

## **Dynamic and static measurements of small strain moduli of toyoura sand**

### **Medidas estáticas y dinámicas del módulo de elasticidad a pequeñas deformaciones en arena *toyoura***

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#### **Abstract**

A series of triaxial tests were performed on specimens made of Toyoura sand having different dry densities. Very small unloading/ reloading cycles were applied to obtain “static” Young’s modulus. Furthermore, as the “dynamic” measurements, two types of wave propagation techniques were applied; one using Bender elements, and the other one using piezoelectric actuators to excite primary and secondary waves. Based on these static and dynamic measurements, we discuss the following aspects: 1) the difference among the three types of dynamic measurements, 2) the relationships between the dynamic and static measurement results and 3) the effects of stress state-induced anisotropy and dry densities of the specimens.

----- **Keywords:** triaxial test, small strain cyclic loading, primary wave velocity, secondary wave velocity, accelerometer, bender element, Young’s modulus, shear modulus.

#### **Resumen**

Se realizó una serie de ensayos triaxiales en arena de *Toyourea* con distintas densidades secas. Se aplicaron ciclos muy pequeños de carga y descarga para obtener el módulo de Young “estático”. Además, como medidas “dinámicas”, se emplearon dos tipos de técnicas de propagación de ondas; una utilizando elementos Bender, y la otra utilizando actuadores piezoeléctricos para pro-

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vocar ondas primarias y secundarias. Basados en estas medidas estáticas y dinámicas, se discute: 1) la diferencia entre los tres tipos de medidas dinámicas empleadas, 2) la relación entre los resultados de las medidas estáticas y dinámicas y 3) el efecto del estado de esfuerzos en la anisotropía inducida y la densidad seca del material

----- *Palabras clave:* ensayo triaxial, ciclos de carga en pequeñas deformaciones, velocidad de onda primaria, velocidad de onda secundaria, acelerómetros, Bender element, módulo de Young, módulo cortante.

## Introduction

It is well known that the ground deformation under the normal working condition is generally less than 0.1% of strain. In soil mechanics, it is normally assumed that the ground consists of a continuum and that its behavior is linear and recoverable within a very small strain range i.e. less than  $10^{-3}$  % [1]. Therefore, “elastic” deformation properties of soil like Young’s modulus and shear modulus play an important role in designing civil engineering structures. In order to obtain these parameters through in-situ tests, it is common to use PS logging by cross hole, down hole and suspension sonde methods, while as laboratory tests to evaluate these properties, resonant column, torsional shear and triaxial tests as well as Bender elements are commonly used.

In this study, Toyoura sand specimens having different dry densities were subjected to triaxial tests. Very small unloading/reloading cycles were applied at several stress states, and strains were measured locally using local deformation transducers at the side surface of the specimen. This method is called as “static” herein. For the “dynamic” measurement, two types of wave propagation techniques were applied. One was using Bender elements which have been widely used for laboratory testing. The other was composed of piezoelectric actuators and accelerometers made of ceramics, which was developed by Anh Dan et al. [2].

Based on these static and dynamic measurements, elastic moduli are compared with each other, especially focusing on the following issues; 1) the difference among the three types of dynamic measurements, 2) the relationships between the dynamic and static measurement results and 3) the effects of stress state-induced anisotropy and dry densities of the specimens.

### Tested material, equipment and test procedures

#### *Specimen preparation and apparatus*

The tested material was Toyoura sand (mean grain diameter,  $D_{50} = 0.21$  mm, density of solid

particles,  $\rho_s = 2.656$  g/cm<sup>3</sup>, maximum void ratio,  $e_{\max} = 0.992$ , and minimum void ratio,  $e_{\min} = 0.632$ ) having relatively uniform gradation as shown in Figure 1. Air dried Toyoura sand particles were pluviated through the air to prepare a cylindrical specimen with dimensions of 5 cm diameter and 10 cm height. Specimens having relative densities,  $D_r$ , of about 50 and 80 % (the dry density  $\rho_d$  equal to 1.454 and 1.553-1.568 g/cm<sup>3</sup>, respectively) were prepared to study the effect of density.

For the triaxial apparatus employed in this study, an AC servo motor was used in the loading system so that very small unloading/ reloading cycles (cyclic loading) under strain control could be applied accurately to the specimen in the vertical direction. To measure the vertical stress,  $\sigma_1$ , a load cell is located just above the top cap inside the triaxial cell in order to eliminate the effects of piston friction [3]. The vertical strain  $\varepsilon_1$  was measured not only with the external displacement transducer but also with a pair of vertical local deformation transducers (LDTs) [4] located on opposite sides of the specimen. The horizontal stress  $\sigma_3$  was applied through the air in the cell, which was measured with a high capacity differential pressure transducer (HCDPT).

### Dynamic measurement using piezoelectric actuators and accelerometers

To generate primary wave (P wave) and secondary wave (S wave), a special type of wave source (called trigger) was employed (Figure 2a). It was composed of a multi-layered piezoelectric actuator made of ceramics (dimensions of 10 mm x 10 mm x 20 mm, mass of 35 grams and a natural frequency of 69 kHz), and a U-shaped thick steel bar to provide reaction force. Triggers were used in pairs to apply large excitation equally [5]. To receive the dynamic waves, piezoelectric accelerometers (cylindrical in shape with a diameter of 3.6 mm, a height of 3 mm, a mass of 0.16 grams, and natural frequency of 60 kHz) as shown in Figure 2b were used.

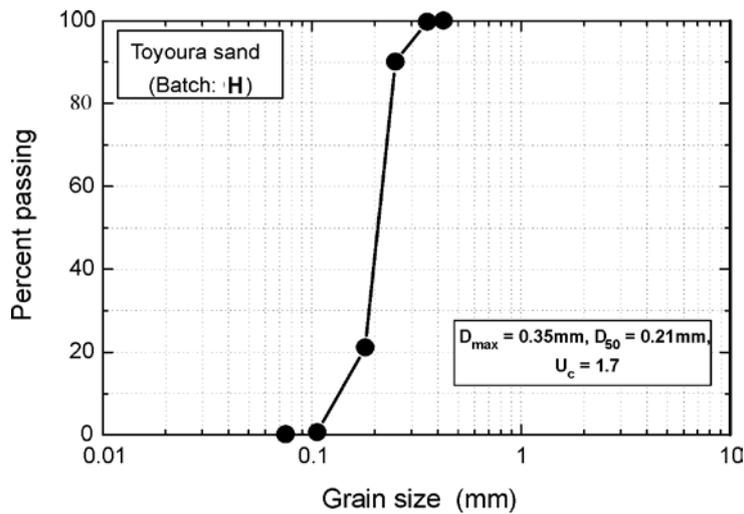


Figure 1 Grain size distribution of Toyoura sand

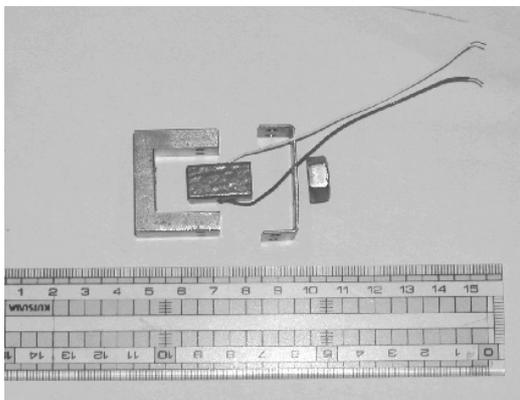


Figure 2a. Components of a trigger

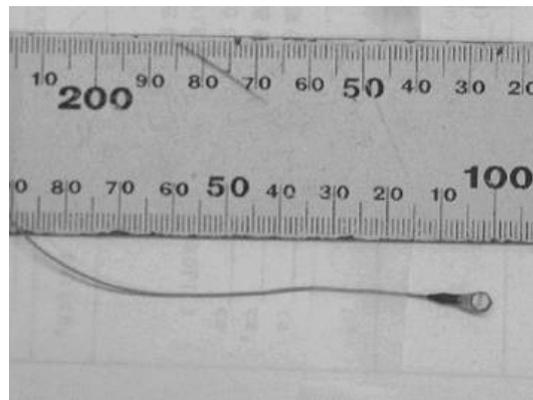


Figure 2b. Accelerometer

P wave is a vertically-transmitting compression wave given by a couple of triggers set on the top cap, as denoted “PT” in Figure 3. S wave is a vertically-transmitting torsional wave given by the other couple of triggers set to the opposite sides of the top cap, “ST” in Figure 3. These dynamic waves were received by three pairs of accelerometers which were glued on the side surface of the specimen at two different heights as shown in Figure 3 (“P” and “S” in the circles, respectively).

One pair on “side A” was for P wave measurement to evaluate Young’s modulus, and the other two pairs on both sides A and B were for S wave measurement to evaluate shear modulus.

### Dynamic measurement using Bender elements

Bender elements are small piezo-electrical transducers which either bend as an applied voltage is changed or generate a voltage as they are bent (figure 4a and figure 4b). In this study, electrical signal was sent to the Bender element fixed to the top cap to use it as a trigger. It was set vertically, shown as “BE” in figure 3, so that it generates vertically-transmitting horizontally-polarized waves, equivalent to the S wave triggered by the piezoelectric actuators, and they were received by the other Bender element fixed to the pedestal.

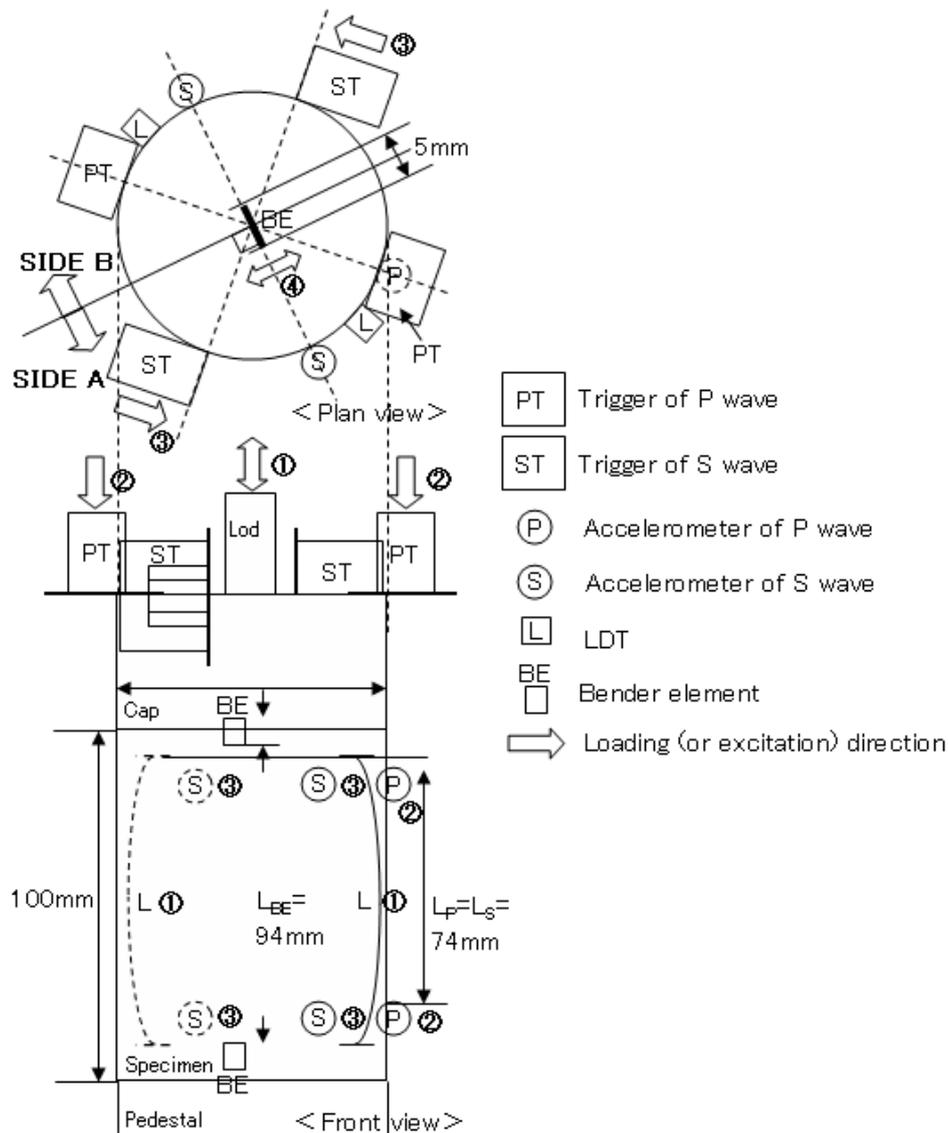


Figure 3 Schematic of specimen and location of all the equipment

### Recording techniques of dynamic waves

A digital oscilloscope was employed to record electrical outputs from the accelerometers and Bender elements with an interval of  $10^{-6}$  sec. To obtain clear signals, a stacking (averaging) technique which had been originally installed in the oscilloscope was introduced instead of using any filtering methods [6]. The number of stacking was 256 with the Bender elements and 128 with the accelerometers.

