Active vibration control of a rotor-bearing system based on dynamic stiffness

Control activo de vibraciones en un sistema rotor-chumaceras basado en la rigidez dinámica

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Abstract

This paper presents an active vibration control scheme to reduce unbalance-induced synchronous vibration in rotor-bearing systems supported on two ball bearings, one of which can be automatically moved to control the effective rotor length and, as an immediate consequence, the rotor stiffness. This dynamic stiffness control scheme, based on frequency analysis, speed control and acceleration scheduling, is used to avoid resonant vibration of a rotor system when it passes (run-up or coast down) through its first critical speed. Algebraic identification is used for on-line unbalance estimation at the same time that the rotor is taken to the desired operating speed. Some numerical simulations and experiments are included to show the unbalance compensation properties and robustness of the proposed active vibration control scheme when the rotor is started and operated over the first critical speed.

----- *Keywords:* active vibration control, eccentricity identification, Jeffcott rotor

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Resumen

En este artículo se presenta un esquema de control activo de vibraciones para atenuar las amplitudes de vibración síncrona inducidas por el desbalance en un sistema rotor-chumaceras; donde una de las chumaceras puede ser desplazada automáticamente para modificar la longitud efectiva del rotor, y como consecuencia, la rigidez del rotor. El control de la rigidez dinámica se basa en un análisis de la respuesta en frecuencia, control de velocidad y en el uso de esquemas de aceleración, para evadir las amplitudes de la vibración en la resonancia mientras el sistema rotatorio pasa (acelerado o desacelerado) a través de una velocidad crítica. Se utiliza identificación algebraica para estimar el desbalance en línea, mientras el rotor es llevado a la velocidad de operación deseada. Algunas simulaciones numéricas y resultados experimentales son incluidos para mostrar las propiedades de la compensación del desbalance y la robustez del esquema de control activo de vibraciones propuesto, cuando el rotor se opera a una velocidad por encima de la primera velocidad crítica.

----- Palabras clave: control activo de vibraciones, identificación de excentricidad, rotor Jeffcott.

Introduction

Rotating machinery is commonly used in many mechanical systems, including electrical motors, machine tools, compressors, turbo machinery and aircraft gas turbine engines. Typically these systems are affected by exogenous or endogenous vibrations produced by unbalance, misalignment, resonances, material imperfections and cracks.

Vibration caused by mass unbalance is a common problem in rotating machinery. Rotor unbalance occurs when the principal inertia axis of the rotor does not coincide with its geometrical axis and leads to synchronous vibrations and significant undesirable forces transmitted to the mechanical elements and supports. Many methods have been proposed to reduce the unbalance-induced vibration, where different devices such as electromagnetic bearings, active squeeze film dampers, lateral force actuators, active balancers and pressurized bearings have been developed [1-5]. Passive and active balancing techniques are based on the unbalance estimation to attenuate the unbalance response in the rotating machinery. The Influence Coefficient Method has been used to estimate the unbalance while the rotating speed

of the rotor is constant [6, 7]. This method has been used to estimate the unknown dynamics and rotor-bearing system unbalance during the speed-varying period [8]. On the other hand, there is a vast literature on identification methods [9-11], which are essentially asymptotic, recursive or complex, which generally suffer of poor speed performance.

This paper presents an active vibration control scheme to reduce unbalance-induced synchronous vibration in rotor-bearing systems supported on two ball bearings, one of which can be automatically moved along the shaft to control the effective rotor length and, as an immediate consequence, the rotor stiffness. This dynamic stiffness control scheme, based on frequency analysis, speed control and acceleration scheduling, is used to avoid resonant vibration of a rotor system when it passes (run-up or coast down) through its first critical speed. Algebraic identification is used for on-line unbalance estimation at the same time that the rotor is taken to the desired operating speed. The proposed results are strongly based on the algebraic approach to parameter identification in linear systems reported [12], which requires a priori knowledge of the mathematical model of the system. This

