weights above the matrix $M(1A)$ are the centralizer orders $|C_{\overline{G}}(x_j)|$ of the four classes $[x_j]$ of $\overline{G}$ coming from the identity coset N and the weights below are the values m_j. Whereas, the row weights to the left of the matrix $M(1A)$ represent the centralizer orders $|C_{H_i}(1A)|$ of the inertia factors

H_i on the identity element $1A$. Applying the remaining Fischer matrix $M(g)$ properties in Section 2 to the above matrix, the entries of $M(1A)$ are completed and shown below.

$$M(1A) = \begin{array}{c} \\ 51840 \\ 432 \\ 384 \\ 51840 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 26542080 & 26542080 & 110592 & 98304 \end{array} \\ \left(\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 120 & 120 & 8 & \text{-}8 \\ 135 & 135 & \text{-}9 & 7 \\ 16 & \text{-}16 & 0 & 0 \end{array} \right) \\ \begin{array}{cccc} 1 & 1 & 240 & 270 \end{array} \end{array}$$

$[x]_{\overline{G}}$	1A	$[x_2]$	$[x_3]$	$[x_4]$
$\lvert C_{\overline{G}}(x)\rvert$	26542080	26542080	110592	98304
χ_1	1	1	1	1
χ_{26}	120	120	8	-8
χ_{40}	135	135	-9	7
χ_{60}	$16c$	$-16c$	0	0

TABLE 1. The partial character table of $\overline{G}$ for coset N

Table 1 is the partial ordinary character table of $\overline{G}$ on the classes $1A$, $[x_2]$, $[x_3]$ and $[x_4]$ of $\overline{G}$ coming from N , where each of the 4 lines of Table 1 corresponds to the first row of entries of the sub-matrices $C_i(1A)M_i(1A), i = 1, 2, 3, 4$. $M_i(1A)$ and $C_i(1A)$ correspond to the rows of the Fischer-Clifford matrix $M(1A)$ and columns of the character tables (ordinary or projective) of the inertia factors H_i, respectively, which are associated with the classes $[1A]_{H_i}$ of the inertia factors H_i which fuse into the class $[1A]_G$ of G. Also, note that the character values in the 1st column of Table 1 are the degrees of the ordinary irreducible characters $\chi_1, \chi_{26}, \chi_{40}$ and χ_{60} of $\overline{G}$. The characters $\chi_1, \chi_{26}, \chi_{40}$ and χ_{60} occupy the first position for each block of characters coming from an inertia subgroup $\overline{H_i}$ of $\overline{G}$. Also note that $\overline{H_1}$, $\overline{H_2}$ and $\overline{H_3}$ will contribute 25, 14 and 20 ordinary irreducible characters, respectively, towards the set $\mathrm{Irr}(\overline{G})$. The reason for this is that it was found earlier that we will use the ordinary irreducible characters of the inertia factors H_1, H_2 and H_3 in the construction of the set $\mathrm{Irr}(\overline{G})$ and it only remains to determine whether we will use the ordinary character table of H_4 or the set $\mathrm{IrrProj}(H_4, \alpha)$ of irreducible projective characters with factor set α of order 2. It can be readily being verified in GAP that the Schur multiplier $M(H_4) \cong 2$ of H_4 is a cyclic group of order 2 and hence will have two sets of irreducible projective character tables, i.e. $\mathrm{Irr}(H_4)$ and $\mathrm{IrrProj}(H_4, \alpha)$. Now $\deg(\chi_1) = 1$, $\deg(\chi_{26}) = 120$ and $\deg(\chi_{40}) = 135$ and $\deg(\chi_{60}) = 16c$ (see Table 1) are the degrees of the ordinary irreducible characters of $\overline{G}$ which occupy the first position in each block of the set $\mathrm{Irr}(\overline{G})$ which corresponds to the inertia groups $\overline{H_1}$, $\overline{H_2}$, $\overline{H_3}$ and $\overline{H_4}$. The number c is the degree of one of the irreducible characters which is contained in either $\mathrm{Irr}(H_4)$ or $\mathrm{IrrProj}(H_4, \alpha)$.

A small part of the ordinary character table of $U_6(2){:}2$ (see ATLAS or GAP library) is found in Table 2, which contains the values of the irreducible characters $22a$ and $231a$ on the classes $1A$, $2A$, $2B$, $2D$ and $4A$ of $U_6(2){:}2$.

$[y]_{U_6(2):2}$	$1A$	$2A$	$2B$	$2D$	$4A$
$\lvert C_{U_6(2):2}(y)\rvert$	18393661440	26542080	294912	2903040	110592
$22a$	22	-10	6	8	6
$231a$	231	39	7	21	23

Table 2. The partial character table of $U_6(2){:}2$

Taking into account how the centralizer orders of the classes $1A$, $[x_2]$, $[x_3]$ and $[x_4]$ of $\overline{G}$ (see Table 1) can divide those of the classes of $U_6(2){:}2$, we obtain that the classes $1A$, $2A$, $2B$, $2D$ and $4D$ of $U_6(2){:}2$ (see Table 2) are the only candidates for the classes $1A$, $[x_2]$, $[x_3]$ and $[x_4]$ of $\overline{G}$ to fuse into. Notice that $[x_4]$ can either fuse into $2A$ or $2B$ of $U_6(2){:}2$. Now it is obvious that the other two non-trivial classes $[x_2]$ and $[x_3]$ of $\overline{G}$ will fuse into the classes $2A$ and $4A$ of $U_6(2){:}2$, respectively. Suppose that $[x_4]$ will fuse into $2A$ of $U_6(2){:}2$, then the inner product $< (22a)_N, 1_N >_N = -2,4375$ of the restriction of $22a \in \mathrm{Irr}(U_6(2){:}2)$ to N with the identity character 1_N of N will give us a negative rational number which is impossible. Now, if we assume that $[x_4]$ will fuse into $2B$ of $U_6(2){:}2$ then $< (22a)_N, 1_N >_N = 6$ and this shows that $[x_4]$ will fuse into $2B$ of $U_6(2){:}2$. See Table 3 for the fusion map of classes of $\overline{G}$ coming from the identity coset N into the classes of $U_6(2){:}2$.

$\lvert C_{\overline{G}}(x_j)\rvert$	$[x_j]_{\overline{G}}$ $\longrightarrow$	$[y]_{U_2(2):2}$	$\lvert C_{U_6(2):2}(y)\rvert$
26542080	$1A$	$1A$	18393661440
26542080	$[x_2]$	$2A$	26542080
110592	$[x_3]$	$4A$	110592
98304	$[x_4]$	$2B$	294912

Table 3. The fusion map of classes of $\overline{G}$ from N into classes of $U_6(2){:}2$

By obtaining the orders of the elements in the classes $[x_2]$, $[x_3]$ and $[x_4]$ of $\overline{G}$ and also their fusion into $U_6(2){:}2$ we can now proceed to decompose the ordinary irreducible character $22a$ of $U_6(2){:}2$ with degree of 22 into the set $\mathrm{Irr}(N)$ which is represented in Table 1. Now

$$\begin{aligned}(22a)_N =& < (22a)_N, 1_N > (\chi_1)_N + < (22a)_N, (\chi_{26})_N > (\chi_{26})_N + < (22a)_N, (\chi_{40})_N > (\chi_{40})_N + \\ & < (22a)_N, (\chi_{60})_N > (\chi_{60})_N = 6 \times 1_N + 0 \times (\chi_{26})_N + 0 \times (\chi_{40})_N + c \times (\chi_{60})_N \\ &= 6 \times 1_N + c \times (\chi_{60})_N\end{aligned}$$

Since the deg($22a$)=22=6deg(1_N)+$c$deg(χ_{60}) =6(1)+c(16c)=6+16c^2, it follows that $c = 1$ because c is the degree of one of the irreducible characters belonging either to $\mathrm{Irr}(H_4)$ or $\mathrm{IrrProj}(H_4, \alpha)$. Therefore it shows that we will use the ordinary irreducible character table of H_4. Hence each of the irreducible characters of $\mathrm{Irr}(N)$ extends to an ordinary irreducible character of its inertia group $\overline{H_i}$.