### Testing procedures

A flow chart of the procedures for each measurement is shown in **Figure 5**. Each specimen was kept air-dried and subjected to isotropic or anisotropic consolidation. During anisotropic consolidation, the ratio of the horizontal stress to the vertical one was kept at 0.5. After the vertical stress,  $\sigma_v (= \sigma_1)$ , reached to 50, 100, 200, and 400 kPa, the dynamic and static measurements were conducted.

First, small cyclic loading (11 cycles, triangular wave, axial strain rate,  $\dot{\epsilon}_v \sigma$ , equal to 0.05%/min) was applied to evaluate vertical Young's modulus statically. Second, electric voltage of +20 volt in the form of single pulse were supplied to the triggers for exciting P and S waves to evaluate Young's modulus and shear modulus, respectively. Finally, single pulse wave (+20 volt) and single sinusoidal waves ( $\pm 10$

volt) at different frequencies which range from 3 to 30 kHz were supplied to the Bender element to evaluate shear modulus.

Under these conditions, three tests were performed. The first test was under isotropic consolidation with Dr of about 50%, named "IC50"; the second was under isotropic consolidation with Dr of about 80% (IC80), and the last was under anisotropic consolidation with Dr of about 80% (AC80).

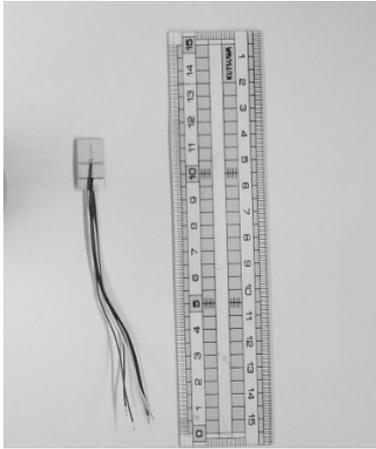


Figure 4a Bender element



Figure 4b Set-up condition of Bender element (at pedestal)

## Evaluation procedures of static and dynamic moduli

### Evaluating elastic modulus

Typical stress-strain relationship during a small vertical loading cycle is shown in figure 6. At each stress state, the stress-strain relationship was fitted by a linear function, and the small-strain vertical Young's modulus,  $E_s$ , was evaluated from its inclination.

In computing dynamic Young's modulus  $E_D$ , we assumed that unconstrained compression waves were generated because the top cap touched the whole cross-section of the specimen that was with free side boundaries. The P wave velocity  $V_p$  is directly related to the small-strain vertical Young's modulus of the material by the dynamic measurement as:

$$E_D = \rho V_p^2 \quad (1)$$

Where  $\rho$  is the dry density of the specimen. In the same manner, dynamic shear modulus  $G_D$  was evaluated from the S wave velocity  $V_s$  as:

$$G_D = \rho V_s^2 \quad (2)$$

The compression wave velocity  $V_p$ , shear wave velocities measured with the accelerometers and the Bender element,  $V_{S\_Acc.}$  and  $V_{S\_BE}$ , respectively were calculated by the same equation as follows:

$$V = L/t \quad (3)$$

Here the wave velocity "V" can be replaced with " $V_p$ ", " $V_{S\_Acc.}$ " or " $V_{S\_BE}$ "; while substituting the distance "L" and the corresponding travel time "t" with  $L_p$ ,  $L_s$ , or  $L_{BE}$  and  $t_p$ ,  $t_s$ , or  $t_{BE}$ , respectively. The distances  $L_p$ ,  $L_s$ , and  $L_{BE}$  are the differences in the vertical locations between each pair of accelerometers or Bender elements as defined in Figure 3.