approach has been employed for parameter and signal estimation in nonlinear and linear vibrating mechanical systems, where numerical simulations and experimental results show that the algebraic identification provides high robustness against parameter uncertainty, frequency variations, small measurement errors and noise [13, 14]. In addition, algebraic identification is combined with integral reconstruction of time derivatives of the output (GPI Control) using a simplified mathematical model of the system, where some nonlinear effects (stiffness and friction) were neglected; in spite of that, the experimental results show that the estimated values represent good approximations of the real parameters and high performance of the proposed active vibration control scheme, which means that the algebraic identification and GPI control methodologies could be used for some industrial applications, when at least a simplified mathematical model of the system is available [14].

Some numerical simulations and experiments are included to show the unbalance compensation properties and robustness when the rotor is started and operated over the first critical speed.

System description

Mathematical model

The Jeffcott rotor system consists of a planar and rigid disk of mass m mounted on a flexible shaft of negligible mass and stiffness k at the mid-span between two symmetric bearing supports (see figure 1(a) when $a = b$). Due to rotor unbalance the mass center is not located at the geometric center of the disk S but at the point G (center of mass of the unbalanced disk); the distance u between these two points is known as disk eccentricity or static unbalance [15, 16]. An end view of the whirling rotor is also shown in figure 1(b), with coordinates that describe its motion.

In our analysis the rotor-bearing system is modeled as the assembly of a rigid disk, flexible shaft and two ball bearings. This system differs from

the classical Jeffcott rotor because the effective shaft length can be increased or decreased from its nominal value. In fact, this adjustment is obtained by enabling longitudinal motion of one of the bearing supports (right bearing in figure 1.a) to different controlled positions into a small interval by using some servomechanism, which provides the appropriate longitudinal force. With this simple approach one can modify the shaft stiffness; moreover, one can actually control the rotor natural frequency, during run-up or coast-down, to evade critical speeds or at least reduce rotor vibration amplitudes. Our methodology combines some ideas on variable rotor stiffness [17] and rotor acceleration scheduling [18] but completing the analysis and control for the Jeffcott-like rotor system.
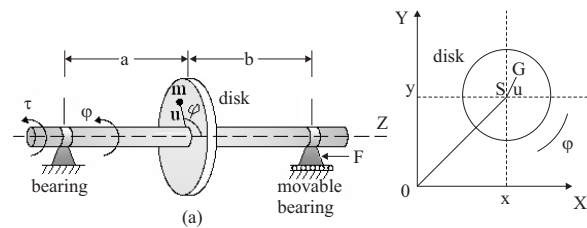

Figure 1 Rotor-bearing system: (a) Schematic diagram of a rotor-bearing system with one movable (right) bearing and (b) end view of the whirling rotor

For simplicity, the following assumptions are considered: flexible shaft with attached disk, gravity loads neglected (insignificant when compared with the actual dynamic loads), equivalent mass for the base-bearing m_b, linear viscous damping c_b between the bearing base and the linear sliding, force actuator to control the shaft stiffness F, angular speed $\omega = \dfrac{d\omega}{dt} = \varphi$ controlled by means of an electrical motor with servodrive and local Proportional Integral (PI) controller to track the desired speed scheduling in presence of small dynamical disturbances. The mathematical model of the four degree-of-freedom Jeffcott-like rotor is obtained using Newton equations as follows.

$$m\ddot{x} + c\dot{x} + kx = m(u\ddot{\varphi}\sin\varphi + u\dot{\varphi}^2\cos\varphi) \qquad (1)$$

$$m\ddot{y} + c\dot{y} + ky = m(u\dot{\varphi}^2\sin\varphi - u\ddot{\varphi}\cos\varphi) \qquad (2)$$

$$(J_z + mu^2)\ddot{\varphi} + c_\varphi\dot{\varphi} = \tau - p = m(\ddot{x}u\sin\varphi - \ddot{y}u\cos\varphi) \qquad (3)$$

$$m_b\ddot{b} + c_b\dot{b} = F \qquad (4)$$

where k and c are the stiffness and viscous damping of the shaft, J_z is the polar moment of inertia of the disk and $\tau(t)$ is the applied torque (control input) for rotor speed regulation. In addition, x and y denote the orthogonal coordinates that describe the disk position and $\dot{\varphi} = \omega$ is the rotor angular velocity. The coordinate b denotes the position of the movable (right) bearing, which is controlled by means of the control force $F(t)$ (servomechanism).

In our analysis the stiffness coefficient for the rotor-bearing system is given by [19]

$$k = \frac{3EIl(a^2 - ab + b^2)}{a^3b^3} \qquad (5)$$

where $l = a + b$ is the total length of the rotor between both bearings with b the coordinate to be controlled, $I = \frac{\pi D^4}{64}$ is the moment of inertia of a shaft of diameter D and E is the Young's modulus of elasticity ($E = 2.11 \times 10^{11} N/m^2$) for AISI 4140 steel). The natural frequency of the rotor system is then obtained as follows [19]

$$\omega_n = \sqrt{k/m} \qquad (6)$$

In such a way that, controlling b by means of the control force F one is able to manipulate ω_n to evade appropriately the critical speeds during rotor operation.

The proposed control objective is to reduce as much as possible the rotor vibration amplitude, denoted in adimensional units by

$$R = \sqrt{\frac{x^2 + y^2}{u}} \qquad (7)$$

for run-up, coast-down or steady state operation of the rotor system, even in presence of small exogenous or endogenous disturbances. Note, however, that this control problem is quite difficult because of the 8th order nonlinear model, many couplings terms, underactuation and uncontrollability properties from the two control inputs (τ, F).

Experimental results

Some experimental results were performed in open loop in a Rotor-Kit experimental platform of Bently Nevada®. The positions of the inertial disk, sensors and bearing supports in the rotor-kit are shown in figure 2. The experimental results were performed to show how the natural frequency can be modified to different positions of the right bearing. Furthermore, the rotor is started and operated over the first critical speed where the speed operating condition for the rotor is given as $\varphi = 418.9$ *rad/s* (4000 *rpm*) and the acceleration for the speed ramp was 1.74 *rad/s²*.

Figure 2 Positions of the inertial disk, sensors and bearing supports in the rotor-bearing system

The response for the Jeffcott rotor configuration (right bearing position at point B, see figure 2) is shown in figure 3 and it was recorded by the sensor S3 (see figure 2). The natural frequency for the Jeffcott rotor configuration was about $\omega_n = 2200$ *rpm*.

Figure 3 Unbalance response for a Jeffcott Rotor (right bearing position at point B)

Experimental results for the bearing position at points A ($l = 0.4$ m) and C ($l = 0.5$ m), are shown in figures 4 and 5, respectively. Here, the open-loop responses show that a smaller length $l = 0.4$ m leads to a higher natural frequency and a bigger length $l = 0.5$ m leads to a smaller natural frequency. Hence to get a minimal unbalance response, the rotor length should start at $l = 0.4$ m and then abruptly change to $l = 0.5$ m. This change of the bearing position must occur exactly when the response for $l = 0.4$ m crosses the response for $l = 0.5$ m.
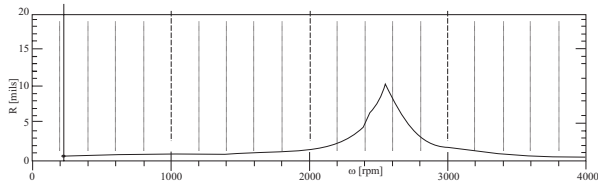
Figure 4 Rotor unbalance response for the right bearing position at point A
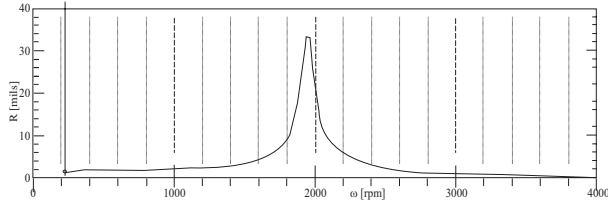
Figure 5 Rotor unbalance response for the right bearing position at point C

Speed control with trajectory planning

In order to control the speed of the Jeffcott-like rotor system, consider $\omega = z_6 = \dot{\varphi}$ equation (3), under the temporary assumption that the eccentricity u is perfectly known and that $c \approx 0$ to simplify the analysis. Then, the following local PI controller is designed to track desired reference trajectories of speed $\omega^*(t)$ and acceleration scheduling $\omega^*(t)$ for the rotor:

$$\tau = J_z v_1 + c_\varphi \omega + kux \sin \varphi - kuy \cos \varphi$$
$$v_1 = \omega^*(t) - \alpha_1 (\omega - \omega^*(t)) - \alpha_0 \int_0^t (\omega - \omega^*(t))dt \quad (8)$$

The use of this controller yields the following closed-loop dynamics for the trajectory tracking error $e_1 = \omega - \omega^*(t)$:

$$\ddot{e}_1 + \alpha_1 \dot{e}_1 + \alpha_0 e_1 = 0 \quad (9)$$

Therefore, selecting the design parameters $[\alpha_1, \alpha_0]$ so that the associated characteristic polynomial for equation (9) is a *Hurwitz* polynomial, one guarantees that the error dynamics is asymptotically stable.

The prescribed speed and acceleration scheduling for the planned speed trajectory is given by

$$\omega^*(t) = \begin{cases} \omega_i & 0 \le t < t_i \\ \sigma(t, t_i, t_f) \, \omega_f & t_i \le t < t_i \\ \omega_f & t > t_f \end{cases} \quad (10)$$

where ω_i and ω_f are the initial and final speeds at the times t_i and t_f, respectively, passing through the first critical frequency, and $\sigma(t, t_i, t_f)$ is a Bézier polynomials, with $\sigma(t, t_i, t_i) = 0$ and $\sigma(t, t_i, t_f) = 1$, described by

$$\sigma(t, t_i, t_f) \, \omega_f = \left(\frac{t - t_i}{t_f - t_i} \right)$$

$$\left[\gamma_1 - \gamma_2 \left(\frac{t - t_i}{t_f - t_i} \right) + \gamma_3 \left(\frac{t - t_i}{t_f - t_i} \right)^2 - ... + \gamma_6 \left(\frac{t - t_i}{t_f - t_i} \right)^5 \right] \quad (11)$$

with $\gamma_1 = 252$, $\gamma_2 = 1050$, $\gamma_3 = 1800$, $\gamma_4 = 1575$, $\gamma_5 = 700$, $\gamma_6 = 126$, in order to guarantee a sufficiently smooth transfer between the initial and final speeds.

The fundamental problem with the proposed feedback control in equation (8) is that the eccentricity u is not known, except for the fact that it is constant. The Algebraic identification methodology is proposed to on-line estimate the eccentricity u, which is based on the algebraic approach to parameter identification in linear systems [12].

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Algebraic identification of eccentricity

Consider equation (3) with perfect knowledge of the moment of inertia J_z and the shaft stiffness k, and that the position coordinates of the disk (x, y) and the control input τ are available for the identification process of the eccentricity u.

Multiplying equation (3) by t and integrating by parts the resulting expression once with respect to time t, one gets

$$\int_0^t (tz_6)dt = \frac{1}{J_z}\int_0^t t(t - c_\varphi z_6)dt +$$
$$\frac{ku}{J_z}\int_0^t t(z_3 - \cos z_5 - z_1 \sin z_5)dt \quad (12)$$

Solving for the eccentricity u in equation (12) leads to the following on-line algebraic identifier for the eccentricity:

$$\hat{u} = \frac{J_z t\omega - \int_0^t (J_z\omega + t\tau - tc_\varphi\omega)dt}{k\int_0^t t(y \cos \varphi - x \sin \varphi)dt}, \forall t \in [0,\delta)] \quad (13)$$

where δ is a positive and sufficiently small value.