4. Fischer matrices and conjugacy classes of $\overline{G}$

In this section, the Fischer matrices and the conjugacy classes of $\overline{G}$ will be determined from those of a subgroup $\overline{G}_1$ of $O^+_{10}(2)$ with shape $2^8{:}(U_4(2){:}2)$ (see [6]) which is an isomorphic copy of the quotient group $Q = \frac{\overline{G}}{Z(2^{1+8}_+)}$. Also, the fusion of $\overline{G}$ into the group $\overline{G}_2$ will help to determine the orders of the classes of $\overline{G}$.

In [11] and [13] the Fischer matrices of the maximal subgroups $\overline{G}_2 = 2^{1+8}_+{:}Sp_6(2)$ and $\overline{G}_3 = 2^{1+22}_+{\cdot}Co_2$ of the sporadic simple groups Co_2 and the Baby Monster B, respectively, were computed. It was mentioned in these publications (see also Remark 7 in [2]) that the Fischer matrices of their quotients groups $Q_2 = \frac{\overline{G}_1}{Z(2^{1+8}_+)} \cong 2^8{:}Sp_6(2)$ and $Q_3 = \frac{\overline{G}_3}{Z(2^{1+22}_+)} \cong 2^{22}.Co_2$ (see proof of Lemma 7 in [13]) can be obtained by removing the first column and last row of each Fischer matrix of $\overline{G}_2$ and $\overline{G}_3$. In both cases, as in our case, the ordinary irreducible characters of 2^{1+8}_+ and 2^{1+22}_+ extend to ordinary irreducible characters of their inertia subgroups in $\overline{G}_2$ and $\overline{G}_3$. Therefore, only the ordinary character tables of the inertia factors are involved in the construction of the character tables of $\overline{G}_2$ and $\overline{G}_3$. Since the action of our group $\overline{G}_1$ on $\mathrm{Irr}(2^{1+8}_+)$ follows a similar pattern as the actions of $\overline{G}_2$ and $\overline{G}_3$ on $\mathrm{Irr}(2^{1+8}_+)$ and $\mathrm{Irr}(2^{1+22}_+)$, respectively, the results obtained in [11] and [13] will be applicable to $\overline{G}$. Also, an isomorphic copy of $\overline{G}$ sits maximally inside $\overline{G}_2$ and so, the Fischer matrices of $\overline{G}$ can be obtained by adding a first row and a last column to the Fischer matrices of $\overline{G}_1$. The nature of these rows and columns are described in the two lemmas below which were taken from [13] and adjusted for $\overline{G}$. The proofs of these lemmas for the case of $\overline{G}$ will follow the exact pattern as that for $\overline{G}_3$ [13] with differences in notation. For the notation use in Lemma 4.1 and Lemma 4.2 the reader is referred to Section 2 of this paper.

Lemma 4.1. *For every $c(g) \times c(g)$ Fischer matrix $M(g)$ of $\overline{G}$ the sum of the first $c(g) - 1$ rows equals the (componentwise) square of the last row.*

Proof. See proof of Lemma 6 of [13]. ☑

Lemma 4.2. *For each $M(g)$ of $\overline{G}$, the x_j's in the set $X(g)$ (in Section 2) can be ordered in such a way that the last row of each $M(g)$ is of the form $[q_j, -q_j, 0, ..., 0]$ with q_j a power of 2 and we may choose $x_2 = zx_1$ with z the central involution in $\overline{G}$. Also $a^1_{(i,y_k)} = a^2_{(i,y_k)} = \frac{|C_G(g)|}{|C_{H_i}(y_k)|}$ for $1 \le i \le 3$, $1 \le k \le r$.*

Proof. See proof of Lemma 7 of [13]. ☑

From Lemma 4.1 and Lemma 4.2 the first two columns and last row of each matrix $M(g)$ of $\overline{G}$ are known and so are the values of all $\chi \in \mathrm{Irr}(\overline{G})$ on the classes $[x_1]$ and $[x_2]$ of $\overline{G}$ coming from a coset Ng. Note that the character $\chi_{60} \in \mathrm{Irr}(\overline{G})$ in Table 1 is the extension of the unique faithful irreducible character θ_{257} of N of degree sixteen. Also, χ_{60} is the equivalent of the character $\eta \in \mathrm{Irr}(\overline{G}_3)$ used in the proofs of Lemma 6 and Lemma 7 in [13]. Moreover, $\left(\chi_{60}^2\right)_N = \theta_{257}^2$ is the lifting of the regular character of $N/Z(N) \cong N_1$ and hence the sum of the 256 linear characters of N. Observe that

$$\chi_{60}^2 = \chi_1 + \chi_{26} + \chi_{40},$$

where χ_1, χ_{26} and χ_{60} (see Table 1) are the extensions ψ_i, $i = 1, 2, 3$, of the representatives of the three orbits of $\overline{G}$ on the linear characters of N to their respective inertia groups $\overline{H_i}$, which are induced to $\overline{G}$. This shows that ψ_i, $i = 1, 2, 3$, are uniquely determined linear characters of the inertia subgroups $\overline{H_i}$, $i = 1, 2, 3$, of $\overline{G}$ and hence we will not consider any projective characters of the inertia groups $\overline{H_i}$ in the construction of the ordinary character table of $\overline{G}$ (as it was established in Section 3). In addition, the ordinary irreducible character χ_{60} of $\overline{G}$ is made completely known by Lemma 4.2 and therefore also all the faithful irreducible characters of $\overline{G}$.

Since $\overline{G}_1$ is a split extension of an elementary abelian group 2^8 by $U_4(2){:}2$ we have that if the first column and last row of a Fischer matrix $M(g)$ of $\overline{G}$ is removed then we are left with the Fischer matrix $\widetilde{M}(g)$ of $\overline{G}_1$. Having computed the Fischer matrices of $\overline{G}_1$ in Table 4 of [6] , we can just add to the Fischer matrices of $\overline{G}_1$ a first column and a last row (as described in Lemma 4.2) to obtain the Fischer matrices of $\overline{G}$. The fusion maps for the inertia factors H_2 and H_3 into H_1 are available in [6]. For example, consider the Fischer matrix $\widetilde{M}(2D)$ of $\overline{G}_1$ corresponding to the coset $N_1 g$ of 2^8 in $\overline{G}_1$, where g is a representative of the class $2D$ of involutions in $U_4(2){:}2$ (see [6]). The coset $N_1 g$ splits into four classes $\{2H, 4F, 4G, 4H\}$ of $\overline{G}_1$ with their respective centralizer orders indicated in the row above the matrix $\widetilde{M}(2D)$.

$$\widetilde{M}(2D) = \begin{array}{c} \\ 96 \\ 12 \\ 96 \\ 12 \end{array} \begin{array}{c} \begin{array}{cccc} 1536 & 1536 & 256 & 192 \end{array} \\ \left(\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 8 & -8 & 0 & 0 \\ 1 & 1 & 1 & -1 \\ 6 & 6 & -2 & 0 \end{array}\right) \end{array}$$

Using Lemma 4.2, a first column and a last row are inserted to the matrix $\widetilde{M}(2D)$ to obtained the required Fischer matrix $M(2D)$ (see below) of $\overline{G}$ corresponding to the class $2D$ of $U_4(2){:}2$.

$$M(2D) = \begin{array}{c} \\ 96 \\ 12 \\ 96 \\ 12 \\ 96 \end{array} \begin{array}{c} \begin{array}{ccccc} 3072 & 3072 & 1536 & 256 & 192 \end{array} \\ \left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 8 & 8 & -8 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 6 & 6 & 6 & -2 & 0 \\ 4 & -4 & 0 & 0 & 0 \end{array} \right) \end{array}$$

Notice that the coset Ng corresponding to the class $2D$ of $U_4(2){:}2$ splits into five classes $[x_1]$, $[x_2]$, $[x_3]$, $[x_4]$ and $[x_5]$ of $\overline{G}$ with their centralizer orders found in the row on top of the matrix $M(2D)$, respectively. These centralizer orders $|C_{\overline{G}}(x_j)|$ were computed using the column orthogonality relation (d) for Fischer matrices in Section 2. The next step is to find the orders of the elements contain in the five classes. For this purpose, we will make use of the permutation character $\chi(\overline{G}_2|\overline{G}) = 1a + 27a$ of $\overline{G}_2$ on the conjugacy classes of $\overline{G}$ together with Proposition 7.5.1 in [12]. Moreover, the elements of the above five classes will have orders $o(x_j) \in \{2, 4, 8\}$, since 2_+^{1+8} is an extra-special 2-group.

