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Quadratic functions for efficient load balancing in the terminals of a substation of a three-phase asymmetric network with power loss reduction capabilities

Funciones cuadráticas para el balance de carga eficiente en los terminales de una subestación de una red asimétrica trifásica con capacidades de reducción de pérdidas de potencia

Research article

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Abstract

This research addresses the problem of optimal load balancing in terminals of the three-phase substation by proposing three quadratic objective functions. These objective functions are formulated considering active, reactive, and apparent power consumptions aggregated at the terminals of the substation. The proposed formulation belongs to the mixed-integer quadratic models' family, which can be solved globally with specialized mixed-integer convex tools. To evaluate the effect of load redistribution in the substation terminals, the 15- and 35-bus grids are tested using each of the proposed quadratic functions. In addition, Broyden's unbalanced power flow method is used to determine the extent of power loss reduction and enhancement of voltage profile. Numerical results confirm the effectiveness of the proposed mixed-integer quadratic model in enhancing electrical performance in three-phase asymmetric networks through load balancing at the substation terminals. After solving each quadratic function for the 15-bus grid, power losses were reduced between 12.9624% and 17.2550%, and these reductions were between 5.0771% and 7.7389% in the 35-bus grid.

Keywords: mixed-integer quadratic models, load redistribution, asymmetric three-phase networks, Broyden's power flow method.

Resumen

Esta investigación aborda el problema del balanceo óptimo de carga en terminales de la subestación trifásica proponiendo tres funciones objetivo cuadráticas. Estas funciones objetivo se formulan considerando los consumos de energía activa, reactiva y aparente agregados en los terminales de la subestación. La formulación propuesta pertenece a la familia de modelos cuadráticos de enteros mixtos, que pueden resolverse globalmente con herramientas convexas de enteros mixtos especializadas. Para evaluar el efecto de la redistribución de carga en las terminales de la subestación, se prueban las redes de 15 y 35 barras utilizando cada una de las funciones cuadráticas propuestas. Además, el método de flujo de potencia desequilibrado de Broyden se utiliza para determinar el grado de reducción de la pérdida de potencia y la mejora del perfil de tensión. Los resultados numéricos confirman la efectividad del modelo cuadrático entero mixto propuesto para mejorar el rendimiento eléctrico en redes asimétricas trifásicas, a través del equilibrio de carga en las terminales de la subestación. Después de resolver cada función cuadrática para la red de 15 nodos, las pérdidas de energía se redujeron entre 12,9624% y 17,2550%, y estas reducciones fueron entre 5,0771% y 7,7389% en la red de 35 nodos.

Palabras clave: modelos cuadráticos enteros mixtos, redistribución de carga, redes trifásicas asimétricas, método de flujo de potencia de Broyden.

1. Introduction

Energy efficiency and optimization in the operation of electrical energy distribution systems have always been a topic of great importance (Claeys *et al*., 2021). Furthermore, with the continuous growth of the demand for electrical energy and the need to reduce electrical losses in distribution networks (Wilms *et al*., 2017), it has become imperative to develop strategies and technologies that allow for improving the operation of these networks by reducing their losses while maintaining their operating limits (Marini *et al*., 2019; Soltani *et al*., 2017).

In this context, the balanced operation of electrical energy distribution systems plays a crucial role in the efficient functioning of electrical systems (Bina & Kashefi, 2011). Therefore, carrying out load balancing at the terminals of a substation is important because various elements and circuits are interconnected. In unbalanced three-phase distribution systems, load imbalance between phases can cause a series of problems, such as overloads on certain equipment, additional losses of electrical energy, and degradation of the quality of the electrical supply (Shen *et al*., 2018; Garcés-Ruiz, 2022). Therefore, these challenges motivate us to propose innovative solutions that allow for mitigating load imbalance at substation terminals (Jimenez *et al*., 2022). In the quest to enhance operational efficiency and minimize power losses in electrical networks, it is essential to investigate novel methodologies and technologies that enable the optimization of load distribution among phases intelligently and effectively (Huangfu *et al*., 2024).

In the specialized literature, various methodologies have been proposed to address the issue of phase balance or load redistribution. The work by Al-Kharsan *et al.,* (2020), applied a particle swarm optimization (PSO) to optimize the balance of phases in unbalanced three-phase distribution systems. The PSO algorithm demonstrated its capability to explore optimal phase configurations to minimize the imbalance of the unbalanced three-phase system. The above was achieved by adjusting the inertia weight to identify the most efficient configuration and, ultimately, enhance the performance of the PSO algorithm.

The study by Cortés-Caicedo *et al*., (2021), implemented a methodology for optimal balancing in unbalanced three-phase distribution systems using a discretized vortex search algorithm (DVSA). The DVSA aimed to minimize power losses in a three-phase system by determining the optimal configuration of phase connections for the system's demand nodes. This algorithm was evaluated in IEEE 8-, 25-, and 37-node test systems and compared with the Chu-Beasley genetic algorithm. The numerical results showed the efficiency of the DVSA in reducing power losses in unbalanced three-phase distribution systems.

The work by Montoya et al. (2021), proposed an optimal balancing of phases in three-phase electrical distribution systems using a mixed-integer quadratic convex formulation, which minimized the sum of the squared currents through the distribution lines. This work was tested in three radial test systems and compared with three metaheuristic techniques. The mixed-integer quadratic convex formulation exhibited superior performance compared to the other techniques.

The authors of Bohórquez-Álvarez *et al*., (2023), addressed the challenge of minimizing power losses in unbalanced distribution networks using a convex approximation. The proposed optimization approach aimed to minimize total power losses in a three-phase network by utilizing the concept of electric momentum. The above was achieved by simplifying the power balance restrictions and transforming the objective function into a strictly convex form, resulting in efficient loss minimization. The GAMS software, with its solvers CPLEX, SBB, and XPRESS, was used to compare the performance of the proposed approximation.

Other studies have also explored the optimal phase balance in three-phase distribution networks with asymmetric loads using methods such as specialized genetic algorithm (Granada-Echeverri *et al*., 2012), smart meter data-based algorithm (Grigoraș *et al*., 2020), modified PSO algorithm (Tuppadung & Kurutach, 2006), a heuristic algorithm that uses pole measurement (El Hassan *et al*., 2022), Derivative-Free algorithm (Montoya *et al*., 2021), and genetic algorithms incorporating group theory (Garcés *et al*., 2020), among several other proposed approaches.

In light of the information provided, this research compares three mixed-integer quadratic programming models designed to address the expected imbalance in active, reactive, or apparent power at substation terminals within three-phase asymmetric networks. These optimization models, which belong to the domain of mixed-integer convex models, guarantee solution repeatability in each execution by utilizing the Gurobi solver in the Julia programming language, supplemented by the HiGHS solver in the JuMP optimization framework. These contributions offer an innovative approach to addressing power imbalances in asymmetric networks, providing a robust and reliable tool for the efficient planning and operation of electrical systems.

This paper is organized as follows: Section 2 describes the proposed methodology for optimal load balancing in terminals of the substation. Section 3 presents two test systems to assess the proposed methodology, as well as their main results. Section 4 presents the main concluding remarks of the study.

2. Methodology

In the proposed methodology for the optimal load balancing in terminals of the substation and the power loss minimization, two main elements are considered. The first element corresponds to the mixed-integer quadratic models and the second element is the three-phase power flow formulation via Broyden's formulation to evaluate the final load redistribution. Each one of these elements is widely described below.

A. