ADVANCING PROBABILISTIC FRAME ANALYSIS: A COMPREHENSIVE APPROACH USING MONTE CARLO SIMULATION AND RESPONSE SURFACES

Avanzando en el análisis probabilístico de marcos: un enfoque integral mediante simulaciones Monte Carlo y superficies de respuesta

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ABSTRACT

An ANSYS-based probabilistic finite element analysis has been created, expanding Haldar and Mahadevan's canonical frame approach. The model is parameterized with eleven random variables, including applied loads, cross-section properties, member lengths, elastic modulus, and limit horizontal displacement. The sensitivity of these variables is analyzed, and the structure's failure probability is evaluated using a limit state function for horizontal displacement. Monte Carlo (MC), Monte Carlo with Latin Hypercube sampling (MCLH), and Linear and Quadratic Response Surface (RSM-LIN and RSM-QUAX) methods analyze structural reliability; RSM methods achieved the lowest computational costs. The results are conclusive and future applications involving nonlinearity, dynamic loading, and other extreme scenarios are predicted.

Keywords: design of experiments; finite element; Monte Carlo methods; probabilistic sensitivities; response surface method; structural reliability.

RESUMEN

Se realizó un análisis probabilístico de elementos finitos en ANSYS que amplía el enfoque del pórtico canónico de Haldar y Mahadevan. El modelo se parametriza con once variables aleatorias que incluyen las cargas impuestas, las propiedades geométricas de las secciones, las longitudes de los miembros, el módulo elástico y el desplazamiento horizontal límite. El modelo incorpora un análisis de las sensibilidades probabilísticas de las variables y la evaluación de la probabilidad de falla de la estructura. El criterio de falla se introduce a través de una función de estado límite de desplazamiento horizontal. Los

resultados de la confiabilidad estructural del pórtico utilizando los métodos de Monte Carlo (MC), Monte Carlo con Muestreo por Hipercubo Latino (MCLH) y Superficie de Respuesta con polinomios lineales (RSM-LIN) y cuadráticos (RSM-QUAX) son contundentes y se destacan los bajos costos computacionales logrados con los métodos RSM. Finalmente, se pronostican aplicaciones futuras en pórticos que incluyan no linealidad geométrica y de materiales, carga dinámica y otros escenarios de carga extrema.

Palabras clave: confiabilidad estructural; diseño de experimentos; elementos finitos; método de superficie de respuesta; métodos Monte Carlo; sensibilidades probabilísticas.

1. INTRODUCTION

In civil engineering, the study of the reliability of frames is essential for creating safe, durable, functional, and optimal structural designs. Structural frames are the cornerstone of buildings, bridges, and all infrastructure projects since they provide the necessary support and stability to face various types of loads and environmental conditions.

In practice, reliability analyses or probabilistic analyses of frames involve comprehensive evaluations and analyses to assess the structural integrity, performance, and resistance of the frame under different loading scenarios or conditions, including the uncertainties inherent to the random variables that affect the design. The objective of these studies is to identify possible failure modes, estimate the probability of failure (*Pf*) and its associated reliability index (β), and determine the hierarchy or importance of the factors contributing to the reliability of the structure. The concept of randomness can be included in the material properties, the geometric properties of the sections, the length of the members, the intensity of the loads, and even in the limits established by standards for the control of deflections, internal forces, or stresses of the structural elements, among other aspects.

Randomness is incorporated into the variables through a variety of probability distributions. Notable distributions include normal, lognormal, and extreme value distributions like Gumbel and Weibull. Some renowned authors in the field of reliability present different probability distribution functions for engineering [1]–[4].

Various techniques can be utilized for conducting probabilistic analysis of frames. These consist of first and second-order reliability methods (FORM/SORM), Monte Carlo (MC) simulation methods and their variations, and surrogate model-based approaches.

The FORM/SORM methods rely on approximating the limit state function of a structure by using a series expansion of first or second order at the design point. This design point is the most likely point of failure, located at a minimum multidimensional distance, and dependent on the set of random variables that control the value of the limit state function [1], [4]–[14]. The reliability index, also known as the minimum distance, is calculated through an optimization method. Meanwhile, the probability of failure is the value of the standard normal function evaluated at the negative of the reliability index. The FORM/SORM methods generally have good accuracy and efficiency for low dimensional and weak nonlinearity problems but may have many errors for high dimensional and strong nonlinearity problems [15].

Monte Carlo simulation methods generate many random samples for the input variables. The response of the structure is then evaluated for each sample by assessing the value attained by a predefined limit state function under service or ultimate conditions [10], [11], [16]–[25]. The probability of failure is

determined by counting the instances of detected failures, which occur when the limit state function yields a negative value in each sample. This method is robust and applicable across random spaces of any dimension. However, it does have the drawback of imposing a high computational cost when estimating small failure probabilities or addressing complex problems [26]. The direct Monte Carlo method has been enhanced with various improvements, one of which is the incorporation of Importance sampling (MCI) and The Monte Carlo with Latin Hypercube sampling (MCLH) [27]–[29]. The Latin Hypercube (LH) sampling technique employed in the MCLH method offers a notable advantage by incorporating memory in the sampling process, effectively preventing sample clustering. This feature improves sample distribution and enhances the method's overall accuracy.

Further advancements in Monte Carlo methods include parallelization, which can be implemented on workstations or clusters using parallelized simulation code. Alternatively, parallelized algorithms can be developed in Computational Algebra Systems like MATLAB [12], Fortran, or C++. These advancements in parallelization allow for more efficient and accelerated Monte Carlo simulations.

Alternatively, surrogate model methods involve creating a simplified model that approximates the response of the initial structural system. After constructing the surrogate model, a probabilistic sensitivity analysis is conducted on it. This decreases the number of evaluations needed for the initial structural system, typically represented by finite elements. Selecting the appropriate type and grade of surrogate model is crucial to enhancing the efficiency and accuracy of analysis through these methods [30].

Various techniques to create surrogate models are used, including Response Surface (RSM) [9], [20], [23], [31], kriging [32], [33], as well as statistical learning techniques [9], [34], [35]. For instance, replacing the finite element model in RSM methods consists of two phases: first, a design of the experiment phase [12] selects a set of carefully chosen samples to simulate and generate a database of input and output variables (typically the limit state function). In the second phase, the datdset is used to obtain a regression polynomial that substitutes the finite element model for future Monte Carlo simulations.

Probabilistic sensitivity analysis is a powerful technique that complements the conventional structural reliability assessment based on measuring reliability indices. This method allows analysts to account for the uncertainties of different variables, including material properties, loads, and boundary conditions. This approach enables a concurrent evaluation of all these factors on the structure's reliability, using either direct simulation techniques or surrogate models. In other words, many simulations are executed, each featuring a unique combination of values derived from the defined distributions for each random variable. This process generates a spectrum of potential outcomes, illustrating the variability inherent in the uncertain variables.

Statistical measures, such as sensitivity indices and correlation coefficients, are computed to estimate the sensitivity and significance of each uncertain variable's contribution to overall reliability. These metrics help identify the most critical variables impacting reliability, facilitating their prioritization in design optimization efforts [10], [11], [23].

This study significantly contributes to the detailed application of the Monte Carlo, Monte Carlo with Latin Hypercube sampling, and Response Surface Methods for evaluating structural reliability and identifying probabilistic sensitivities in a frame.

This study proposes a detailed probabilistic analysis for a frame inspired by the structural reliability problem presented by Haldar [16]. This approach aimed to include the highest level of complexity and uncertainty, resulting in a meticulous evaluation of statistical behavior and the probabilistic sensitivities of variables. Furthermore, this approach enabled the calculation of reliability indexes and comparing Monte Carlo and Response Surface methods. By analyzing the findings, we can establish important

criteria and make significant conclusions that aid in developing secure and effective frame designs. This study also serves as the foundation for analyzing the structural reliability of more theoretically and computationally complex portal frame structures. The paper includes the methodology, problem definition, results, conclusions, future work, and references.

2. METHODOLOGY

A parameterized FE and a probabilistic model were developed to calculate failure probabilities, probabilistic sensitivities, and CPU times using the direct Monte Carlo, Monte Carlo with Latin Hypercube Sampling, and Response Surface methods. Probabilistic models for addressing uncertainty and reliability analyses are built using the Ansys Probabilistic Design System (PDS), which operates based on the Parametric Design Language (APDL). This process enables constructing a parametric finite element model, solving it, and extracting important result parameters. A simultaneous variation of all parameters in a probabilistic sensitivity analysis is also performed to assess the limit state function's variability and identify the most sensitive variables [36]. The stages of the process are summarized in Figure 1.
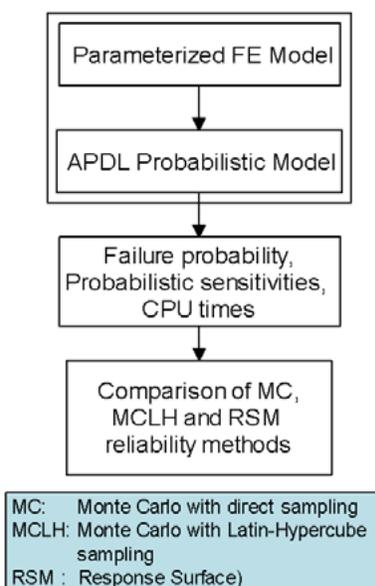

Fig. 1. *Methodology for the probabilistic analysis of the problem.*

The study introduced additional randomness to the cross-section area and moment of inertia of the canonic Haldar-Mahadevan portal frame illustrated in Figure 2 [16]. Stochastic behavior was also applied to the beam span, column height, and the limiting horizontal displacement of the frame. In addition, a Weibull extreme value distribution function (Type III smallest) was established for the live load (LL) and wind load (WW), as shown in Table 1. Thus, the proposed study expands Haldar and Mahadevan's canonical framework approach by including seven new random variables. Figure 2 shows the simulated frame's geometric configuration, constraints, and loads.
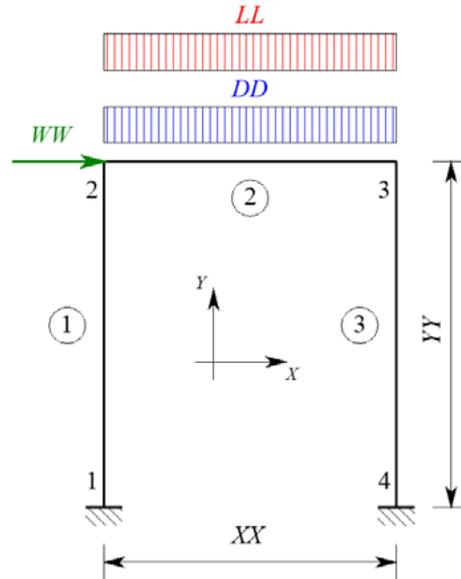
Fig. 2. *Geometry, constraints, and loads of the simulated frame.*

A service limit state function, *gg(X)*, has been defined in the problem according to Equation (1):

$$gg(\boldsymbol{X}) = ux_{lim} - u_2 \quad (1)$$

where **X** represents the set of random variables of the frame, including live, dead, and wind load cases (*DD, LL, WW*), the elastic modulus (*EE*), the beam span and column height (*XX, YY*), cross-section areas and moments of inertia for both beam and columns (A_1, I_1, A_2, I_2), the allowable deflection (ux_{lim}), the frame's horizontal displacement (u_2), and the deflection *u2*, which is calculated at node 2 and governed by the serviceability limit state (ux_{lim}). The deflection *u2* measures the random response of the structure. It is compared against the limit deflection (ux_{lim}) to evaluate a potential simulation failure when *gg(X)* results in negative values. The problem's set of random variables, **X**, is expressed as an 11-variable vector: (*DD, LL, WW, EE, XX, YY,* A_1, I_1, A_2, I_2, ux_{lim})^t.