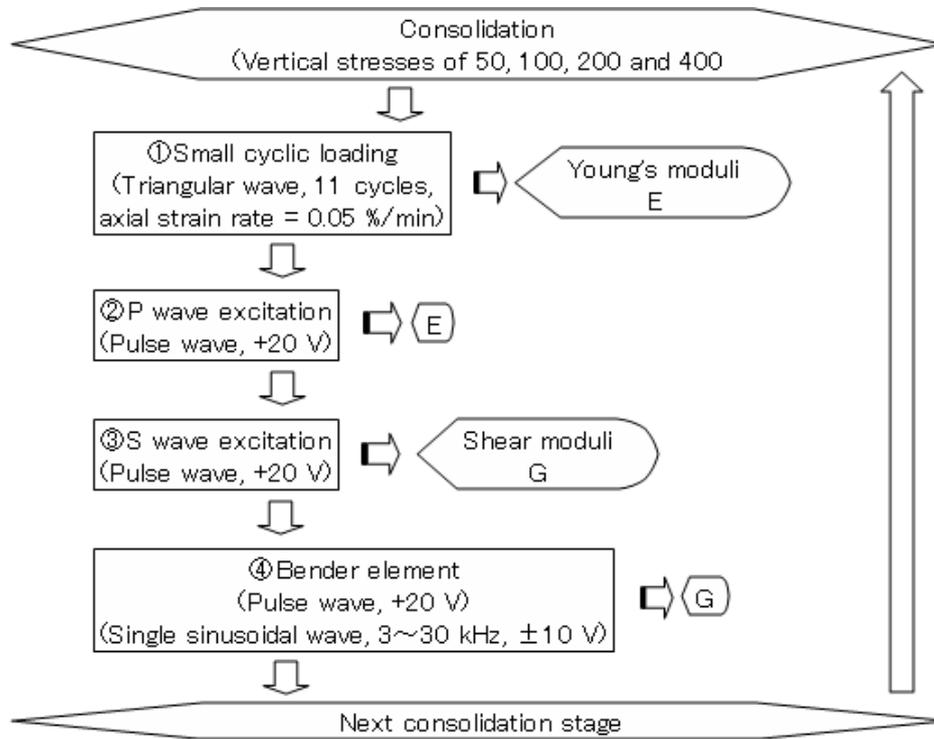


Figure 5 Flow chart of testing procedures for each measurement

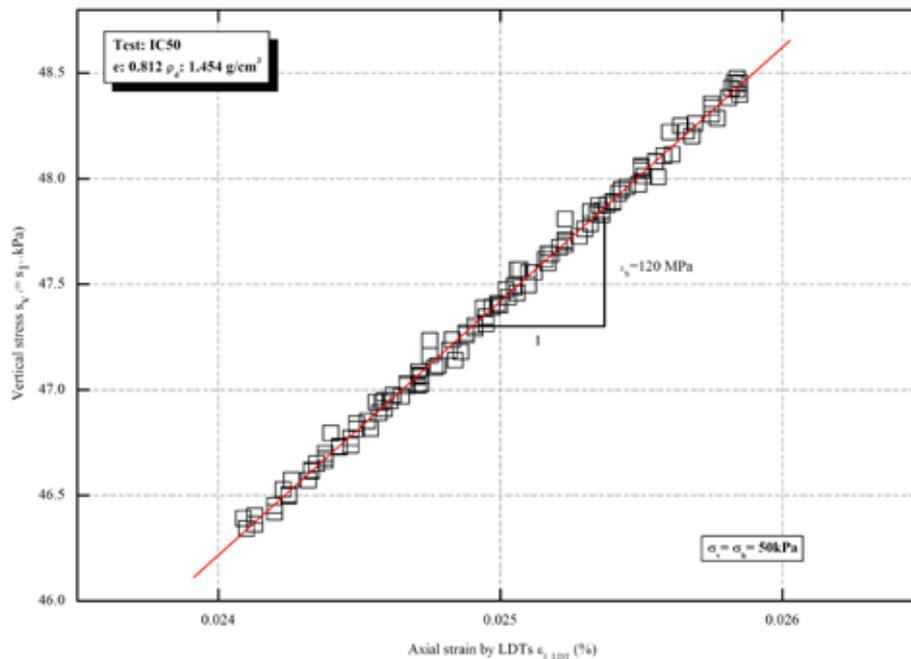


Figure 6 Typical stress-strain relationship during a small vertical loading cycle

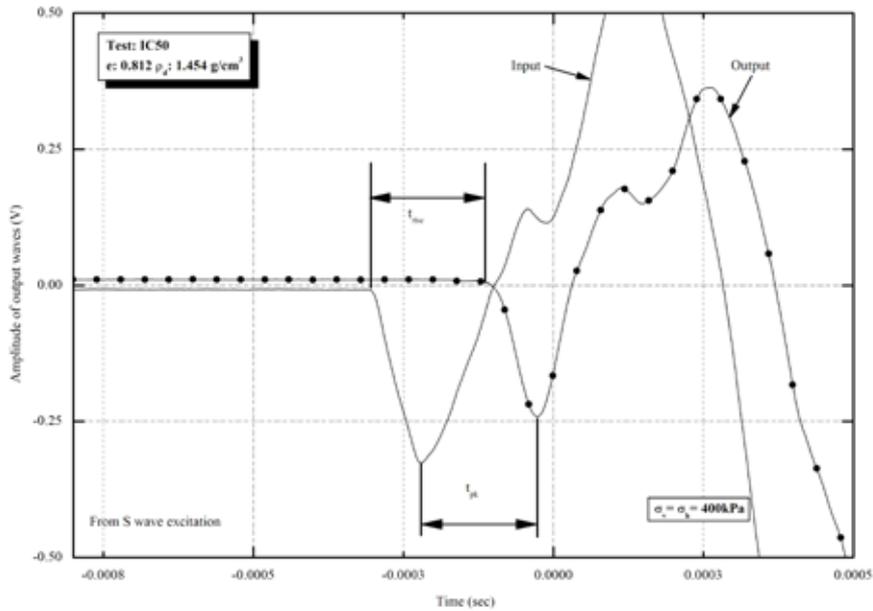
In these evaluations, it was assumed that the dry density was kept constant as the volume change due to consolidation of the air-dried specimen could not be measured. On the other hand, the effect of vertical deformation obtained from LDTs at each stress state was considered in the calculation of the distances  $L_P$ ,  $L_S$ , and  $L_{BE}$ .

### Travel time definitions

Based on the records of accelerometers measuring P and S wave excitations, the time difference in the first wave arrival between the upper (input) and the lower (output) accelerometers was computed. Although there are several ways to detect the travel time [7], in this study two types of simple definitions were employed.

In the first technique, as shown in Figure 7, “ $t$ ” was computed based on the first rising points of the input and output waves. It will be hereafter called as “first rise to first rise” technique and denoted as “ $t_{rise}$ ”. In this method, a problem lies with the decision of evaluation of the first rising point in both signal records. The outcome may depend on observer’s judgment.

In the second technique, “ $t$ ” was determined using the first peak points of input and output waves as shown in Figure 7. It will be called “peak to peak” technique and denoted as “ $t_{pk}$ ”. In this technique, the observer has to decide the correct peak points. In most of the cases, it’s easier to do so than deciding the correct rising points, while there were some exceptions in the P wave records (as will be discussed later).



**Figure 7** Definitions of travel time “ $t$ ” using; first rise and first peak techniques

In case of Bender elements, the time difference between the input wave to the Bender element at the top cap and the output one from the Bender element at the pedestal was used, since it was confirmed that the Bender element used as the trigger responds against the input signal without any significant time lag. Travel time definitions are the same with the one explained above.

Figure 8 shows some time-history data from the Bender element under the vertical stress equal to 200 kPa in test AC80. A single pulse and some representative single sinusoidal wave records are included. The upper graph on the left presents all the input wave records, while the others show each output wave record. From the output records, the first rising point can not be identified

clearly. Besides this, although all the input waves were transmitted in the same direction, it seems that the first peak with the pulse wave excitation is on the negative side, while all of the first peaks with the sinusoidal waves are to the opposite, i.e.

positive. In spite of such kinds of difficulties in evaluating the travel time, the first rising points or peak points were determined rather empirically, while referring to the data obtained under different input wave conditions as well.