The class of involutions $2D$ of $U_4(2){:}2$ is the only conjugacy class of $U_4(2){:}2$ that is fusing into the class of involutions $2D$ of $Sp_6(2)$ (see Table 7.7 in [12]). Then it follows from Proposition 7.5.1 in [12] that the classes of $\overline{G}$ coming from the coset Ng, $g \in 2D$ of $U_4(2){:}2$, will fuse into the classes of $\overline{G}_2$ coming from the coset Ng, $g \in 2D$ of $Sp_6(2)$. In Example 3.8.17 of [11], the technique of Fischer matrices is applied to the group $\overline{G}_2$ which is a maximal subgroup of the Conway sporadic simple group Co_2. The computation of the Fischer matrix of $\overline{G}_2$ corresponding to the class $2D$ of $Sp_6(2)$ is left as Exercise 3.8.3 in [11]. Using a permutation representation of $\overline{G}_2$ obtained from the online ATLAS [21] and a similar GAP routine as used in [16], the five classes $[y_j]$, $j = 1, 2, ..., 5$, of $\overline{G}_2$ corresponding to the coset Ng, $g \in 2D$ of $Sp_6(2)$ are computed and the information about them are shown in Table 4 below. Taking in consideration, the sizes $|C_{\overline{G}}(x_j)|$ of the centralizers of the elements in the classes $[x_j]$, $j =, 1, 2, .., 5$, of $\overline{G}$ and those of the corresponding classes $[y_j]$ of $\overline{G}_2$ together with the values of the permutation character $\chi(\overline{G}_2|\overline{G})$ of $\overline{G}_2$ on the classes of $\overline{G}$, we deduce that the orders $o(x_j)$ of the elements in the classes $[x_1]$, $[x_2]$ and $[x_4]$ will be all 4 whereas the orders of elements in conjugacy classes $[x_3]$ and $[x_5]$ will be 2 and 8 respectively. All of the above-mentioned information is summarized in Table 4 below.

$[y_j]_{\overline{G}_2}$	$[y_1]$	$[y_2]$	$[y_3]$	$[y_4]$	$[y_5]$
$o(y_j)$	4	4	2	4	8
$\|C_{\overline{G}_2}(y_j)\|$	12288	12288	6144	1024	768
$[x_j]_{\overline{G}}$	$[x_1]$	$[x_2]$	$[x_3]$	$[x_4]$	$[x_5]$
$o(x_j)$	4	4	2	4	8
$\|C_{\overline{G}}(x_j)\|$	3072	3072	1536	256	192
$\chi(\overline{G}_2\|\overline{G})$	4	4	4	4	4

TABLE 4. The orders $o(x_j)$ of elements of $\overline{G}$ from the coset Ng, $g \in 2D$

In a similar manner, as described above, we obtained the conjugacy classes $[x_j]$ and the Fischer matrices $M(g_i)$ of $\overline{G}$ corresponding to the remaining classes $[g_i]$ of G and this information is listed in Table 5 and Table 6, respectively.

$[g]_G$	$\|C_G(g)\|$	$[x]_{\overline{G}}$	2P	3P	5P	$\|C_{\overline{G}}(x)\|$	$\rightarrow [y]_{U_6(2):2}$	$[g]_G$	$\|C_G(g)\|$	$[x]_{\overline{G}}$	2P	3P	5P	$\|C_{\overline{G}}(x)\|$	$\rightarrow [y]_{U_6(2):2}$
1A	51840	1A	1A	1A	1A	26542080	1A	2A	1440	2C	1A	2C	2C	46080	2D
		2A	1A	2A	2A	26542080	2A			2D	1A	2D	2D	46080	2E
		4A	2A	4A	4A	110592	4A			4B	2B	4B	4B	1536	4H
		2B	1A	2B	2B	98304	2B								
2B	1152	2E	1A	2E	2E	147456	2B	2C	192	2H	1A	2H	2H	6144	2B
		2F	1A	2F	2F	147456	2A			2I	1A	2I	2I	6144	2C
		4C	2A	4C	4C	12288	4B			4E	2B	4E	4E	1024	4C
		2G	1A	2G	2G	8192	2C			4F	2B	4F	4F	512	4F
		4D	2B	4D	4D	1536	4E			4G	2B	4G	4G	512	4D
2D	96	4H	2B	4H	4H	3072	4H	3A	648	6A	3A	2A	6A	5184	6A
		4I	2B	4I	4I	3072	4G			3A	3A	1A	3A	5184	3B
		2J	1A	2J	2J	1536	2E			12A	6A	4A	12A	864	12A
		4J	2B	4J	4J	256	4I								
		8A	4A	8A	8A	192	8D								
3B	216	3B	3B	1A	3B	432	3C	3C	108	3C	3C	1A	3C	3456	3A
		6B	3B	2A	6B	432	6E			6C	3C	2A	6C	3456	6B
										12B	6C	4A	12B	288	12B
										6D	3C	2B	6D	192	6D
4A	96	4K	2H	4K	4K	768	4I	4B	96	4M	2F	4M	4M	3072	4A
		4L	2H	4L	4L	768	4G			4N	2F	4N	4N	3072	4B
		8B	4E	8B	8B	128	8G			4O	2E	4O	4O	1536	4C
										4P	2E	4P	4P	768	4D
										8C	4C	8C	8C	128	8A
4C	32	4Q	2H	4Q	4Q	256	4H	4D	16	4S	2H	4S	4S	128	4F
		4R	2H	4R	4R	256	4I			4T	2H	4T	4T	128	4E
		8D	4E	8D	8D	128	8F			8F	4E	8F	8F	64	8B
		8E	4E	8E	8E	164	8E			8G	4G	8G	8G	32	8C
5A	10	5A	5A	5A	1A	20	5A	6A	72	6E	3A	2E	6E	576	6C
		10A	5A	10A	2A	20	10A			6F	3A	2F	6F	576	6A
										12C	6A	4C	12C	96	12C
6B	36	6G	3C	2D	6G	288	6J	6C	36	6I	3B	2F	6I	72	6E
		6H	3C	2C	6H	288	6H			6J	3B	2E	6J	72	6F
		12D	6D	4B	12D	48	12I								
6D	36	6K	3B	2C	6K	72	6K	6E	36	6M	3C	2E	6M	288	6D
		6L	3B	2D	6L	72	6L			6N	3C	2F	6N	288	6B
										12E	6D	4D	12E	48	12F
6F	24	6O	3B	2H	6O	48	6F	6G	12	12F	6D	4I	12F	96	12G
		6P	3B	2I	6P	48	6G			12G	6D	4H	12G	96	12I
										6Q	3C	2J	6Q	48	6J
										24A	12B	8A	24A	24	24A
8A	8	8H	4O	8H	8H	64	8F	9A	9	9A	9A	3A	9A	18	9A
		8I	4O	8I	8I	64	8G			18A	9A	6A	18A	18	18A
		16A	8C	16A	16B	32	16A								
		8J	4M	8J	8J	32	8D	10A	10	10B	5A	2D	10B	20	10C
		16B	8C	16B	16A	32	16B			10C	5A	2C	10C	20	10B
12A	12	12H	6O	4L	12H	24	12J	12B	12	12J	6F	4N	12J	96	12C
		12I	6O	4K	12I	24	12K			12K	6F	4M	12K	96	12A
										12L	6E	4P	12N	48	12E
										12M	6E	4O	12M	48	12D
										12N	6E	4P	12L	48	12E