It is crucial to note that the variability in the size of the portal frame is a result of the construction conditions. Additionally, the unpredictability in the horizontal limit displacement is linked to the frame's random geometric properties. These two sources of randomness are interrelated and enable evaluating how changes in the boundary condition can affect the probability of failure and, as a result, the reliability index.

The random variables of the problem are described by their statistical parameters in Table 1.

ANSYS defines the Weibull distribution as the Type III smallest, with three parameters known as *Xchr*, *Xmin*, and *m* [37]. *Xchr* represents the characteristic value of the random variable, *Xmin* is the minimum limit of the variable, and *m* is the shape factor. These parameters are determined based on the values of the mean (*μ*), coefficient of variation (*CV*), and standard deviation (*σ*). The steps to obtain these parameters are outlined below.

A value of *Xmin*=0 is assumed to obtain a two-parameter Weibull distribution, which is valid in the current study. Starting from the standard deviation calculated according to Equation (2):

$$\sigma = CV * \mu \quad (2)$$

and starting from expression (3) to calculate j,

$$1 + \left(\frac{\sigma}{\mu - X_{min}}\right)^2 = \frac{\Gamma(1+2j)}{\Gamma^2(1+j)}, j = \frac{1}{m} \quad (3)$$

it is possible to obtain the form factor *m* as the inverse of *j* (e.g., with the Excel Solver tool). However, it's important to note that Equation (3) includes the gamma function, making it inherently complex to solve.

The value of the random variable *Xchr* can be determined by Equation (4), which is derived from Equation (3) with *Xmin*=0:

$$X_{chr} = \frac{\mu}{\Gamma\left(1+\frac{1}{m}\right)} \quad (4)$$

Figure 3 illustrates the plot of function (3) within the predefined search intervals of the parameters for the random variables *LL* and *WW* [22], [38], [39]. Additionally, Table 2 displays the computed values for *m* and *Xchr* parameters.

Table 1. *Random variables and their statistical parameters.*

Variable	Unit	Mean Value	CV	Distribution
DD	k/ft	3.15	0.1	Normal
LL	k/ft	0.91	0.32	Weibull
WW	K	7.425	0.5	Weibull
EE	Ksi	29000	0.06	Normal
XX	Ft	15	0.05	Lognormal
YY	Ft	30	0.05	Lognormal
A_1	in²	10.6	0.05	Lognormal
I_1	in⁴	448	0.05	Lognormal
A_2	in²	20.1	0.05	Lognormal
I_2	in⁴	1830	0.05	Lognormal
ux_{lim}	In	0.9	0.05	Lognormal

Table 2. *Parameters of the Weibull distribution functions for LL and WW variables.*

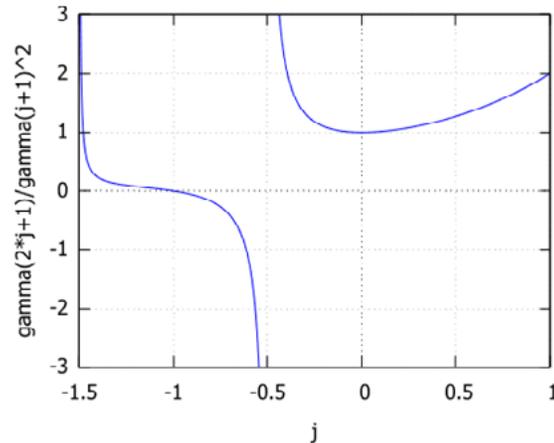| Variable		J	M	Xchr
LL	1.1024	0.289269404	3.456985035	1.01205514
WW	1.2500	0.475885501	2.101345801	8.38329558

Fig. 3. *j factor graph.*

3. RESULTS AND DISCUSSION

The frame model in Figure 4 is discretized into 35 BEAM3 finite elements. This element can support loads in tension, compression, and bending and has three degrees of freedom per node for x-translation, y-translation, and z-rotation.

Table 3 presents the results of the failure probability, reliability index, coefficient of variation of the probability of failure, *CVpf*, and CPU time obtained from the frame simulations for different probabilistic analyses. These methods include direct Monte Carlo, Monte Carlo with Latin Hypercube sampling, and Response Surface with Central Composite Design sampling (CCD) using linear regression (RMS-LIN) and quadratic regression (RMS-QUAX), which incorporates linear and quadratic terms, including cross terms. The CCD technique includes five probability levels for sampling points of each input random variable. These levels are not dependent on the probability density function of the variable [40].

Initially, the four methods were operated with a small number of simulations (identified by suffix 1). Later, they were used with a higher number of simulations (identified by suffix 2) to minimize the coefficients of variation. These coefficients of variation, expressed as a percentage, estimate the potential error in calculating the probability of failure. This percentage also reflects the accuracy of the failure probability and the reliability index assessment.

According to Table 3, the Response Surface Methods with Linear regression (RSM-LIN) and Quadratic regression (RSM-QUAX) are more effective in accurately predicting the probability of failure. This is because they simultaneously exhibit the highest reliability indexes and the lowest coefficient of variation values. The findings also suggest that a Response Surface method of linear order can effectively substitute the implicit finite element model of the frame in question. This implies that the finite element model employed to compute the limit state function, *gg(X)*, in each Monte Carlo simulation can be replaced with a multivariate linear polynomial.