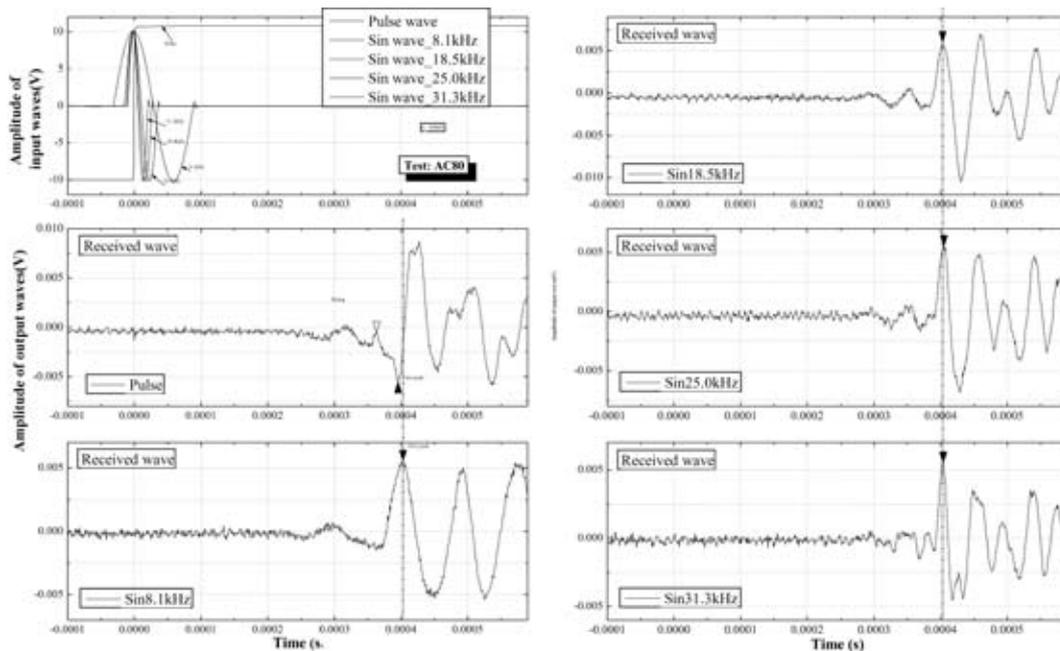


Figure 8 Time-history data from Bender element

### Test results and discussion

To compare among different measurement techniques, the Young’s moduli obtained from small cyclic loading and P wave excitation,  $E_v$ , are converted to shear moduli  $G_{vh}$ . In general, isotropic elasticity modeling is employed for the conversion by using the following equation:

$$G_{vh} = \frac{E_v}{2(1+\nu)} \quad (4)$$

where  $\nu$ : Poisson’s ratio (set equal to the value of  $\nu_0$ , as will be shown later)

On the other hand, another equation based on an anisotropic elasticity modeling was proposed by [8]. Since this model can consider both the inherent and stress state-induced anisotropy, this equation is employed in this study to consider only the stress state-induced anisotropy. The

equation and the settings of each coefficient are as follows:

$$G_{vh} = \frac{E_v}{2(1+\nu_0)} \frac{2(1-\nu_0)}{1+aR^n - 2\sqrt{a} \cdot R^{n/2} \cdot \nu_0} \quad (5)$$

where  $\nu_0$ : reference Poisson’s ratio (set as 0.17, by referring to [9] .

a: coefficient on the degree of inherent anisotropy (set as 1.0, by neglecting the possible effect of inherent anisotropy),

R: stress ratio ( $= \sigma_v/\sigma_h$ )

n: coefficient on the degree of stress-state dependency (set as 0.44, referring to the results from small cyclic loading)

It should be noted that Equation (4) can be drawn from Equation (5) when isotropic conditions are applied to the value of a and R, i.e.  $a = R = 1.0$ .

Relationships between shear moduli and the stress levels based on all the static and dynamic measurements in each test are shown in figures 9 - 11. In the legend of these figures, “Peak” or “Rising” means that “ $t_{pk}$ ” or “ $t_{rise}$ ” was selected as the travel time. The results obtained by Bender elements with sinusoidal waves (denoted as “Bender elements\_Sin\_Peak”) are based on the average value evaluated with clear wave records at different frequencies. As clearly seen in figures 9 - 11, shear moduli increased with the increase in the stress levels, and their relationships were linear in logarithmic scale.

In general, the values of shear moduli of denser specimens in the tests IC80 and AC 80 were larger than those of the looser specimen in the test IC50. It

should be noted that all the values of shear moduli converted from the static Young’s moduli, denoted as “Cyclic loading” in these figures coincided with the lower bound of the dynamic measurements, i.e. all the values by dynamic measurements were larger than those by the static measurements. Detailed discussion on the differences between static and dynamic measurements.

In each test, the shear moduli by S wave excitation measured on both sides A and B (refer to figure 3) can be regarded as similar to each other under each travel time definition. Accordingly, averaged values measured on both sides will be regarded as the representative of whole the specimen and compared with those by other measurements in the latter discussions.

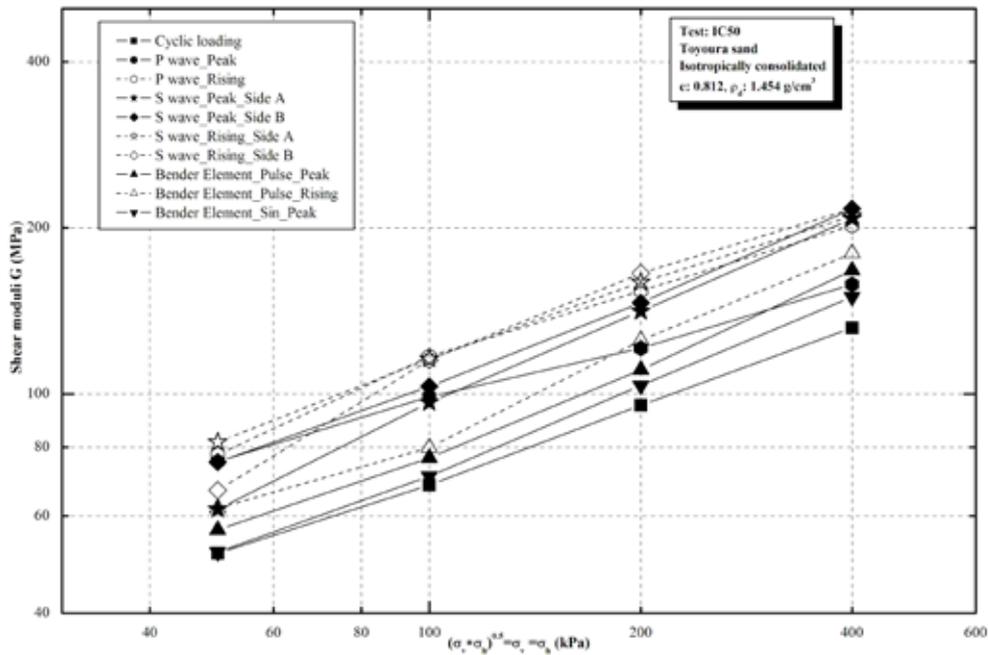
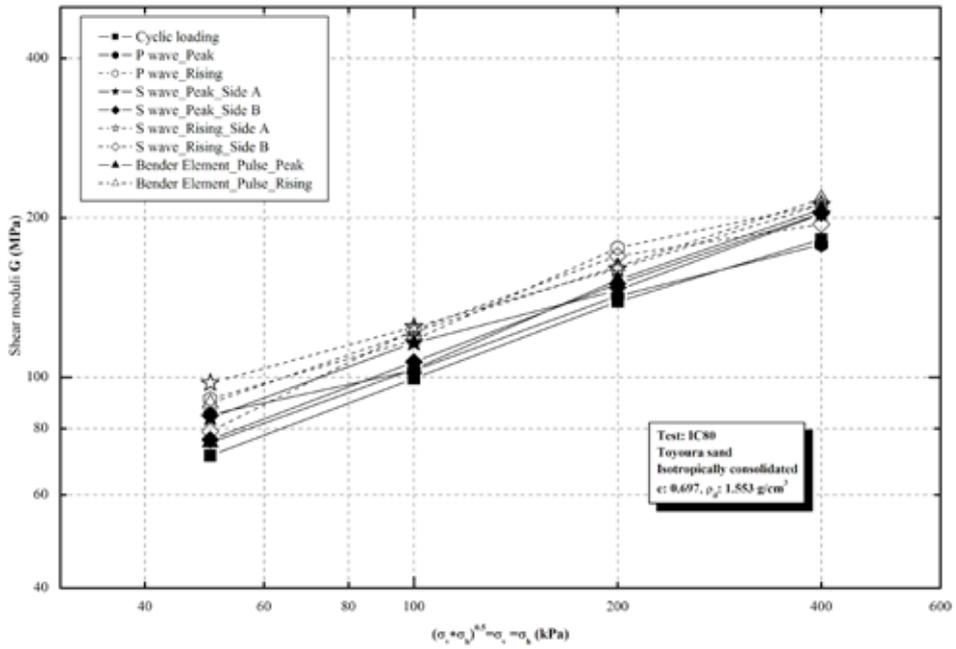