Therefore, when the denominator of the identifier of equation (13) is different to 0, at least for a small time interval $[O, \delta]$ with $\delta < 0$, one can find from equation (13) a closed-form expression to on-line identify the eccentricity.

An adaptive-like controller with algebraic identification

The PI controller given by equation (8) can be combined with the on-line identification of the eccentricity in equation (13), resulting the following certainty equivalence PI control law

$$\tau = J_z v_1 + c_\varphi\omega + kux \sin \varphi - k\hat{u}y \cos \varphi$$
$$v_1 = \dot{\omega}^*(t) -\alpha_1 (\omega - \omega^*(t)) - \alpha_0\int (\omega - \omega^*(t))dt \quad (14)$$

with

$$\hat{u} = \frac{J_z t\omega - \int_0^t (J_z\omega + t\tau - tc_\varphi\omega)dt}{k\int_0^t t(y \cos \varphi - x \sin \varphi)dt}, \forall t \in [0,\delta)]$$

Note that, in accordance with the algebraic identification approach, providing fast identification for the eccentricity, the proposed controller (14) resembles an adaptive control scheme. From a theoretical point of view, the algebraic identification is instantaneous [12]. In practice, however, there are modeling errors and other factors that inhibit the algebraic computation. Fortunately, the identification algorithms and closed-loop system are robust against such difficulties [14].

Simulation results

Some numerical simulations were performed using the parameters listed in table 1.

Table 1 Rotor system parameters

m_r = 0.9 kg	D - 0.01 *m*	α = 0.3 *m*
m_b = 0.4 kg	r_{disk} = 0.04 *m*	$c\varphi$ = 1.5 x 10⁻³ *Nms/rad*
c_b = 10 *Ns/m*	u = 100 µ*m*	z7 = 0.3 ± 0.05 *m*

Figure 6 shows the identification process of eccentricity and the dynamic behavior of the adaptive-like PI controller given by equation (14), which starts using the nominal value u = 0 µ*m*. One can see that the identification process is almost instantaneous. The control objective is to take from the rest position of the rotor to the operating speed ωf = 300 *rad/s*.
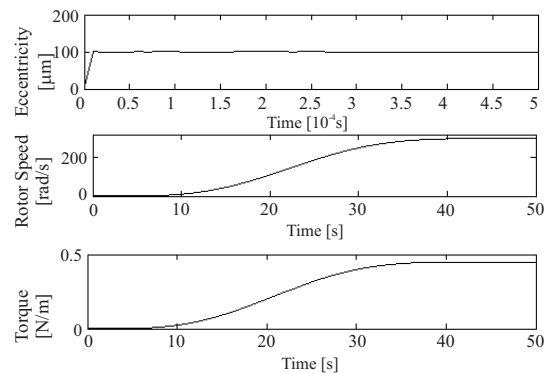
Figure 6 Close loop system response using the PI controller: (a) identification of eccentricity, (b) rotor speed and (c) control input