The Monte Carlo method with Latin Hypercube sampling performs better than the direct Monte Carlo method as it reasonably decreases the computational time. Despite this advantage, neither is as effective as Response Surface methods that radically decrease the CPU time to less than one minute without sacrificing accuracy and precision.
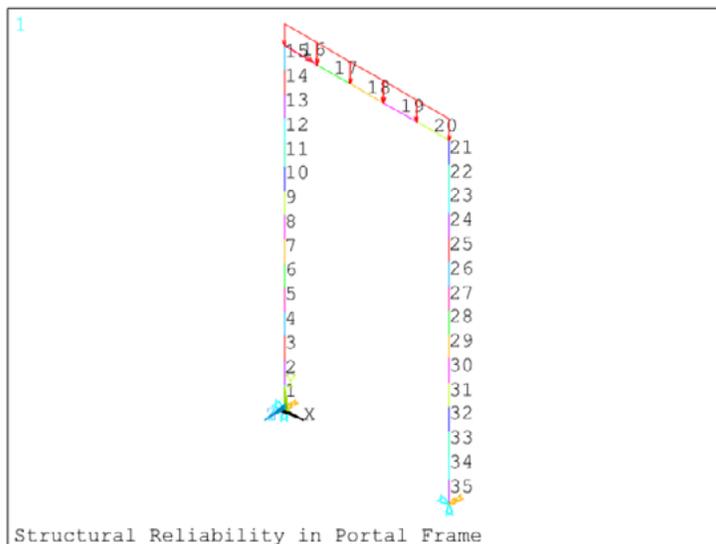
Fig. 4. *Mesh, numbering, restraints, and loading of the simulated frame.*

Table 3. *Reliability and CPU time results.*

Method	Failure probability P_f	Reliability Index (b)	CV_{Pf} (%)	CPU Time (min)
MC1 (10000 sim)	7.937E-02	1.409	3.406	11.233
MC2 (30000 sim)	7.226E-02	1.459	2.069	33.650
MCLH1 (5000 sim)	7.276E-02	1.456	5.049	5.917
MCLH2 (15000 sim)	7.489E-02	1.440	2.870	16.883
RSM-QUAX (100000 sim)	7.234E-02	1.459	1.132	0.333
RSM-LIN (100000 sim)	7.413E-02	1.446	1.118	0.333

In Figure 5, the reliability indices (β) are displayed on the primary vertical axis, while each method's coefficient of variation (CV_{Pf}) is shown on the secondary vertical axis. Based on this figure, it is evident that the RSM-LIN method outperforms other methods with high numbers of simulations, such as MC2 and MCLH2, as it has the lowest *CVPf* value.

In Figure 6, each method is examined concerning the reliability index and the coefficient of variation of the failure probability percentage, along with the corresponding CPU time in minutes and the number of simulations. The size of the circles corresponds to the value of CV_{Pf}, while the color of the circles indicates the CPU time in minutes. This figure demonstrates that based on the size of circles, RSM-LIN, RSM-QUAX, MC2, and MCLH2 are the best-performing methods, in that order. However, when considering the CPU time indicated by the colors, the optimal methods are RSM-LIN, RSM-QUAX, MCLH2, and MC2, orderly. Therefore, RSM and MCLH are the preferred choices for analyzing the reliability of the frame in question. These results emphasize the significance of both accurate calculations and efficient CPU times.

The model size can be small when dealing with a small portal frame, as in this case; nevertheless, the computational time can significantly increase when working on frames for medium-rise buildings. This effect becomes even more pronounced when dealing with high-rise frames. Therefore, it is essential to prioritize the selection of a fast and reliable method to conduct performance evaluations of these structures.

Figures 7 to 10 show the approximate distribution of the *gg(X)* limit state function calculated with the MC2, MCLH2, RSM-QUAX, and RSM-LIN versions through histogram plots. The service limit state function, *gg(X)*, is denoted as GG, hereafter.
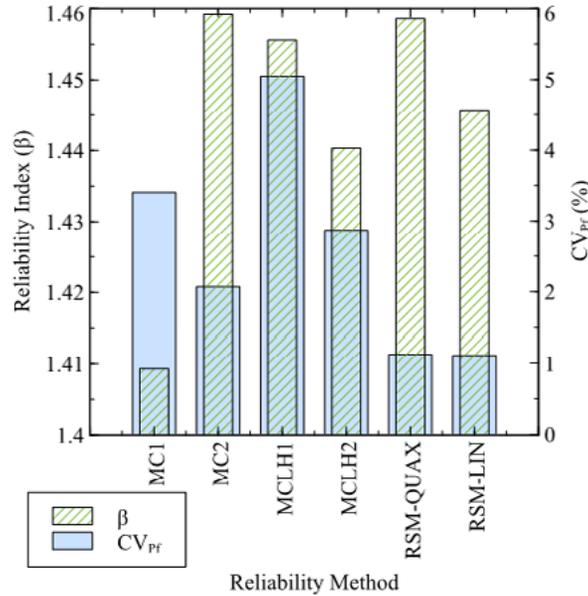