Figure 9 Relationships between shear moduli and stress levels (test IC50)

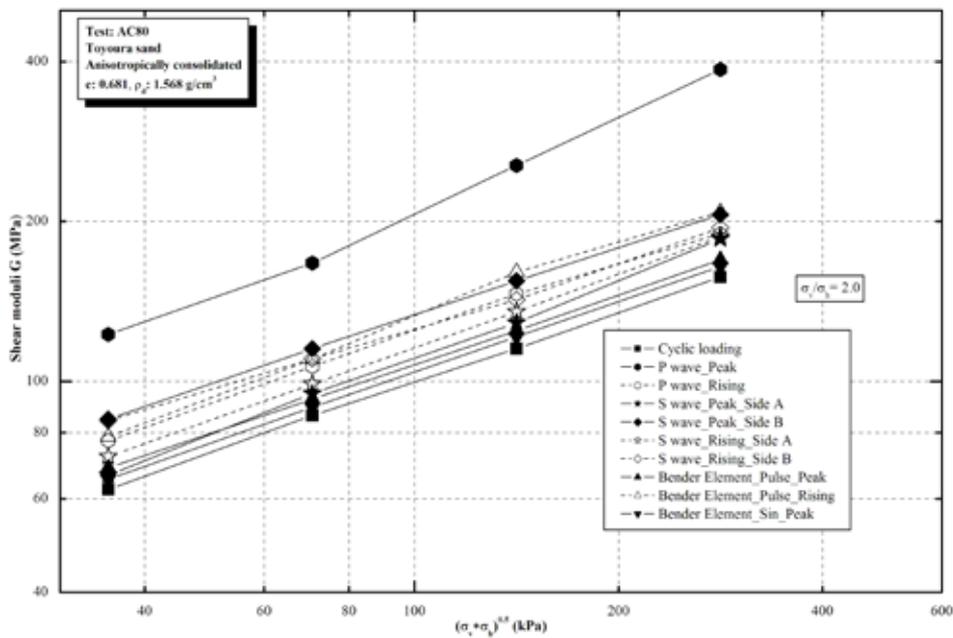
### Comparison between S wave excitation using triggers and Bender elements

The shear moduli values obtained by S wave excitation using the triggers,  $G_{Sw}$  (indicated with a notation “Sw”), and the Bender elements,  $G_{BE}$  (indicated with a notation “BE”), in all the tests

are compared in figure 12. The general trends on stress state dependency of  $G_{Sw}$  and  $G_{BE}$  were consistent to each other. However, it could be seen that the  $G_{Sw}$  values were almost similar at the same stress levels irrespective of the different densities of the specimens, while the  $G_{BE}$  values of denser specimens in the tests IC80 and AC80 were larger than those of the looser one in the test IC50.



**Figure 10** Relationships between shear moduli and stress levels (test IC80) will be made later.



**Figure 11** Relationships between shear moduli and stress levels (test AC80)

As a result, the  $G_{Sw}$  and  $G_{BE}$  values showed good agreement with each other with the denser specimens, while the difference between the  $G_{Sw}$  and  $G_{BE}$  values amounted up to 40 % for the the looser specimen at each stress level.

Comparison between P wave and S wave measurements

The  $G_{Sw}$  values and the shear moduli values,  $G_{Pw}$ , that are converted from vertical Young's moduli obtained by P wave excitation are compared in figure 13. It was observed that stress state dependencies of  $G_{Pw}$  calculated based on  $t_{pk}$ , denoted as "Pw\_Peak" in this figure, were not consistent with the others. In addition, the  $G_{Pw}$

values under this definition in the test AC80 were much larger than the other results.

As an example, time-history data from P wave measurement at each stress level in the test AC80 are presented in Figure 14. On both input and output records, some disturbances in the wave forms at the peak in the first half cycle could be observed, and their shapes changed continuously as the confining pressure increased. These properties should have affected the travel time evaluations and thus caused the above inconsistent values of  $G_{Pw}$ . The reason for these disturbances of the first half cycles would be wave generation by the couple of triggers. Therefore, the results from dynamic measurements based on  $t_{rise}$  are employed in the following discussions.

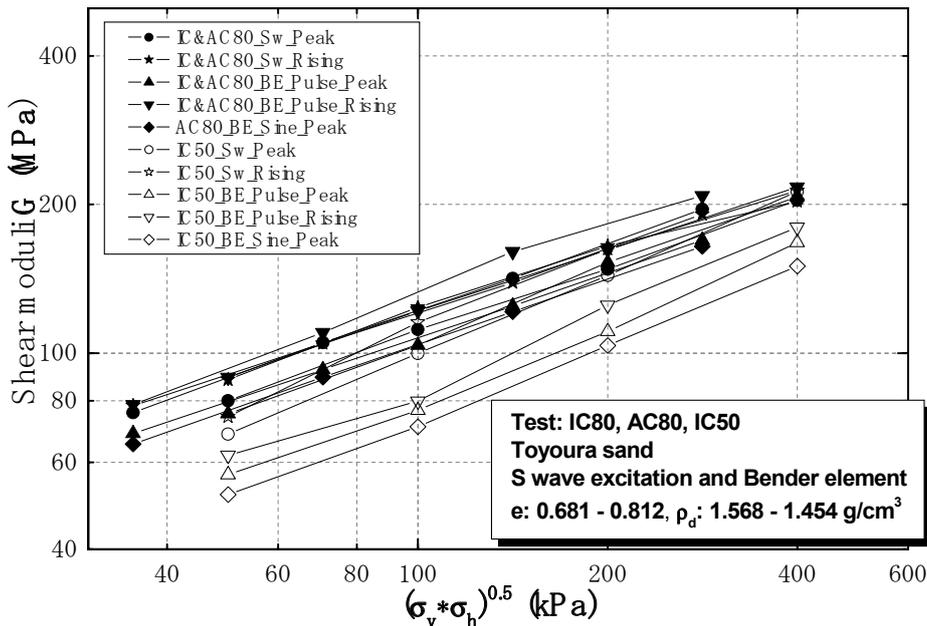


Figure 12. Comparison between S wave excitation using actuators and Bender elements

### Comparison between static and dynamic measurements

For comparison between the dynamic and static measurements, the vertical Young's moduli obtained by P wave excitation,  $E_{Pw}$  calculated with  $t_{rise}$  (denoted as "Pw\_Rising"), and the static measurements,  $E_s$  (denoted as "Cyclic loading"),

are shown in Figure 15. It could be clearly observed that the  $E_s$  values under the same dry density of the specimens in the tests IC80 and AC80 were almost equal to each other at each vertical stress level irrespective of the different horizontal stresses, and the  $E_s$  values of the looser specimen in the test IC50 were consistently smaller than those of denser specimens. On

the other hand, the  $E_{pw}$  values were almost similar at each vertical stress level irrespective of the different densities of the specimens and horizontal stresses. Consequently, the dynamic

moduli of denser specimens were larger than the static moduli by about 20 %, while the difference between the dynamic and static moduli of the looser one became as large as 60 %.

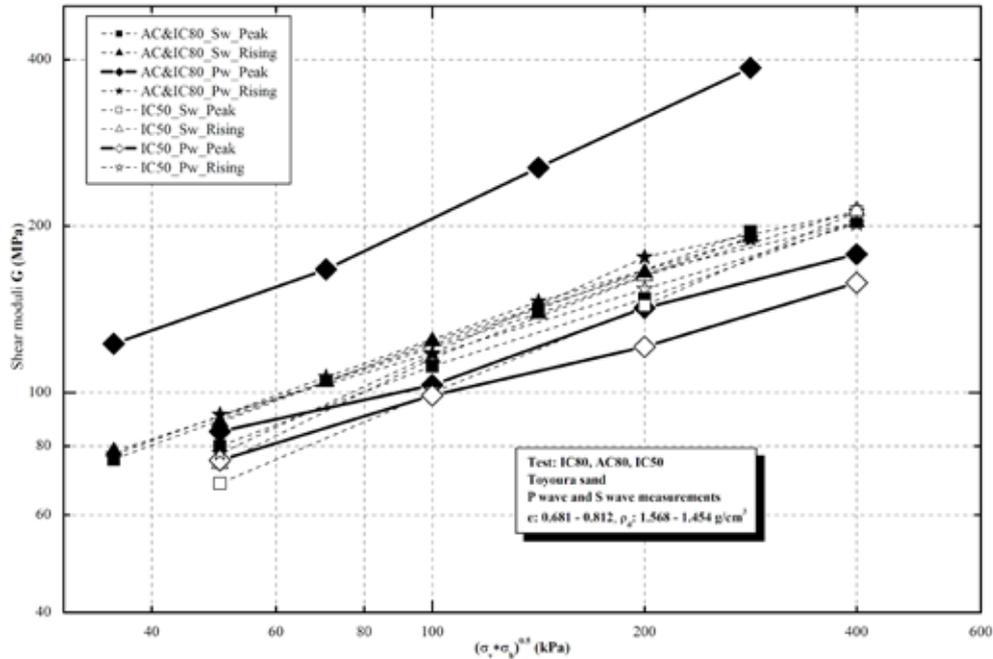


Figure 13 Comparison between P wave and S wave measurements

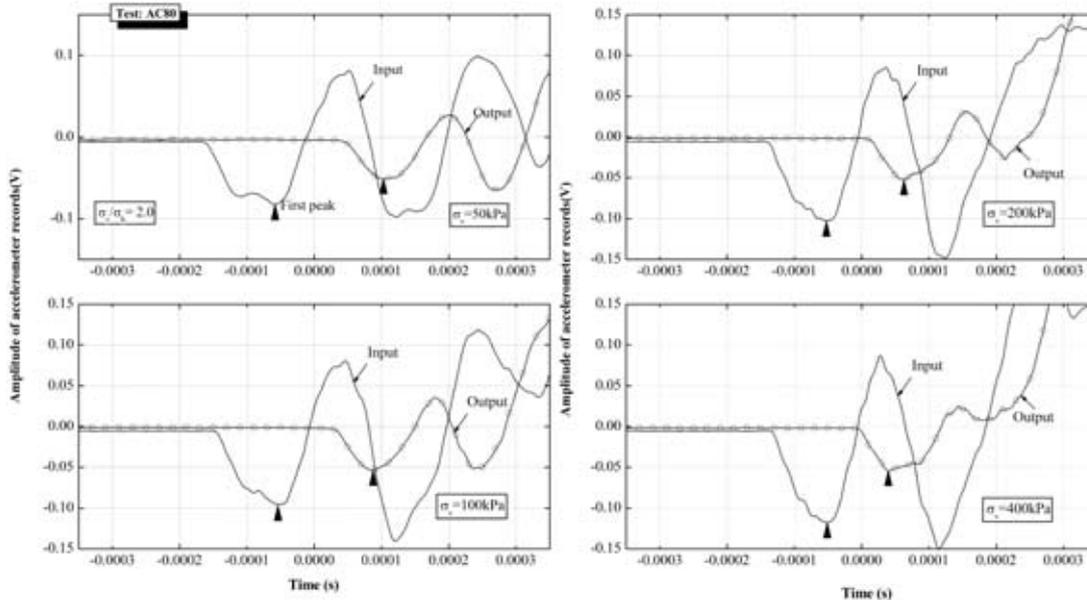


Figure 14 Time-history data from P wave measurement in the test AC80

The reason for these phenomena would be that the dynamic wave does not reflect the overall cross-sectional property of the specimen but travels through the shortest path made by interlocking of soil particles, resulting into larger elastic moduli as compared to those by the static measurement [10, 11]. With this inference, it would be also possible that the difference in the elastic moduli would increase with the decrease in the dry density, since the looser specimens would have larger structural or microscopic heterogeneity than denser ones. However, it is generally recognized that the differences between dynamic

and static measurements on fine uniform sands like Toyoura sand are very small and negligible as compared to those on well graded materials having large particles.