TABLE 5. The classes of $\overline{G}$

$M(g)$	$M(g)$	$M(g)$
$M(1A)=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 120 & 120 & 8 & -8 \\ 135 & 135 & -9 & 7 \\ 16 & -16 & 0 & 0 \end{bmatrix}$	$M(2A)=\begin{bmatrix} 1 & 1 & 1 \\ 15 & 15 & -1 \\ 4 & -4 & 0 \end{bmatrix}$	$M(2B)=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 24 & 24 & 8 & -8 & 0 \\ 3 & 3 & 3 & 3 & -1 \\ 36 & 36 & -12 & 4 & 0 \\ 8 & -8 & 0 & 0 & 0 \end{bmatrix}$
$M(2C)=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & -1 & -1 \\ 6 & 6 & -2 & -2 & 2 \\ 6 & 6 & -2 & 2 & -2 \\ 4 & -4 & 0 & 0 & 0 \end{bmatrix}$	$M(2D)=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 8 & 8 & -8 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 6 & 6 & 6 & -2 & 0 \\ 4 & -4 & 0 & 0 & 0 \end{bmatrix}$	$M(3A)=\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ 2 & -2 & 0 \end{bmatrix}$
$M(3B)=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(3C)=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 6 & 6 & 2 & -2 \\ 9 & 9 & -3 & 1 \\ 4 & -4 & 0 & 0 \end{bmatrix}$	$M(4A)=\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ 2 & -2 & 0 \end{bmatrix}$
$M(4B)=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 4 & 4 & -4 & 0 \\ 8 & 8 & -8 & 0 & 0 \\ 3 & 3 & 3 & 3 & -1 \\ 4 & -4 & 0 & 0 & 0 \end{bmatrix}$	$M(4C)=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 2 & -2 & 0 & 0 \end{bmatrix}$	$M(4D)=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 2 & -2 & 0 & 0 \end{bmatrix}$
$M(5A)=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(6A)=\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ 2 & -2 & 0 \end{bmatrix}$	$M(6B)=\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ 2 & -2 & 0 \end{bmatrix}$
$M(6C)=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(6D)=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(6E)=\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ 2 & -2 & 0 \end{bmatrix}$
$M(6F)=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(6G)=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 2 & -2 & 0 & 0 \end{bmatrix}$	$M(8A)=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 \\ 2 & -2 & 0 & 0 & 0 \end{bmatrix}$
$M(9A)=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(10A)=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(12A)=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
	$M(12B)=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 \\ 2 & -2 & 0 & 0 & 0 \end{bmatrix}$	

Table 6. The Fischer Matrices of $\overline{G}$

5. The character table of $\overline{G}$

Using the information of the classes of $\overline{G}$ in Table 5, the ordinary irreducible characters of the inertia factors H_i and the Fischer matrices in Table 6, the ordinary character table of $\overline{G}$ (see Table 7) is successfully constructed using the outline given in Section 2 of this paper. Consistency and accuracy checks of the character table of $\overline{G}$ have been carried out the aid of Programme E in [19] together with the computation of the class multiplication coefficients of the classes of $\overline{G}$. The set of ordinary irreducible characters of $\overline{G}$ will be partitioned into 4 blocks $\triangle_1 = \{\chi_j | 1 \leq j \leq 25\}$, $\triangle_2 = \{\chi_j | 26 \leq j \leq 39\}$, $\triangle_3 = \{\chi_j | 40 \leq j \leq 59\}$ and $\triangle_4 = \{\chi_j | 60 \leq j \leq 84\}$ corresponding to the inertia factor groups H_1, H_2, H_3 and H_4, respectively, where $\chi_j \in Irr(\overline{G})$. Since $\overline{G} \cong 2^{\cdot}\overline{G_1}$ is a two-fold cover of $\overline{G}_1$, the ordinary characters of $\overline{G_1} = 2^8{:}(U_4(2){:}2)$ (see Table 5 in [6]) are found in blocks $\triangle_1$ to $\triangle_3$ and a set $\mathrm{IrrProj}(G_1, \alpha)$ of irreducible projective characters with factor set α of order 2 for $\overline{G}_1$ can be obtained from block $\triangle_4$.

Using Programme E in GAP, the unique p-power maps of the elements of $\overline{G}$ are computed (see Table 5) from our Table 7. Also, using the power maps of $\overline{G}$ and $U_6(2){:}2$, the permutation character $\chi(U_6(2){:}2|\overline{G}) = 1a + 252a + 440a$ of $U_6(2){:}2$ on the classes of $\overline{G}$ and the restriction of some characters of small degrees of $U_6(2){:}2$ to the set $\mathrm{Irr}(\overline{G})$ in Table 7, the fusion map of the classes of $\overline{G}$ into the classes of $U_6(2){:}2$ is computed (see last column of Table 5).