Fig. 5. *Bar graph of the reliability index and coefficient of variation (%) for all methods.*
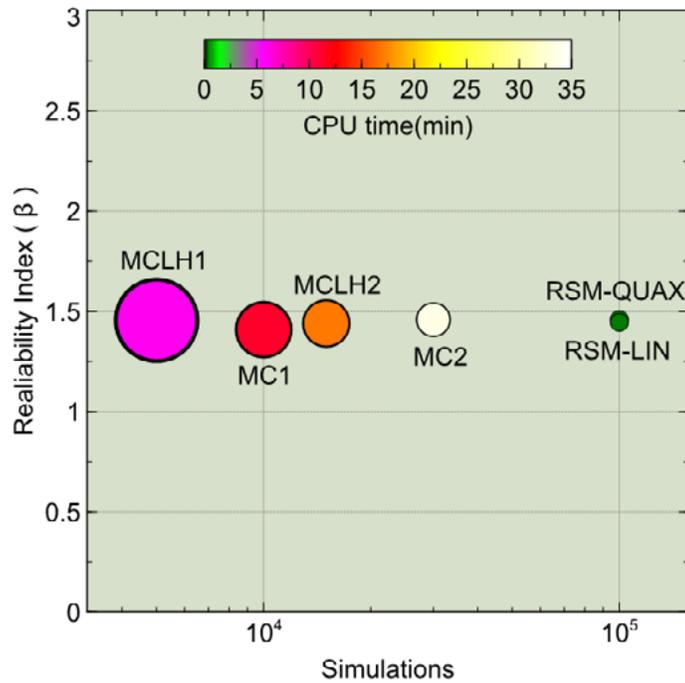

Fig. 6. *Multivariate plot of simulations, reliability index, CPU time, and coefficient of variation of the failure probability.*
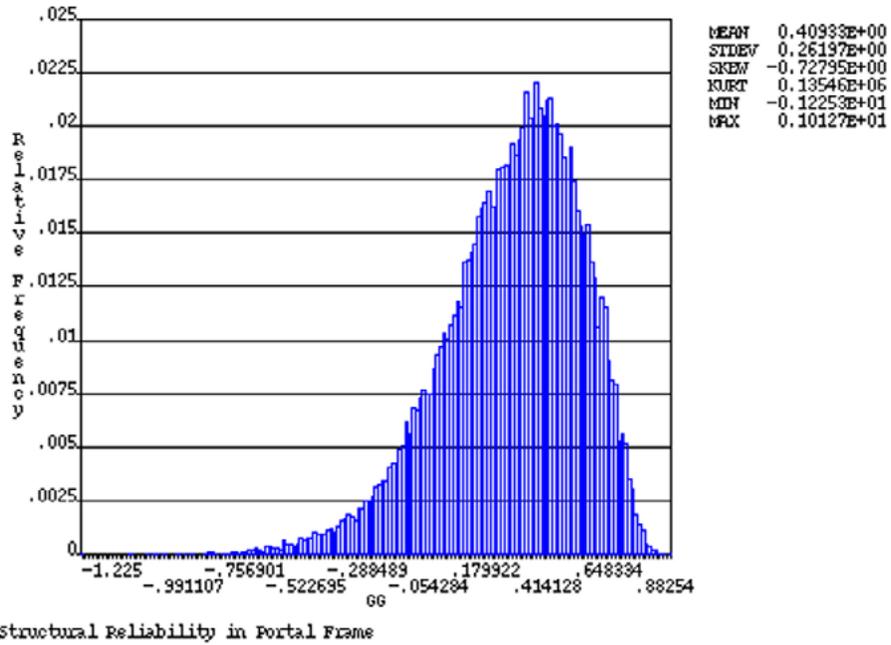
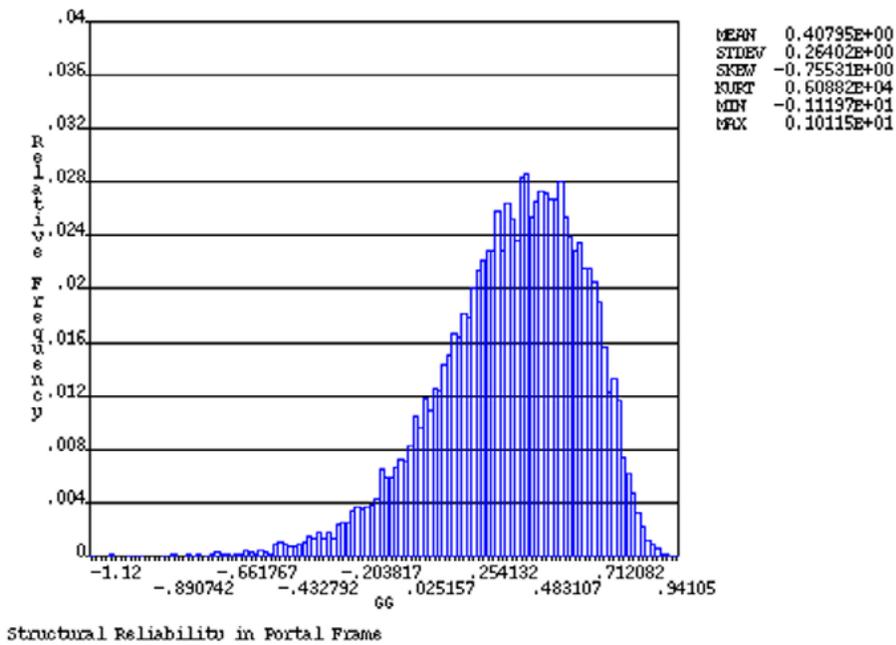**Fig. 7.** *Histogram of the limit state function GG - Monte Carlo (MC2).*

Fig. 8. *Histogram of the GG limit state function - Monte Carlo with Latin Hypercube sampling (MCLH2).*
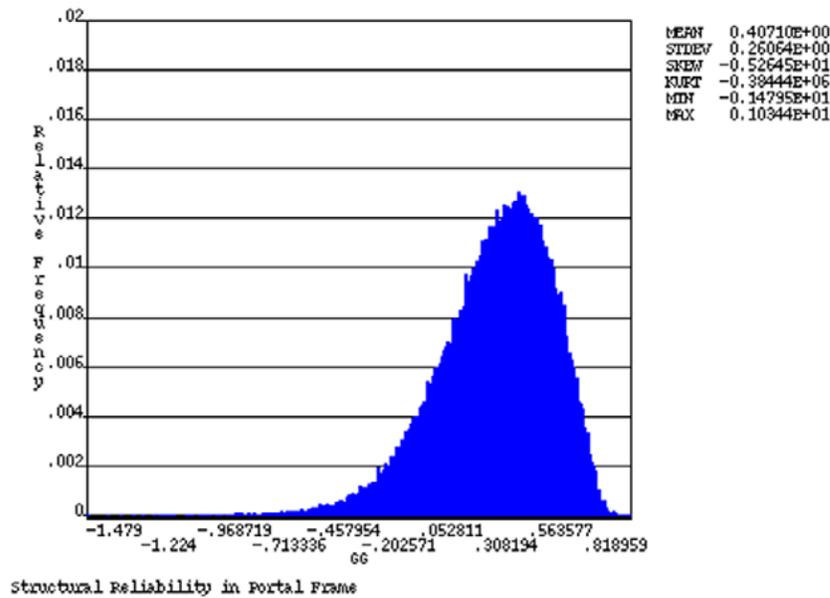
Fig. 9. *Histogram of the limit state function GG - Response Surface with full quadratic regression (RSM-QUAX).*
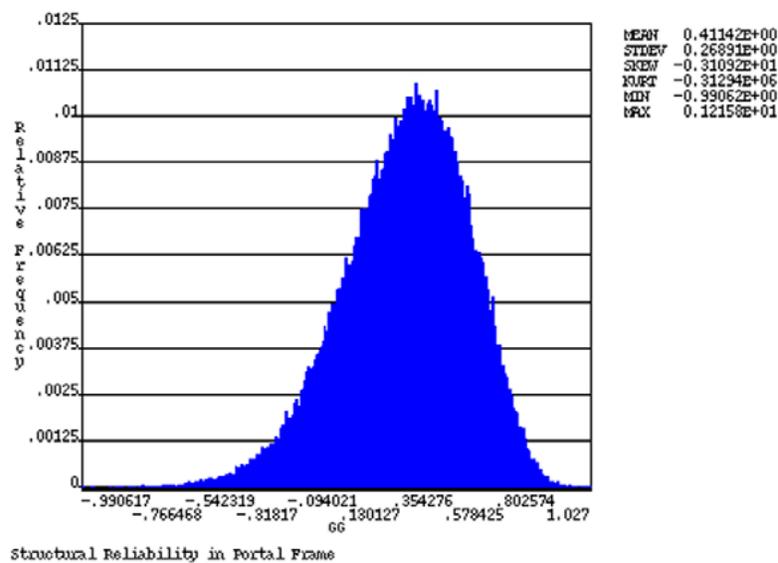

Fig. 10. *Histogram of the limit state function GG - Response Surface with Linear Regression (RSM-LIN).*

The histograms in Figures 7 to 10 also show negative skewness coefficients (SKEW), thus indicating a right-skewed distribution; that is, the values tend to cluster towards the right of the mean. This behavior can be attributed to using two Weibull-type extreme value distribution variables (LL and WW) in the limit state function distribution, which induces skewness.

The probabilistic sensitivities were calculated using Spearman's order correlation coefficient in all methods. This measurement determines the significance level of a random variable about the limit state function [10], [23], [40]. The ANSYS Probabilistic Design System (PDS) presents significant variables through clear, easy-to-understand bars and pie charts. The PDS uses a hypothesis test based on Student's t distribution to identify the most sensitive variables [37]. The graph provides a list of both significant and insignificant variables. Meanwhile, the bar chart illustrates the most sensitive random variables in

order of importance and their corresponding sensitivity value. These variables are arranged from left to right on the graph, with the most sensitive variable on the left and the least sensitive on the right.

Comprehending the correlation between independent and dependent variables is crucial when conducting a probabilistic structure analysis. A positive sensitivity value establishes a direct relationship between these variables, while a negative value represents an inverse relationship. For this case, the limit state function is the dependent variable, and the independent variables include the eleven random variables examined.

The pie chart displays sensitivities in a ratio that is always positive and relative to each other. The plot starts at 12 o'clock and moves clockwise and the most significant data points come first [10], [40].

The probabilistic sensitivities between the random variables and the limit state function for the MC2, MCLH2, and RMS-LIN methods are compared in Figures 11 to 13 to verify the impact of the sampling method and its type. The UXLIM variable is the limit deflection (ux_{lim}), hereafter.

After analyzing the graphs obtained from the MC2, MCLH2, and RMS-LIN methods, it is evident that there are no substantial variations in the evaluation of sensitivities at the quantitative level or in the ranking of variable sensitivity. This observation highlights the effectiveness and practicality of using the RSM method for computing the probabilistic sensitivities in the analyzed problem.

From sensitivity graphs, it was found that the wind load (WW) is the most significant variable as it directly affects the horizontal displacement of the frame and is controlled by the limit state function. The second most significant variables are the column height (YY) and the UXLIM, which are related to the limit control criteria for horizontal deflection used in building seismic design standards. According to their level of significance, the next variables are the modulus of elasticity (EE), followed by the moment of inertia of the column (I2), the moment of inertia of the beam (I1), and the beam span. The moment of inertia of the column (I2) is more significant because it directly influences the control of lateral deflection established by the limit state function.
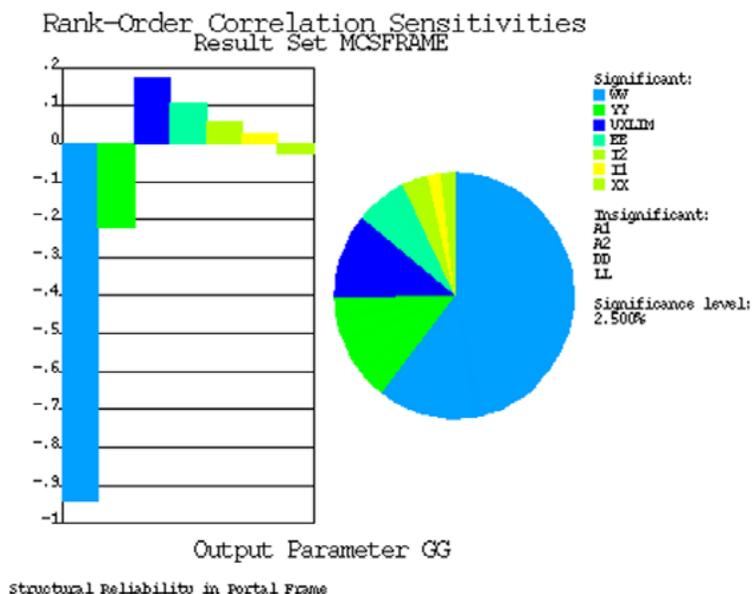

Fig. 11. *Bar and pie chart of probabilistic sensitivities between random input variables and limit state function GG for MC2.*

The variables A1, A2, DD, and LL are insignificant when considering the limit state function. However, it's worth noting that DD and LL, representing vertical loads, may play a substantive role in analyzing the portal frame's vertical deflections.

The influence of the two most sensitive variables on the limit state function can be seen in Figures 14 and 15. Figure 14 displays the response surface for wind load WW using the RSM-QUAX method, while Figure 15 displays the response surface for the height of the portal frame YY using the RSM-LIN method.
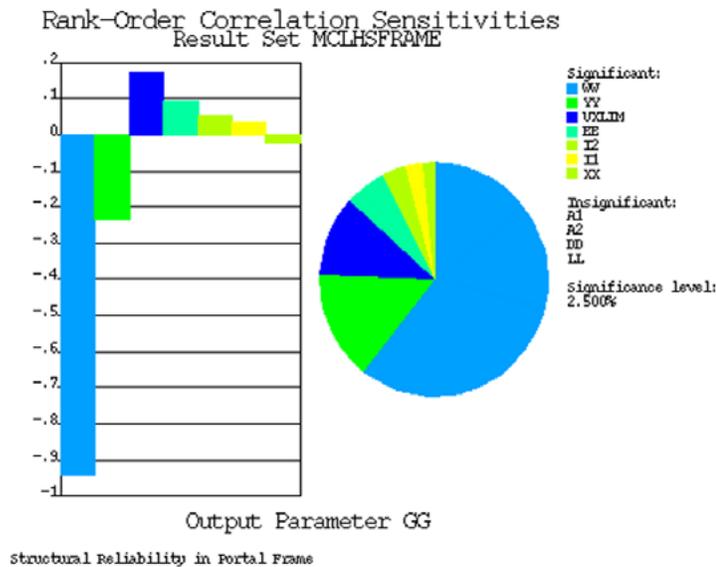

Fig. 12. *Bar and pie chart of probabilistic sensitivities between random input variables and limit state function GG for MCLH2.*
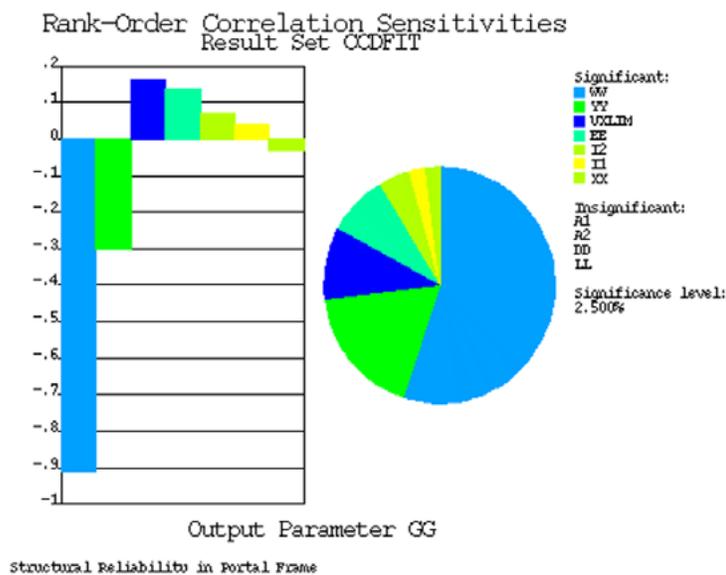