In addition to the above differences between static and dynamic measurements, the dynamic shear moduli by the Bender elements in this study showed different tendency from the other dynamic measurements. To clarify the relationships between the dynamic and static measurements more in detail, further investigations are required including the issue of proper determination of travel time from the records of P wave and Bender elements.

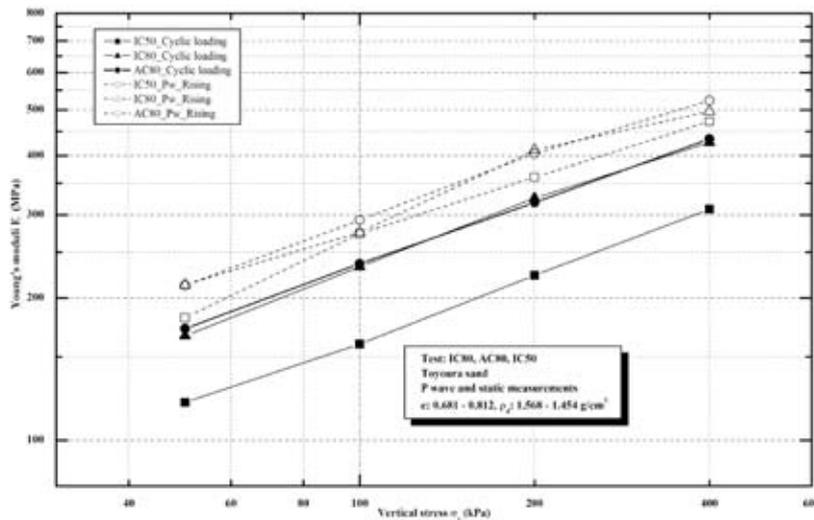


Figure 15 Comparison between static and dynamic measurements

### Effect of stress state-induced anisotropy

To discuss the effect of stress state-induced anisotropy, as shown in figure 16, the shear moduli  $G_{Sw}$  are regarded as the references here (denoted as “Sw\_Rising”), and are compared with the other shear moduli,  $G_S$  and  $G_{Pw}$ , that are converted from statically and dynamically measured Young’s moduli in the tests IC80 and AC80. It could be observed that the  $G_{Pw}$  values in the test IC80 using Equation (4) (denoted as “IC80\_Pw\_Rising\_Eq.(4)”) showed good agreement with the  $G_{Sw}$  values, while the  $G_{Pw}$  values in the test

AC80 using Equation (4) (denoted as “AC80\_Pw\_Rising\_Eq.(4)”) were different from the other dynamic measurements. By employing Equation (5), however, the  $G_{Pw}$  values in the test AC80 (denoted as “AC80\_Pw\_Rising\_Eq.(5)”) coincided with the other results since the stress state-induced anisotropy was properly considered. In the same manner, the  $G_S$  values in the test AC80 became almost the same as the  $G_S$  values in the test IC80 by using Equation (5) instead of Equation (4). Comparisons described above demonstrate the importance of considering the effects of stress state-induced anisotropy in comparing the elastic properties defined on different planes.

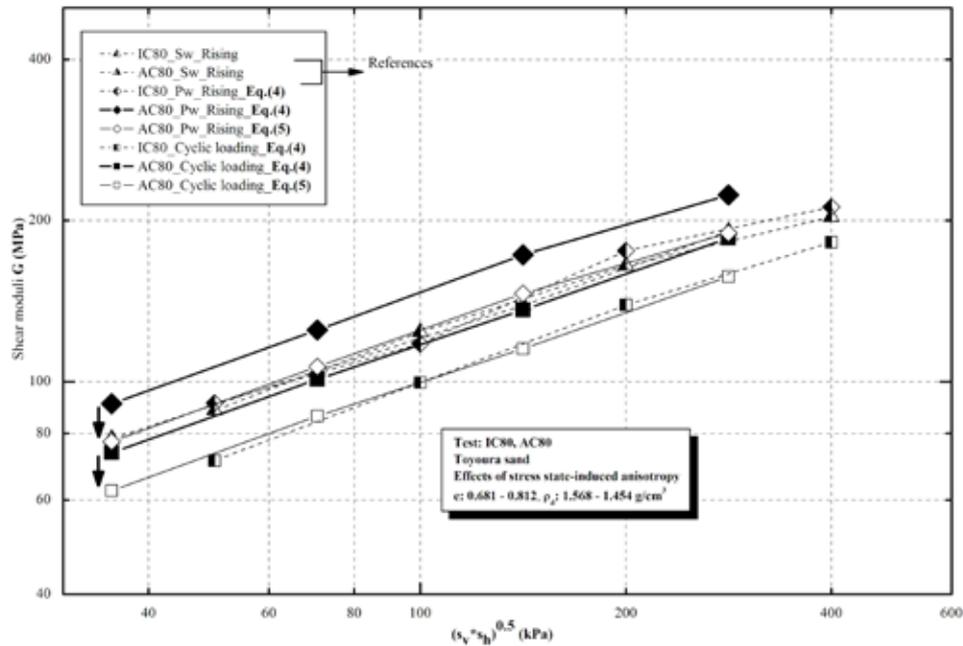


Figure 16 Effect of stress state-induced anisotropy

### Conclusions

The following conclusions can be drawn from the results presented in this study.

1. Dynamic measurement results in terms of shear moduli based on the secondary and primary wave velocities using piezoelectric actuators showed good agreements to each other, and these values were rather similar irrespective of the different densities of the specimens.
2. Dynamic Young's moduli based on the primary wave velocity were larger than those by static measurement, and the difference between them increase with the decrease in the densities of the specimens.
3. Dynamic measurement results obtained by the Bender elements showed the dependency on the densities of the specimens unlike the other dynamic measurement results. To clarify the relationships between dynamic and static measurements more in detail, further investigations are required including

the issue of proper determinations of travel time from the records of P wave and Bender elements.

4. By considering the effect of stress state-induced anisotropy, the shear moduli based on the secondary wave velocity and the converted shear moduli from vertical Young's moduli coincided with each other both on the dynamic and static measurements. It would demonstrate the importance of considering the effects of stress state-induced anisotropy in comparing the elastic properties defined on different planes.

### Acknowledgments

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