$[g]_G$	$1A$				$2A$			$2B$					$2C$					$2D$					$3A$			$3B$	
$[x]_{\overline{G}}$	$1A$	$2A$	$4A$	$2B$	$2C$	$2D$	$4B$	$2E$	$2F$	$4C$	$2G$	$4D$	$2H$	$2I$	$4E$	$4F$	$4G$	$4H$	$4I$	$2J$	$4J$	$8A$	$6A$	$3A$	$12A$	$3B$	$6B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	1	1	1	1	1
χ_3	6	6	6	6	-4	-4	-4	-2	-2	-2	-2	-2	2	2	2	2	2	0	0	0	0	0	-3	-3	-3	3	3
χ_4	6	6	6	6	4	4	4	-2	-2	-2	-2	-2	2	2	2	2	2	0	0	0	0	0	-3	-3	-3	3	3
χ_5	10	10	10	10	0	0	0	-6	-6	-6	-6	-6	2	2	2	2	2	0	0	0	0	0	1	1	1	-2	-2
χ_6	15	15	15	15	-5	-5	-5	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	3	6	6	6	3	3
χ_7	15	15	15	15	-5	-5	-5	7	7	7	7	7	3	3	3	3	3	-1	-1	-1	-1	-1	-3	-3	-3	0	0
χ_8	15	15	15	15	5	5	5	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-3	-3	-3	-3	-3	6	6	6	3	3
χ_9	15	15	15	15	5	5	5	7	7	7	7	7	3	3	3	3	3	1	1	1	1	1	-3	-3	-3	0	0
χ_{10}	20	20	20	20	-10	-10	-10	4	4	4	4	4	4	4	4	4	4	-2	-2	-2	-2	-2	2	2	2	5	5
χ_{11}	20	20	20	20	10	10	10	4	4	4	4	4	4	4	4	4	4	2	2	2	2	2	2	2	2	5	5
χ_{12}	20	20	20	20	0	0	0	4	4	4	4	4	-4	-4	-4	-4	-4	0	0	0	0	0	-7	-7	-7	2	2
χ_{13}	24	24	24	24	-4	-4	-4	8	8	8	8	8	0	0	0	0	0	-4	-4	-4	-4	-4	6	6	6	0	0
χ_{14}	24	24	24	24	4	4	4	8	8	8	8	8	0	0	0	0	0	4	4	4	4	4	6	6	6	0	0
χ_{15}	30	30	30	30	-10	-10	-10	-10	-10	-10	-10	-10	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3
χ_{16}	30	30	30	30	10	10	10	-10	-10	-10	-10	-10	2	2	2	2	2	-2	-2	-2	-2	-2	3	3	3	3	3
χ_{17}	60	60	60	60	-10	-10	-10	-4	-4	-4	-4	-4	4	4	4	4	4	-2	-2	-2	-2	-2	6	6	6	-3	-3
χ_{18}	60	60	60	60	10	10	10	-4	-4	-4	-4	-4	4	4	4	4	4	2	2	2	2	2	6	6	6	-3	-3
χ_{19}	60	60	60	60	0	0	0	12	12	12	12	12	4	4	4	4	4	0	0	0	0	0	-3	-3	-3	-6	-6
χ_{20}	64	64	64	64	-16	-16	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	-8	-8	4	4
χ_{21}	64	64	64	64	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	-8	-8	4	4
χ_{22}	80	80	80	80	0	0	0	-16	-16	-16	-16	-16	0	0	0	0	0	0	0	0	0	0	-10	-10	-10	-4	-4
χ_{23}	81	81	81	81	9	9	9	9	9	9	9	9	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	0	0	0	0	0
χ_{24}	81	81	81	81	-9	-9	-9	9	9	9	9	9	-3	-3	-3	-3	-3	3	3	3	3	3	0	0	0	0	0
χ_{25}	90	90	90	90	0	0	0	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	0	0	0	0	0	9	9	9	0	0
χ_{26}	120	120	8	-8	0	0	0	24	24	8	-8	0	0	0	0	0	0	8	8	-8	0	0	3	3	-1	0	0
χ_{27}	120	120	8	-8	0	0	0	24	24	8	-8	0	0	0	0	0	0	-8	-8	8	0	0	3	3	-1	0	0
χ_{28}	120	120	8	-8	0	0	0	24	24	8	-8	0	0	0	0	0	0	-8	-8	8	0	0	3	3	-1	0	0
χ_{29}	120	120	8	-8	0	0	0	24	24	8	-8	0	0	0	0	0	0	8	8	-8	0	0	3	3	-1	0	0
χ_{30}	240	240	16	-16	0	0	0	48	48	16	-16	0	0	0	0	0	0	0	0	0	0	0	6	6	-2	0	0
χ_{31}	240	240	16	-16	0	0	0	-48	-48	-16	16	0	0	0	0	0	0	0	0	0	0	0	6	6	-2	0	0
χ_{32}	240	240	16	-16	0	0	0	-48	-48	-16	16	0	0	0	0	0	0	0	0	0	0	0	6	6	-2	0	0
χ_{33}	720	720	48	-48	0	0	0	-48	-48	-16	16	0	0	0	0	0	0	0	0	0	0	0	-9	-9	3	0	0
χ_{34}	720	720	48	-48	0	0	0	-48	-48	-16	16	0	0	0	0	0	0	0	0	0	0	0	-9	-9	3	0	0
χ_{35}	720	720	48	-48	0	0	0	-48	-48	-16	16	0	0	0	0	0	0	0	0	0	0	0	-9	-9	3	0	0
χ_{36}	720	720	48	-48	0	0	0	-48	-48	-16	16	0	0	0	0	0	0	0	0	0	0	0	-9	-9	3	0	0
χ_{37}	960	960	64	-64	0	0	0	0	0	0	0	0	0	0	0	0	0	-16	-16	16	0	0	24	24	-8	0	0
χ_{38}	960	960	64	-64	0	0	0	0	0	0	0	0	0	0	0	0	0	16	16	-16	0	0	24	24	-8	0	0
χ_{39}	1440	1440	96	-96	0	0	0	96	96	32	-32	0	0	0	0	0	0	0	0	0	0	0	-18	-18	6	0	0

TABLE 7. The character table of $\overline{G} = 2_+^{1+8}{:}(U_4(2){:}2)$

$[g]_G$	$1A$				$2A$			$2B$					$2C$					$2D$					$3A$			$3B$	
$[x]_{\overline{G}}$	$1A$	$2A$	$4A$	$2B$	$2C$	$2D$	$4B$	$2E$	$2F$	$4C$	$2G$	$4D$	$2H$	$2I$	$4E$	$4F$	$4G$	$4H$	$4I$	$2J$	$4J$	$8A$	$6A$	$3A$	$12A$	$3B$	$6B$
χ_{40}	135	135	-9	7	15	15	-1	39	39	-9	7	-1	15	15	-1	-1	-1	7	7	7	-1	-1	0	0	0	0	0
χ_{41}	135	135	-9	7	-15	-15	1	-33	-33	15	-1	-1	3	3	3	-5	3	5	5	5	-3	1	0	0	0	0	0
χ_{42}	135	135	-9	7	15	15	-1	-33	-33	15	-1	-1	3	3	3	-5	3	-5	-5	-5	3	-1	0	0	0	0	0
χ_{43}	135	135	-9	7	-15	-15	1	39	39	-9	7	-1	15	15	-1	-1	-1	-7	-7	-7	1	1	0	0	0	0	0
χ_{44}	270	270	-18	14	-30	-30	2	6	6	6	6	-2	18	18	2	-6	2	-2	-2	-2	-2	2	0	0	0	0	0
χ_{45}	270	270	-18	14	30	30	-2	6	6	6	6	-2	18	18	2	-6	2	2	2	2	2	-2	0	0	0	0	0
χ_{46}	405	405	-27	21	-45	-45	3	-27	-27	21	5	-3	-3	-3	13	-3	-3	3	3	3	-5	3	0	0	0	0	0
χ_{47}	405	405	-27	21	-45	-45	3	45	45	-3	13	-3	9	9	9	1	-7	-9	-9	-9	-1	3	0	0	0	0	0
χ_{48}	405	405	-27	21	45	45	-3	-27	-27	21	5	-3	-3	-3	13	-3	-3	-3	-3	-3	5	-3	0	0	0	0	0
χ_{49}	405	405	-27	21	45	45	-3	45	45	-3	13	-3	9	9	9	1	-7	9	9	9	1	-3	0	0	0	0	0
χ_{50}	540	540	-36	28	-30	-30	2	60	60	-36	-4	4	-12	-12	4	-4	4	2	2	2	2	-2	0	0	0	0	0
χ_{51}	540	540	-36	28	-30	-30	2	-84	-84	12	-20	4	12	12	-4	4	-4	2	2	2	2	-2	0	0	0	0	0
χ_{52}	540	540	-36	28	30	30	-2	60	60	-36	-4	4	-12	-12	4	-4	4	-2	-2	-2	-2	2	0	0	0	0	0
χ_{53}	540	540	-36	28	30	30	-2	-84	-84	12	-20	4	12	12	-4	4	-4	-2	-2	-2	-2	2	0	0	0	0	0
χ_{54}	810	810	-54	42	0	0	0	-54	-54	42	10	-6	-6	-6	-6	-6	10	0	0	0	0	0	0	0	0	0	0
χ_{55}	810	810	-54	42	0	0	0	90	90	-6	26	-6	18	18	-14	2	2	0	0	0	0	0	0	0	0	0	0
χ_{56}	810	810	-54	42	0	0	0	18	18	18	18	-6	-18	-18	-2	6	-2	-12	-12	-12	4	0	0	0	0	0	0
χ_{57}	810	810	-54	42	0	0	0	18	18	18	18	-6	-18	-18	-2	6	-2	12	12	12	-4	0	0	0	0	0	0
χ_{58}	1080	1080	-72	56	-60	-60	4	-24	-24	-24	-24	8	0	0	0	0	0	4	4	4	4	-4	0	0	0	0	0
χ_{59}	1080	1080	-72	56	60	60	-4	-24	-24	-24	-24	8	0	0	0	0	0	-4	-4	-4	-4	4	0	0	0	0	0
χ_{60}	16	-16	0	0	4	-4	0	8	-8	0	0	0	4	-4	0	0	0	4	-4	0	0	0	2	-2	0	1	-1
χ_{61}	16	-16	0	0	-4	4	0	8	-8	0	0	0	4	-4	0	0	0	-4	4	0	0	0	2	-2	0	1	-1
χ_{62}	96	-96	0	0	-16	16	0	-16	16	0	0	0	8	-8	0	0	0	0	0	0	0	0	-6	6	0	3	-3
χ_{63}	96	-96	0	0	16	-16	0	-16	16	0	0	0	8	-8	0	0	0	0	0	0	0	0	-6	6	0	3	-3
χ_{64}	160	-160	0	0	0	0	0	-48	48	0	0	0	8	-8	0	0	0	0	0	0	0	0	2	-2	0	-2	2
χ_{65}	240	-240	0	0	-20	20	0	-8	8	0	0	0	-4	4	0	0	0	12	-12	0	0	0	12	-12	0	3	-3
χ_{66}	240	-240	0	0	-20	20	0	56	-56	0	0	0	12	-12	0	0	0	-4	4	0	0	0	-6	6	0	0	0
χ_{67}	240	-240	0	0	20	-20	0	-8	8	0	0	0	-4	4	0	0	0	-12	12	0	0	0	12	-12	0	3	-3
χ_{68}	240	-240	0	0	20	-20	0	56	-56	0	0	0	12	-12	0	0	0	4	-4	0	0	0	-6	6	0	0	0
χ_{69}	320	-320	0	0	-40	40	0	32	-32	0	0	0	16	-16	0	0	0	-8	8	0	0	0	4	-4	0	5	-5
χ_{70}	320	-320	0	0	40	-40	0	32	-32	0	0	0	16	-16	0	0	0	8	-8	0	0	0	4	-4	0	5	-5
χ_{71}	320	-320	0	0	0	0	0	32	-32	0	0	0	-16	16	0	0	0	0	0	0	0	0	-14	14	0	2	-2
χ_{72}	384	-384	0	0	-16	16	0	64	-64	0	0	0	0	0	0	0	0	-16	16	0	0	0	12	-12	0	0	0
χ_{73}	384	-384	0	0	16	-16	0	64	-64	0	0	0	0	0	0	0	0	16	-16	0	0	0	12	-12	0	0	0
χ_{74}	480	-480	0	0	-40	40	0	-80	80	0	0	0	8	-8	0	0	0	8	-8	0	0	0	6	-6	0	3	-3
χ_{75}	480	-480	0	0	40	-40	0	-80	80	0	0	0	8	-8	0	0	0	-8	8	0	0	0	6	-6	0	3	-3
χ_{76}	960	-960	0	0	-40	40	0	-32	32	0	0	0	16	-16	0	0	0	-8	8	0	0	0	12	-12	0	-3	3
χ_{77}	960	-960	0	0	40	-40	0	-32	32	0	0	0	16	-16	0	0	0	8	-8	0	0	0	12	-12	0	-3	3
χ_{78}	960	-960	0	0	0	0	0	96	-96	0	0	0	16	-16	0	0	0	0	0	0	0	0	-6	6	0	-6	6
χ_{79}	1024	-1024	0	0	-64	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-16	16	0	4	-4
χ_{80}	1024	-1024	0	0	64	-64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-16	16	0	4	-4
χ_{81}	1280	-1280	0	0	0	0	0	-128	128	0	0	0	0	0	0	0	0	0	0	0	0	0	-20	20	0	-4	4
χ_{82}	1296	-1296	0	0	36	-36	0	72	-72	0	0	0	-12	12	0	0	0	-12	12	0	0	0	0	0	0	0	0
χ_{83}	1296	-1296	0	0	-36	36	0	72	-72	0	0	0	-12	12	0	0	0	12	-12	0	0	0	0	0	0	0	0
χ_{84}	1440	-1440	0	0	0	0	0	-48	48	0	0	0	-24	24	0	0	0	0	0	0	0	0	18	-18	0	0	0

The character table of $\overline{G} = 2_+^{1+8}{:}(U_4(2){:}2)$(continued)

$[g]_G$	3C				4A			4B					4C				4D				5A		6A			6B		
$[x]_{\overline{G}}$	3C	6C	12B	6D	4K	4L	8B	4M	4N	4O	4P	8C	4Q	4R	8D	8E	4S	4T	8F	8G	5A	10A	6E	6F	12C	6G	6H	12D
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	-1	-1	-1
χ_3	0	0	0	0	2	2	2	2	2	2	2	2	-2	-2	-2	-2	0	0	0	0	1	1	1	1	1	2	2	2
χ_4	0	0	0	0	-2	-2	-2	2	2	2	2	2	2	2	2	2	0	0	0	0	1	1	1	1	1	-2	-2	-2
χ_5	4	4	4	4	0	0	0	2	2	2	2	2	0	0	0	0	-2	-2	-2	-2	0	0	-3	-3	-3	0	0	0
χ_6	0	0	0	0	-1	-1	-1	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	0	0	2	2	2	-2	-2	-2
χ_7	3	3	3	3	-3	-3	-3	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
χ_8	0	0	0	0	1	1	1	3	3	3	3	3	1	1	1	1	-1	-1	-1	-1	0	0	2	2	2	2	2	2
χ_9	3	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	0	0	1	1	1	-1	-1	-1
χ_{10}	-1	-1	-1	-1	-2	-2	-2	0	0	0	0	0	-2	-2	-2	-2	0	0	0	0	0	0	-2	-2	-2	-1	-1	-1
χ_{11}	-1	-1	-1	-1	2	2	2	0	0	0	0	0	2	2	2	2	0	0	0	0	0	0	-2	-2	-2	1	1	1
χ_{12}	2	2	2	2	0	0	0	4	4	4	4	4	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
χ_{13}	3	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	2	2	2	-1	-1	-1
χ_{14}	3	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	2	2	2	1	1	1
χ_{15}	3	3	3	3	4	4	4	-2	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{16}	3	3	3	3	-4	-4	-4	-2	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	1
χ_{17}	-3	-3	-3	-3	2	2	2	0	0	0	0	0	2	2	2	2	0	0	0	0	0	0	2	2	2	-1	-1	-1
χ_{18}	-3	-3	-3	-3	-2	-2	-2	0	0	0	0	0	-2	-2	-2	-2	0	0	0	0	0	0	2	2	2	1	1	1
χ_{19}	0	0	0	0	0	0	0	4	4	4	4	4	0	0	0	0	0	0	0	0	0	0	-3	-3	-3	0	0	0
χ_{20}	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	2	2	2
χ_{21}	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	-2	-2	-2
χ_{22}	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0
χ_{23}	0	0	0	0	3	3	3	-3	-3	-3	-3	-3	-1	-1	-1	-1	-1	-1	-1	-1	1	1	0	0	0	0	0	0
χ_{24}	0	0	0	0	-3	-3	-3	-3	-3	-3	-3	-3	1	1	1	1	-1	-1	-1	-1	1	1	0	0	0	0	0	0
χ_{25}	0	0	0	0	0	0	0	2	2	2	2	2	0	0	0	0	2	2	2	2	0	0	-3	-3	-3	0	0	0
χ_{26}	6	6	2	-2	0	0	0	12	12	-4	-4	0	0	0	0	0	0	0	0	0	0	0	3	3	-1	0	0	0
χ_{27}	6	6	2	-2	0	0	0	-4	-4	12	-4	0	0	0	0	0	0	0	0	0	0	0	3	3	-1	0	0	0
χ_{28}	6	6	2	-2	0	0	0	12	12	-4	-4	0	0	0	0	0	0	0	0	0	0	0	3	3	-1	0	0	0
χ_{29}	6	6	2	-2	0	0	0	-4	-4	12	-4	0	0	0	0	0	0	0	0	0	0	0	3	3	-1	0	0	0
χ_{30}	12	12	4	-4	0	0	0	-8	-8	-8	8	0	0	0	0	0	0	0	0	0	0	0	6	6	-2	0	0	0
χ_{31}	12	12	4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	2	0	0	0
χ_{32}	12	12	4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	2	0	0	0
χ_{33}	0	0	0	0	0	0	0	-8	-8	24	-8	0	0	0	0	0	0	0	0	0	0	0	3	3	-1	0	0	0
χ_{34}	0	0	0	0	0	0	0	24	24	-8	-8	0	0	0	0	0	0	0	0	0	0	0	3	3	-1	0	0	0
χ_{35}	0	0	0	0	0	0	0	-8	-8	-8	8	0	0	0	0	0	0	0	0	0	0	0	3	3	-1	0	0	0
χ_{36}	0	0	0	0	0	0	0	-8	-8	-8	8	0	0	0	0	0	0	0	0	0	0	0	3	3	-1	0	0	0
χ_{37}	-6	-6	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	-6	-6	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	2	0	0	0

The character table of $\overline{G} = 2_+^{1+8}$:$(U_4(2)$:$2)$(continued)

$[g]_G$	$3C$				$4A$			$4B$					$4C$				$4D$				$5A$		$6A$			$6B$		
$[x]_{\overline{G}}$	$3C$	$6C$	$12B$	$6D$	$4K$	$4L$	$8B$	$4M$	$4N$	$4O$	$4P$	$8C$	$4Q$	$4R$	$8D$	$8E$	$4S$	$4T$	$8F$	$8G$	$5A$	$10A$	$6E$	$6F$	$12C$	$6G$	$6H$	$12D$
χ_{40}	9	9	-3	1	3	3	-1	3	3	3	3	-1	3	3	-1	-1	3	3	-1	-1	0	0	0	0	0	3	3	-1
χ_{41}	9	9	-3	1	3	3	-1	3	3	3	3	-1	-1	-1	3	-1	-3	-3	1	1	0	0	0	0	0	-3	-3	1
χ_{42}	9	9	-3	1	-3	-3	1	3	3	3	3	-1	1	1	-3	1	-3	-3	1	1	0	0	0	0	0	3	3	-1
χ_{43}	9	9	-3	1	-3	-3	1	3	3	3	3	-1	-3	-3	1	1	3	3	-1	-1	0	0	0	0	0	-3	-3	1
χ_{44}	-9	-9	3	-1	0	0	0	6	6	6	6	-2	-4	-4	4	0	0	0	0	0	0	0	0	0	0	3	3	-1
χ_{45}	-9	-9	3	-1	0	0	0	6	6	6	6	-2	4	4	-4	0	0	0	0	0	0	0	0	0	0	-3	-3	1
χ_{46}	0	0	0	0	3	3	-1	-3	-3	-3	-3	1	3	3	-1	-1	1	1	-3	1	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	-3	-3	1	-3	-3	-3	-3	1	1	1	-3	1	-1	-1	3	-1	0	0	0	0	0	0	0	0
χ_{48}	0	0	0	0	-3	-3	1	-3	-3	-3	-3	1	-3	-3	1	1	1	1	-3	1	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	3	3	-1	-3	-3	-3	-3	1	-1	-1	3	-1	-1	-1	3	-1	0	0	0	0	0	0	0	0
χ_{50}	9	9	-3	1	-6	-6	2	0	0	0	0	0	2	2	2	-2	0	0	0	0	0	0	0	0	0	3	3	-1
χ_{51}	9	9	-3	1	6	6	-2	0	0	0	0	0	-2	-2	-2	2	0	0	0	0	0	0	0	0	0	3	3	-1
χ_{52}	9	9	-3	1	6	6	-2	0	0	0	0	0	-2	-2	-2	2	0	0	0	0	0	0	0	0	0	-3	-3	1
χ_{53}	9	9	-3	1	-6	-6	2	0	0	0	0	0	2	2	2	-2	0	0	0	0	0	0	0	0	0	-3	-3	1
χ_{54}	0	0	0	0	0	0	0	-6	-6	-6	-6	2	0	0	0	0	2	2	2	-2	0	0	0	0	0	0	0	0
χ_{55}	0	0	0	0	0	0	0	-6	-6	-6	-6	2	0	0	0	0	-2	-2	-2	2	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	6	6	-2	6	6	6	6	-2	2	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	0	0	0	0	-6	-6	2	6	6	6	6	-2	-2	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	-9	-9	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	1
χ_{59}	-9	-9	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	-1
χ_{60}	4	-4	0	0	2	-2	0	4	-4	0	0	0	2	-2	0	0	2	-2	0	0	1	-1	2	-2	0	2	-2	0
χ_{61}	4	-4	0	0	-2	2	0	4	-4	0	0	0	-2	2	0	0	2	-2	0	0	1	-1	2	-2	0	-2	2	0
χ_{62}	0	0	0	0	4	-4	0	8	-8	0	0	0	-4	4	0	0	0	0	0	0	1	-1	2	-2	0	4	-4	0
χ_{63}	0	0	0	0	-4	4	0	8	-8	0	0	0	4	-4	0	0	0	0	0	0	1	-1	2	-2	0	-4	4	0
χ_{64}	16	-16	0	0	0	0	0	8	-8	0	0	0	0	0	0	0	-4	4	0	0	0	0	-6	6	0	0	0	0
χ_{65}	0	0	0	0	-2	2	0	12	-12	0	0	0	-2	2	0	0	-2	2	0	0	0	0	4	-4	0	-4	4	0
χ_{66}	12	-12	0	0	-6	6	0	-4	4	0	0	0	2	-2	0	0	2	-2	0	0	0	0	2	-2	0	2	-2	0
χ_{67}	0	0	0	0	2	-2	0	12	-12	0	0	0	2	-2	0	0	-2	2	0	0	0	0	4	-4	0	4	-4	0
χ_{68}	12	-12	0	0	6	-6	0	-4	4	0	0	0	-2	2	0	0	2	-2	0	0	0	0	2	-2	0	-2	2	0
χ_{69}	-4	4	0	0	-4	4	0	0	0	0	0	0	-4	4	0	0	0	0	0	0	0	0	-4	4	0	-2	2	0
χ_{70}	-4	4	0	0	4	-4	0	0	0	0	0	0	4	-4	0	0	0	0	0	0	0	0	-4	4	0	2	-2	0
χ_{71}	8	-8	0	0	0	0	0	16	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0
χ_{72}	12	-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	4	-4	0	-2	2	0
χ_{73}	12	-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	4	-4	0	2	-2	0
χ_{74}	12	-12	0	0	8	-8	0	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	-2	2	0
χ_{75}	12	-12	0	0	-8	8	0	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	2	-2	0
χ_{76}	-12	12	0	0	4	-4	0	0	0	0	0	0	4	-4	0	0	0	0	0	0	0	0	4	-4	0	-2	2	0
χ_{77}	-12	12	0	0	-4	4	0	0	0	0	0	0	-4	4	0	0	0	0	0	0	0	0	4	-4	0	2	-2	0
χ_{78}	0	0	0	0	0	0	0	16	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	6	0	0	0	0
χ_{79}	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	4	-4	0
χ_{80}	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	-4	4	0
χ_{81}	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	-4	0	0	0	0
χ_{82}	0	0	0	0	6	-6	0	-12	12	0	0	0	-2	2	0	0	-2	2	0	0	1	-1	0	0	0	0	0	0
χ_{83}	0	0	0	0	-6	6	0	-12	12	0	0	0	2	-2	0	0	-2	2	0	0	1	-1	0	0	0	0	0	0
χ_{84}	0	0	0	0	0	0	0	8	-8	0	0	0	0	0	0	0	4	-4	0	0	0	0	-6	6	0	0	0	0

The character table of $\overline{G} = 2_+^{1+8}{:}(U_4(2){:}2)$(continued)