Fig. 13. *Bar and pie chart of probabilistic sensitivities between random input variables and limit state function GG for RSM-LIN.*
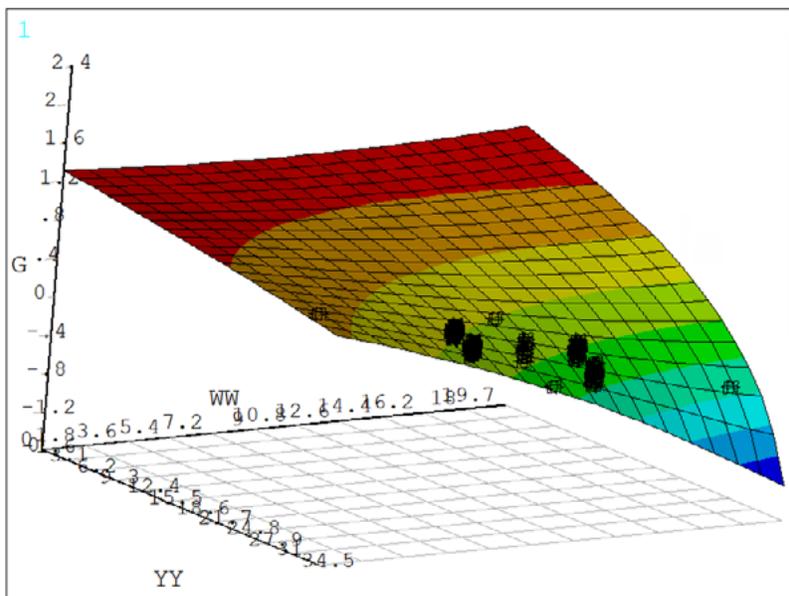
Fig. 14. *Explicit limit state function for RSM-QUAX full quadratic regression.*
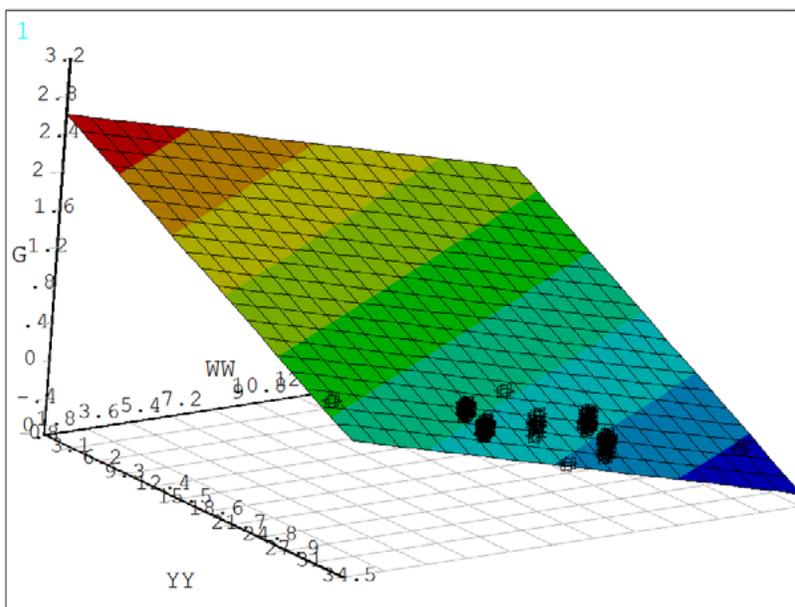

Fig. 15. *Explicit limit state function for RSM-LIN linear regression.*

The response surface in Figure 14 using the RSM-QUAX method confirms the presence of quadratic curvature, whereas the response surface in Figure 15 obtained through the RSM-LIN method suggests a plane behavior. Despite its linear approximation, the RSM-LIN method is enough to yield satisfactory outcomes in just 0.333 minutes.

Based on the balance between accuracy and computational cost, it is clear that the RSM method is the most efficient approach for structural reliability analysis. Following closely behind is the MCLH method, with the Monte Carlo method being the least efficient option.

4. CONCLUSIONS

This study presents an in-depth probabilistic analysis of a structural portal frame's response. The model incorporates 11 parameters as random variables, with a limit state function controlling the horizontal deflection. Various probabilistic analyses were conducted using Monte Carlo, Monte Carlo with Latin Hypercube sampling, and Response Surface methods to address the problem at hand. Furthermore, probabilistic sensitivity analyses were performed to gauge the impact of uncertainty in random variables on the frame's response. The failure probabilities, reliability indices, coefficients of variation, and CPU times to determine the best-performing probabilistic analysis method were also calculated. Finally, the multivariate probabilistic approach allowed for assessing each variable's importance in the frame's performance. After a thorough analysis of the performance of the evaluated methods for calculating the structural reliability of the portal, the following was determined:

· The most advantageous method is the RSM. A significant reduction in computational cost is achieved by replacing the finite element model with a polynomial obtained from experimental design and regression. With just 20 seconds of CPU time, up to 100,000 simulations can be performed with highly satisfactory results in structural reliability. A coefficient of variation of failure probability close to 1% is considered accurate, and coefficients below 3% are recommended.

· The second most efficient method is the Monte Carlo method with Latin Hypercube sampling. Although it is slower than the RSM, its intelligent sampling approach with memory reduces the number of simulations without compromising accuracy beyond a normal limit.

· The direct Monte Carlo method or direct sampling is recommended only for small structures or when software and hardware adapted to computational parallelization are available.

The influence of random variables on the probability of failure of the frame was examined by calculating probabilistic sensitivities. These quantify the impact of each independent variable relative to the other variables. Consequently, the order of significance of the variables was determined as follows: wind load (WW), portal height (YY), horizontal displacement limit (UXLIM), modulus of elasticity (EE), column moments of inertia (I2), beam moment of inertia (I1), and portal beam span (XX).

Based on the probabilistic sensitivity analysis, it was found that the most important variables affecting the limit state function are WW (55.04%), YY (18%), UXLIM (9.87%), EE (8.30%), and I2 (4.40%). These statistics were obtained through the RSM method using a polynomial derived from linear regression and 100,000 simulations. Conversely, the cross-sectional area of the beam (A1), area of the columns (A2), dead load (DD), and live load (LL), representing the two vertical loads, were determined as insignificant factors for the limit state function.

Future research on structural reliability in 2D and 3D frames should include additional randomness variables such as dynamic seismic loads, geometric and material nonlinearity, and extreme load cases.

AUTHORS' CONTRIBUTION

Wilson Rodríguez-Calderón: Conceptualization; Methodology; Formal analysis; Investigation; Writing-original draft; Writing-review and editing. **Myriam-Rocío Pallares-Muñoz:** Conceptualization;

Methodology; Formal analysis; Investigation; Writing-original draft; Writing-review and editing. **Carlos Jurado-Cabañes:** Conceptualization; Methodology; Formal analysis; Investigation; Writing-review and editing.

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