The desired speed profile runs up the rotor in a very slow and smooth trajectory while passing through the first critical speed. This control scheme is appropriate to guarantee stability and tracking. The resulting rotor vibration amplitude (system response when $\tau = 0$) is shown in figure 7, for three different and constant positions of the right bearing (i.e., $b = 0.25$ *m*, 0.30 *m*, 0.35 *m*), using the PI controller.
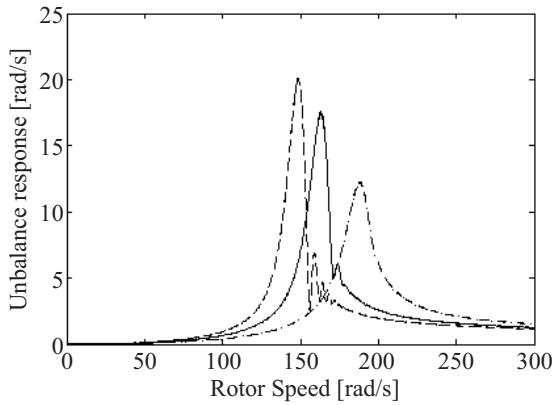
Figure 7 Unbalance response R for different and constant positions of the movable bearing: (dashed line), (solid line) and (dash-dotted line)

The purpose of these simulations is to illustrate how the position of the bearing truly affects the rotor vibration amplitudes for the desired speed profile. The nominal length of the shaft is $l = 0.60$ *m*. A smaller length $l = 0.55$ *m* leads to a higher natural frequency and a bigger length $l = 0.65$ *m* leads to a smaller natural frequency (see figure 7). Hence to get a minimal unbalance response, the rotor length should start at $l = 0.55$ *m* and then abruptly change to $l = 0.65$ *m*. This change of the bearing position must occur exactly when the response for $l = 0.55$ crosses the response for $l = 0.65$, in order to evade the resonance condition, because the rotor speed is different from the natural frequency of the rotor-bearing system.

Position control of the bearing support

It is evident from equations (5) and (6) that controlling the position of the movable (right) bearing b applying the control force F and

according to a pre-specified speed profile $\omega^*(t)$ the modification of the rotor amplitude response to the unbalance is possible. As a matter of fact this methodology is equivalent to a dynamic stiffness control for the Jeffcott-like rotor system, enabling smooth changes on coordinate b.

To design a controller for position reference tracking, consider equation (4). Then, one can propose the following Generalized Proportional Integral (GPI) controller for asymptotic and robust tracking to the desired position trajectory $b^*(t)$ for the bearing position and velocity, which employs only position measurements of the bearing. For more details on GPI control see [20].

$$F = m_b v_2 - c_b \dot{b}$$
$$v_2 = \ddot{b}^*(t) - \beta_2(\hat{\dot{b}} - \dot{b}^*(t)) \quad (15)$$
$$- \beta_1(b - b^*(t)) - \beta_0 \int (b - b^*(t))dt$$

where $\hat{\dot{b}}$ is an integral reconstructor of the bearing velocity, which is given by

$$\hat{\dot{b}} = \frac{c_b}{m_b}b + \frac{1}{m_b}\int_0^t F(\sigma)d\sigma \quad (16)$$

The use of the GPI controller given yields the following closed-loop dynamics for the trajectory tracking error $e_2 = b - b^*(t)$:

$$e_2^{(3)} + \beta_2\ddot{e}_2 + \beta_1\dot{e}_2 + \beta_0 e_2 = 0 \quad (17)$$

Therefore, selecting the design parameters $\{\beta_0, \beta_1, \beta_2,\}$ such that the associated characteristic polynomial for equation (17) be *Hurwitz*, one guarantees that the error dynamics be globally asymptotically stable. The desired trajectory planning $b^*(t)$ for the bearing position and velocity is also based on Bézier polynomials similar to equation (10).

Results and discussion

The proposed methodology for the active vibration control of the transient run-up or coast-down of the rotor-bearing system consists of the following steps:

1. Define the trajectory planning for the speed trajectory profile $\omega^*(t)$ to be asymptotically tracked by the use of the adaptive-like PI controller with the algebraic identifier of the eccentricity, i.e., $lim_{t\to\infty}\omega^*(t) = \omega^*(t)$.

2. Establish an appropriate smooth switching on the position of the movable bearing $b^*(t)$ to be asymptotically tracked by the application of the GPI controller, i.e., $lim_{t\to\infty} b(t) = b^*(t)$. The switching time has to be at the crossing point leading to minimal unbalance response in figure 7.