$[g]_G$	$6C$		$6D$		$6E$			$6F$		$6G$				$8A$					$9A$		$10A$		$12A$		$12B$				
$[x]_{\overline{G}}$	$6I$	$6J$	$6K$	$6L$	$6M$	$6N$	$12E$	$6O$	$6P$	$12F$	$12G$	$6Q$	$24A$	$8H$	$8I$	$16A$	$8J$	$16B$	$9A$	$18A$	$10B$	$10C$	$12H$	$12I$	$12J$	$12K$	$12L$	$12M$	$12N$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	1	1	1	1	1
χ_3	1	1	-1	-1	-2	-2	-2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	-1	-1	-1	-1	-1
χ_4	1	1	1	1	-2	-2	-2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	-1	-1	-1	-1	-1
χ_5	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	-1	-1	-1	-1	-1
χ_6	-1	-1	1	1	2	2	2	-1	-1	0	0	0	0	1	1	1	1	1	0	0	0	0	-1	-1	0	0	0	0	0
χ_7	-2	-2	-2	-2	1	1	1	0	0	-1	-1	-1	-1	1	1	1	1	1	0	0	0	0	0	0	-1	-1	-1	-1	-1
χ_8	-1	-1	-1	-1	2	2	2	-1	-1	0	0	0	0	-1	-1	-1	-1	-1	0	0	0	0	1	1	0	0	0	0	0
χ_9	-2	-2	2	2	1	1	1	0	0	1	1	1	1	-1	-1	-1	-1	-1	0	0	0	0	0	0	-1	-1	-1	-1	-1
χ_{10}	1	1	-1	-1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	-1	-1	0	0	1	1	0	0	0	0	0
χ_{11}	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	0	0	0	0	0	-1	-1	0	0	-1	-1	0	0	0	0	0
χ_{12}	-2	-2	0	0	-2	-2	-2	2	2	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	1	1	1	1	1
χ_{13}	2	2	2	2	-1	-1	-1	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
χ_{14}	2	2	-2	-2	-1	-1	-1	0	0	1	1	1	1	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0
χ_{15}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
χ_{16}	-1	-1	1	1	-1	-1	-1	-1	-1	1	1	1	1	0	0	0	0	0	0	0	0	0	-1	-1	1	1	1	1	1
χ_{17}	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0
χ_{18}	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
χ_{19}	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
χ_{20}	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0
χ_{21}	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
χ_{22}	2	2	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0
χ_{23}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	-1	-1	0	0	0	0	0	0	0
χ_{24}	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	0	0	1	1	0	0	0	0	0	0	0
χ_{25}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1
χ_{26}	0	0	0	0	0	0	0	0	0	2	2	-2	0	2	2	0	-2	0	0	0	0	0	0	0	3	3	-1	-1	-1
χ_{27}	0	0	0	0	0	0	0	0	0	-2	-2	2	0	2	2	0	-2	0	0	0	0	0	0	0	-1	-1	-1	3	-1
χ_{28}	0	0	0	0	0	0	0	0	0	-2	-2	2	0	-2	-2	0	2	0	0	0	0	0	0	0	3	3	-1	-1	-1
χ_{29}	0	0	0	0	0	0	0	0	0	2	2	-2	0	-2	-2	0	2	0	0	0	0	0	0	0	-1	-1	-1	3	-1
χ_{30}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	2	-2	2
χ_{31}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	A	0	$-A$	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-A$	0	A	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-3	1
χ_{34}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	1	1	1
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	B	1	$\overline{B}$
χ_{36}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	$\overline{B}$	1	B
χ_{37}	0	0	0	0	0	0	0	0	0	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where $A = 2\sqrt{2}i$, $B = -1 - 2\sqrt{3}i$

The character table of $\overline{G} = 2_+^{1+8}{:}(U_4(2){:}2)$(continued)

$[g]_G$	$6C$		$6D$		$6E$			$6F$		$6G$				$8A$					$9A$		$10A$		$12A$		$12B$				
$[x]_{\overline{G}}$	$6I$	$6J$	$6K$	$6L$	$6M$	$6N$	$12E$	$6O$	$6P$	$12F$	$12G$	$6Q$	$24A$	$8H$	$8I$	$16A$	$8J$	$16B$	$9A$	$18A$	$10B$	$10C$	$12H$	$12I$	$12J$	$12K$	$12L$	$12M$	$12N$
χ_{40}	0	0	0	0	3	3	-1	0	0	1	1	1	-1	1	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	3	3	-1	0	0	-1	-1	-1	1	1	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	3	3	-1	0	0	1	1	1	-1	-1	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	3	3	-1	0	0	-1	-1	-1	1	-1	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	0	0	0	0	-3	-3	1	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	0	0	0	0	-3	-3	1	0	0	-1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	0	0	0	0	-3	-3	1	0	0	-1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	0	0	0	0	-3	-3	1	0	0	-1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	0	0	-3	-3	1	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	0	-3	-3	1	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	0	0	0	0	3	3	-1	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	0	0	0	3	3	-1	0	0	-1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	1	-1	1	-1	2	-2	0	1	-1	2	-2	0	0	2	-2	0	0	0	1	-1	1	-1	1	-1	2	-2	0	0	0
χ_{61}	1	-1	-1	1	2	-2	0	1	-1	-2	2	0	0	-2	2	0	0	0	1	-1	-1	1	-1	1	2	-2	0	0	0
χ_{62}	1	-1	-1	1	-4	4	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	1	-2	2	0	0	0
χ_{63}	1	-1	1	-1	-4	4	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	-1	-2	2	0	0	0
χ_{64}	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	-2	2	0	0	0
χ_{65}	-1	1	1	-1	4	-4	0	-1	1	0	0	0	0	2	-2	0	0	0	0	0	0	0	-1	1	0	0	0	0	0
χ_{66}	-2	2	-2	2	2	-2	0	0	0	-2	2	0	0	2	-2	0	0	0	0	0	0	0	0	0	-2	2	0	0	0
χ_{67}	-1	1	-1	1	4	-4	0	-1	1	0	0	0	0	-2	2	0	0	0	0	0	0	0	1	-1	0	0	0	0	0
χ_{68}	-2	2	2	-2	2	-2	0	0	0	2	-2	0	0	-2	2	0	0	0	0	0	0	0	0	0	-2	2	0	0	0
χ_{69}	1	-1	-1	1	2	-2	0	1	-1	2	-2	0	0	0	0	0	0	0	-1	1	0	0	1	-1	0	0	0	0	0
χ_{70}	1	-1	1	-1	2	-2	0	1	-1	-2	2	0	0	0	0	0	0	0	-1	1	0	0	-1	1	0	0	0	0	0
χ_{71}	-2	2	0	0	-4	4	0	2	-2	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	2	-2	0	0	0
χ_{72}	2	-2	2	-2	-2	2	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0
χ_{73}	2	-2	-2	2	-2	2	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0
χ_{74}	-1	1	-1	1	-2	2	0	-1	1	-2	2	0	0	0	0	0	0	0	0	0	0	0	1	-1	2	-2	0	0	0
χ_{75}	-1	1	1	-1	-2	2	0	-1	1	2	-2	0	0	0	0	0	0	0	0	0	0	0	-1	1	2	-2	0	0	0
χ_{76}	-1	1	-1	1	-2	2	0	1	-1	2	-2	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0
χ_{77}	-1	1	1	-1	-2	2	0	1	-1	-2	2	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0
χ_{79}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	1	0	0	0	0	0	0	0
χ_{80}	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	-1	0	0	0	0	0	0	0
χ_{81}	2	-2	0	0	4	-4	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0
χ_{82}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	-1	1	0	0	0	0	0	0	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	1	-1	0	0	0	0	0	0	0
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0

The character table of $\overline{G} = 2_+^{1+8}{:}(U_4(2){:}2)$(continued)

Acknowledgements. A start-up fund (cost centre:GB85 and account number:4920) from Nelson Mandela University made it possible to carry out the research work which is contained in this paper. I am most grateful to my Lord Jesus Christ.

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(Recibido en octubre de 2021. Aceptado en enero de 2023)

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