Non-linear joint transform correlator for micro-displacement measurement using speckle patterns

Medida de desplazamientos micrométricos mediante correlación de transformada conjunta no lineal de patrones de *speckle*

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Abstract

This work presents a procedure for measuring displacements of tens micrometer, in one and two dimensions, based on the correlation between two speckle patterns. For this purpose, two objective speckle patterns are recorded by a digital camera –one before and one after the object has been moved- and placed in the input plane of nonlinear joint transform correlator.

Nonlinear transformation of the joint power spectrum allows a sharper correlation peak and a high signal to noise ratio. The autocorrelation peak coordinates of the first pattern are set as a reference for measuring shifts of the successive cross-correlation peak. One criterion for reliable measure is proposed. Results related with different distances sample-sensor and illuminating wavelengths at 632.8 nm and 543.5 nm are presented.

------ *Keywords:* Laser speckle, micro-displacements, nonlinear joint transform correlator

Resumen

Se presenta un procedimiento para medir desplazamiento en el rango de micrones, en una y dos dimensiones, basado en la correlación conjunta de patrones de *speckle* (moteado característico de la superficie iluminada por

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un láser). Para este propósito, dos patrones de *speckle* por propagación libre, registrados por una cámara digital, uno antes y otro después del movimiento de la superficie objeto, se colocan en el plano de entrada de un correlador de transformada conjunta.

Una transformación no lineal de la densidad espectral de energía logra una alta razón señal a ruido en el plano de salida del correlador. Las coordenadas del pico de autocorrelación, del primer patrón se colocan como origen para medir los desplazamientos sucesivos del pico de correlación cruzada. Se propone un criterio para determinar el rango de medida confiable y se presentan algunos resultados para diferentes distancias superficie-sensor y longitud de onda del láser.

------ *Palabras clave:* Moteado laser, correlación de transformada conjunta no lineal, desplazamientos micrométricos

Introduction

Currently there is great variety of processes and systems for measuring displacements in the range of microns. These procedures are based on principles capacitive, inductive, resistive, magnetic, ultrasonic and optical [1]. The last ones are characterized by non-contact, promptness and immunity to electromagnetic interference. The procedures for optically measuring displacements are based in triangulation, time-of-flight diffraction (TOFD), moiré and interference [2, 3]. Before the widespread use of digital cameras, laser-speckle photography was the procedure, based on speckle, used to measure the magnitude and direction of micro-displacements [4]. In this paper, we propose measure displacements, in the range of 20μm – 450 μm, using joint discrete correlation of two digital speckle patterns corresponding to two positions of a diffusing surface. This involves, defining the optical system to form the speckle pattern, establish the optimal sampling rate, development of software for processing images and the calculation of the joint correlation, and finally, associate the result of the correlation with micro-displacements.

Some aspects of this procedure has been previously discussed in a brief technical note [5], here we introduce new elements, such as, the criterion to determine the reliable range of measure, measurements in two dimensions and the use of different wavelengths.

The paper is organized as follow, a brief definition of the speckle phenomenon, followed by the mathematical formalism necessary to understand the non linear joint transform correlator, the experimental, results and conclusions.

Laser speckle

The speckle is a granular structure resulting from the interference of multiple beams when light, highly coherent, is scattered, reflected and refracted by optically rough surfaces. These beams have random amplitudes and phases, and its superposition produces bright and dark spots due to constructive and destructive interference. The speckle exits even without forming an image of the object, in this case is called objective speckle pattern, converse when the image of the speckle pattern is formed by a lens is called subjective speckle pattern [6]. The theoretical models show that the speckle “grain” exhibits a geometrical structure 3D depending on the coherence properties of the light source and the characteristics of the object’s surface [7]. In order to obtain an appropriate image of the speckle pattern, according the sampling theorem, the picture elements of the camera should be at least the half of the size of speckle grain. The average grain size of an objective speckle pattern is given by equation (1). [8]

$$b_s = 1.22\left(\frac{\lambda z}{D}\right) \qquad (1)$$

In the above equation, the 1.22 factor is a result of diffraction, λ is the wavelength of the beam that illuminates the object, *z* is the distance between the diffusing surface and the viewing screen, and *D* is the diameter of the laser beam at *z* distance.

Speckle pattern is suitable for measuring micro-displacements because it contains coded information about the position and shape of the diffusing surface [9].

Joint transform correlator

The joint transform correlator (JTC), also called linear joint transform correlator, was introduced by Weaver and Goodman in 1966; it is often used in digital, optical or hybrid optical-digital object recognition and classification. It has the following advantages:

- No need synthesis of a complex filter in Fourier space.
- It is appropriate for real time applications.
- It is robust against vibration and misalignment compared with filter-based correlator (filters placed in the Fourier plane).

One of the many modifications to the classical joint transform correlator is the nonlinear transformation of joint power spectrum. Nonlinear joint transform correlator exhibits sharp peaks in the exit plane decreasing the uncertainty in locating the target of interest [10]. But, JTC and nonlinear JTC present some disadvantages, such as:

- Noticeable decrease of actual output bandwidth-space.
- Presence of wide side lobes that degrade the signal to noise ratio and increase uncertainty in the location of peak correlation.
- Its performance is affected strongly with rotations of the target.

The input plane of JTC, u_1 (x_1, y_1), consists of a reference h_1 (x_1, y_1), in this case one speckle pattern, placed joint the input scene g_1 (x_1, y_1), other speckle pattern corresponding to displacement of the sample. This can be written according (2),

$$u_1(x_1, y_1) = h\left(x_1, y_1 - \frac{Y}{2}\right) + g\left(x_1, y_1 + \frac{Y}{2}\right) \qquad (2)$$

The equation (3) depicts the intensity of Fourier transform of the correlator input plane, when speckle patterns h (x_1, y_1) and g (x_1, y_1) are located in *Y/2* and – *Y/2*, respectively.

$$\begin{aligned} I(x_2, y_2) &= c^2\left|H(cx_2, cy_2)\right|^2 + c^2\left|G(cx_2, cy_2)\right|^2 + \\ & c^2 H(cx_2, cy_2)G^*(cx_2, cy_2)\,exp(-j2\pi c y_2 Y) + \\ & c^2 H^*(cx_2, cy_2)G(cx_2, cy_2)\,exp(j2\pi c y_2 Y) \end{aligned} \qquad (3)$$

In equation above, $I(x_2, y_2)$ is the classical or lineal joint power spectrum (JPS); $H(cx_2, cy_2)$ and $G(cx_2, cy_2)$ are the Fourier transforms, scaled by a *c* factor, of the reference $h(x_1, y_1)$ and the input scene $g(x_1, y_1)$ respectively.

Amplitude distribution of the output plane of linear JTC, $u_3(x_3, y_3)$, is the Fourier transform of $I(x_2, y_2)$ and is given by expression (4),

$$\begin{aligned} u_3(x_3, y_3) = c[&h(x_3, y_3) \otimes h^*(-x_3, -y_3) + g(x_3, y_3) \otimes g^*(-x_3, -y_3) + \\ &h(x_3, y_3) \otimes g^*(-x_3, -y_3) \otimes \delta(x_3, y_3 - Y) + \\ &h(x_3, y_3) \otimes g^*(x_3, y_3) \otimes \delta(x_3, y_3 + Y)] \end{aligned} \qquad (4)$$

First and second terms in equation (4) are the direct component of the correlation. The last two terms are the cross-correlations between *g* and *h* functions, centered in (0, *Y*) and (0, –*Y*) positions in the output plane of correlator. The symbols, ⊗ and δ are the convolution and delta function respectively.

On the other hand, in nonlinear JTC joint spectral density can be transformed according to equation (5),

$$T(I) = \begin{cases} 1; & I > V_T \\ -1; & I < V_T \\ 0; & I = V_T \end{cases}; \qquad (5)$$

Where V_T is a threshold value, which could be chosen to maximize the intensity of autocorrelation peak and reduce the height of the peaks of higher order.

This joint spectral density, in equation (5) can be seen as a signal coded by pulses with width *d*, and position, Δ.

An expansion in a series of harmonic terms of JPS nonlinear leads to (6),

$$T(y_2) = 2\frac{d}{L} - 1 + \sum_{k=1}^{\infty} \frac{4}{\pi k} sen\left(\pi k \frac{d}{L}\right) \cos\left(\pi k \left(\frac{2y_2 - 2\Delta - d}{L}\right)\right) \tag{6}$$

In equation (6), *L* is the period of the JPS along y_2 axis. Maximizing the ratio between the first-order autocorrelation peak respects to the direct component, require defining a threshold, V_T, which is the same value of the direct component of the JPS, and is described in (7),

$$V_T = c^2|H|^2 + c^2|G|^2 \tag{7}$$

Finally, the output of the correlator u_3 (x_3, y_3) is proportional to the Fourier transform of the cosine term in equation (4). With *k*=1, and V_T given by equation 7, the correlation output result in the expression (8),

$$u_3(x_3, y_3) \approx cte(\delta(x_3, y_3 - Y) + \delta(x_3, y_3 + Y)) \tag{8}$$

Equation 8 shows that the correlation consists of two delta functions located at positions (*0,Y*) and (*0,-Y*) at the output plane.

Experimental

Coherent light beam of a He-Ne laser was used to illuminate the surface of the sample, located on a micro-displacement station (*x, y*), as shown in figure 1.

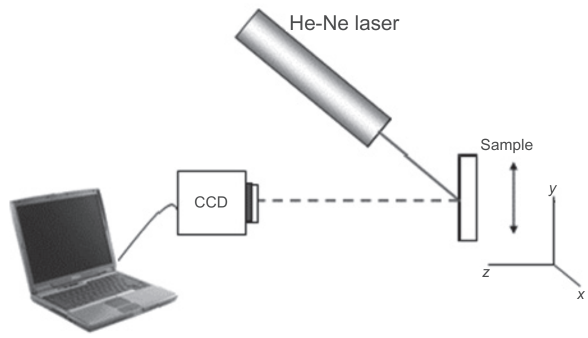

Figure 1 Experimental setup

The diffuse component of light, reflected from the sample, is recorded by the sensor of a monochrome charge-coupled device (CCD) camera at *z* distance. The origin of coordinate system for measurements was established at the position of the autocorrelation peak of the speckle pattern (zero displacement of the sample).

To obtain a calibration curve, theoretical micro-displacements were made by moving the station with the sample using a step by step motor at intervals of 20-μm, after each displacement, one objective speckle pattern was recorded and after placed in the input plane of nonlinear joint transform correlator. Cross-correlation was calculated, with the aim to obtain the respective shift of the first order cross-correlation peak and relate with the displacement of the sample. The experiment was done first using He-Ne laser at wavelength and 632.8 nm and after 543.5 nm. Multiple sensor-sample distances were considered keeping the wavelength of the laser constant.

Data processing involved routines of images treatment and correlation on moderately-fast computers systems, which made possible the almost real-time measurements of the motion of speckle patterns. The software used was *Matlab 7.0*. Roughly ten correlations per second were obtained; this quantity result to be small compared with the potential of optical correlation that is estimated typically in 400 correlations per second; but this setup could compete with hybrid optical-digital setups made with current special-purpose processors that handle typically 40 correlations per second [11].

Results

Behavior of cross-correlation intensity in arbitrary units (A.U) against displacements in microns (μm), see figure 2 (a) and (b), evidence that measure is reliable until 400 (μm) for horizontal displacement and 200 μm for vertical displacement, respectively; this is corroborated in figure 2 (c) and (d), that show calculated displacement versus theoretical displacement.

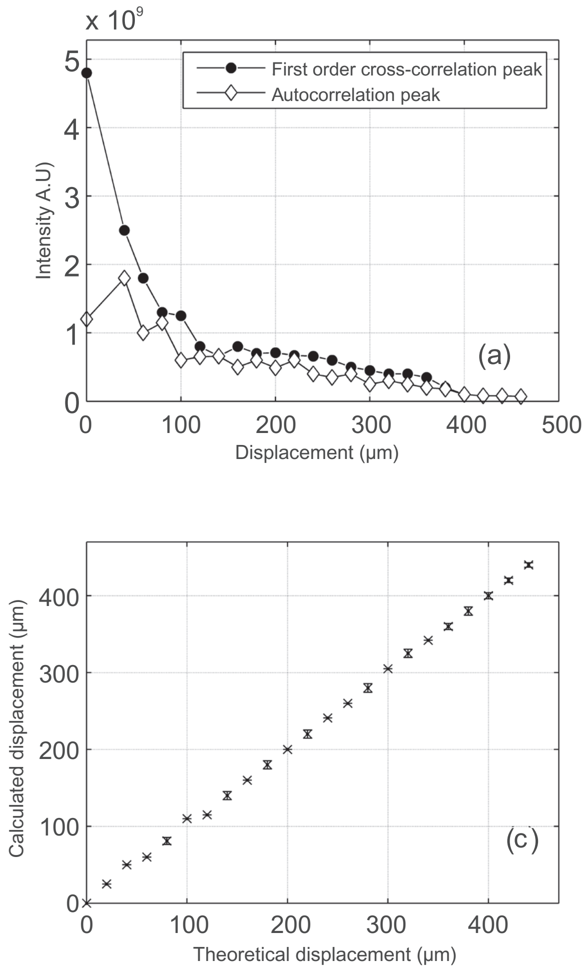

Figure 2 (a)-(b) Criterion for reliable range of measure in horizontal and vertical sense respectively. (c)-(d) Calculated versus theoretical displacements in horizontal and vertical directions correspondingly. Data taken from reference [12]

The graphics suggest a criterion for determining the reliable range of measure, based on the comparison of the maximum of intensity of the first order cross-correlation and the intensity of the peak in the origin (which coincides with the coordinates of the autocorrelation peak). When these intensities are equal or the autocorrelation peak is higher than the first order cross-correlation peak, false measurement values for micro-displacements arise.

Note that, the results are repeatable; this can be appreciated in the symmetric error bars of unit standard deviation of measures. The error increases when the displacements exceed the criterion mention above. Moreover, when the displacement is under the criterion, the resolution is fixed by digital camera (in this case 16-μm. Finally, rectangular shape of imaging sensor determines a different range in the horizontal and vertical measure. Finally, rectangular shape of imaging sensor determines a different range in the horizontal and vertical measure.

On other hand, table 1 (see column 'Max. range') presents reliability range for micro-displacement measure using different wavelengths and different sample-sensor distance [12]. The best result is obtained for distance sample-sensor 7 cm, when speckle pattern is produced using a laser beam with wavelength of 543.5 nm and 6 cm using a laser beam with wavelength of 632.8 nm which is according with the optimal sampling criteria. In all cases, if the displacement is under the condition of ('Max range') the resolution is fixed by digital camera.

Table 1 Maximum ranges of measurement for different wavelengths and different sample-sensor distance

λ=543.5 (nm)		*λ=632.8 (nm)*	
z (cm)	*Max range* μm ±16 μm	*z (cm)*	*Max range* μm ±16 μm
7	448	6	448
9	294	7	420
11	224	11	392

Finally, tracking two-dimensional of the sample can also be made following this setup. In figure 3, a circular path of 225-µm of radius is shown. The trajectory is sampled at interval of 30°.

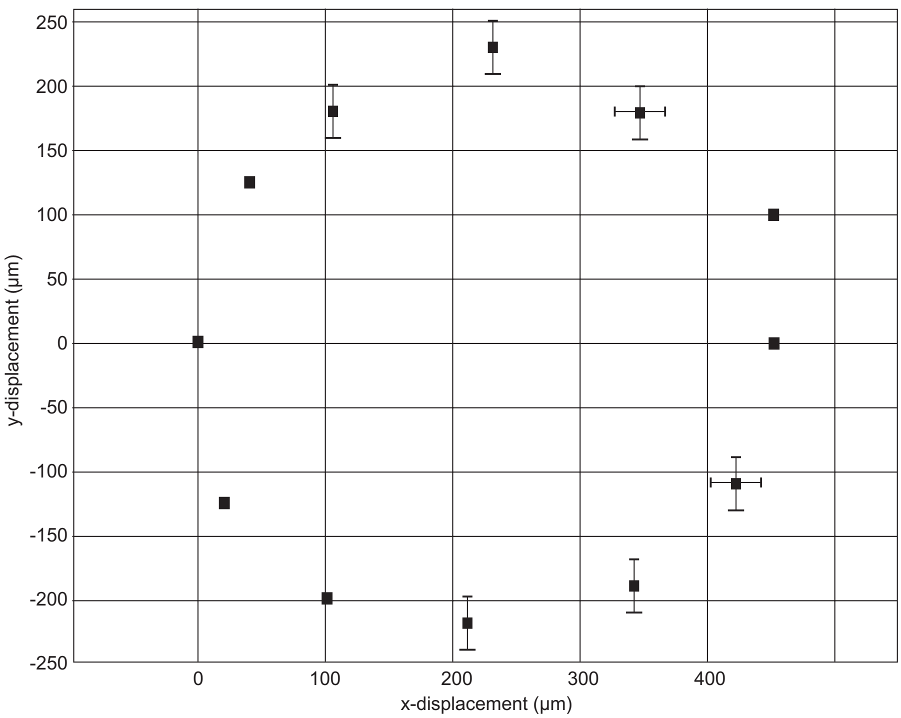

Figure 3 Tracking circular path with angular intervals of 30°. Data taken from reference [12]

Using this technique, there is no ambiguity in determining the sense of the micro-displacements, thus overcoming the double-exposure speckle photography.

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Conclusions

The results show that the nonlinear joint transform correlator is adequate for the measurement of micro-displacement in the range of tens of micrometers. It is possible to establish a reliable range for measure, by comparing the intensities of autocorrelation with cross-correlation peak. The ability of the nonlinear JTC to improve the ambiguity problem of the sense of displacement enables the plotting of 2D-trajectories. The range and resolution of measure is related with the laser wavelength, distance sample-camera and the pixel size of the sensor.

We hope that the increased capabilities of all-digital and hybrid optical-digital systems reduce computation time, potentially increasing the use of the proposed technique in an industrial environment, outside laboratory conditions.

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