Figure 8 shows the unbalance response of the rotor-bearing system when rotor speed PI controller with algebraic identification of eccentricity and GPI control of the bearing position are simultaneously used. Note that the switching of the bearing position leads to small transient oscillations due to inertial and centrifugal effects on the overall rotor system.
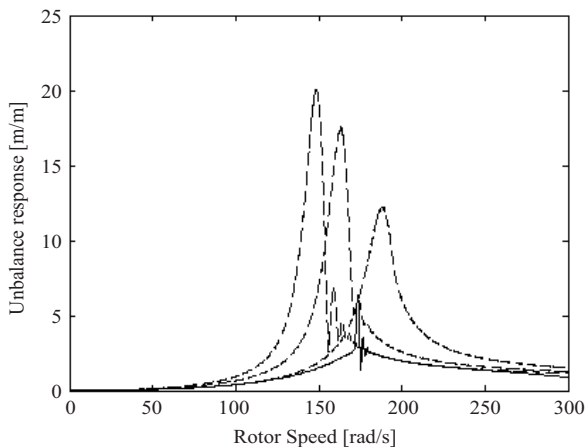
Figure 8 Rotor vibration amplitude response using active vibration control (solid line)

First of all, the speed trajectory planning and control torque shown in figure 6 are similarly used. The smooth switching for the bearing position is implemented in such a way that the run-up of the rotor system starts with the position $b_i = 0.25\ m$ (i.e., $l = 0.55\ m$) and changes to $b_f = 0.35\ m$ (i.e., $l = 0.65\ m$) exactly at the crossing point shown in the corresponding response in

figure 7. The switching time occurs when $\omega = 170.6\ rad/s$, that is, $t = 23.9\ s$. The desired position of the bearing $b(t)$ is illustrated in figure 9 together with the applied control force F. A comparison of the open-loop response and the closed-loop response in figure 8 results in important unbalance reductions about 64%.
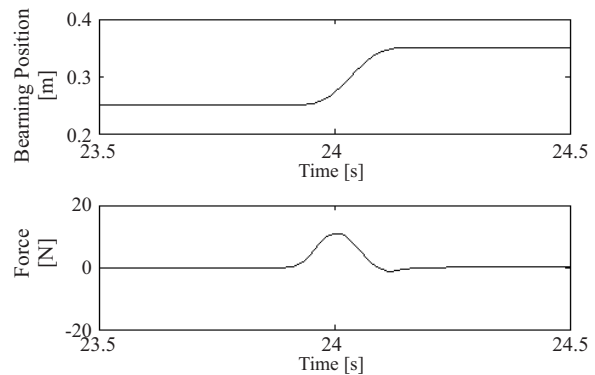
Figure 9 Response of the bearing support using GPI controller: (a) position of the movable bearing and (b) control force

Conclusions

The active vibration control of a Jeffcott-like rotor through dynamic stiffness control and acceleration scheduling is addressed. The control approach consists of a servomechanism able to move one of the supporting bearings in such a way that the effective rotor length is controlled. As a consequence, the rotor stiffness and natural frequency are modified according to an off-line and smooth trajectory planning of the rotor speed/ acceleration in order to reduce the unbalance response when passing through the first critical speed. The vibration control scheme results from the combination of passive and active control strategies, leading to robust and stable performance in presence of the synchronous disturbances associated to the normal operation of the rotor and some small parameter uncertainties. Since this active vibration control scheme requires information of the eccentricity, a novel algebraic identification approach is proposed

for on-line estimation of the eccentricity. From a theoretical point of view, the algebraic identification is practically instantaneous and robust with respect to parameter uncertainty, frequency variations, small measurement errors and noise. The proposed active vibration control scheme, used to reduce unbalance-induced synchronous vibration, is restricted to use in small rotating machinery (e.g., tools machines, motors and generators).

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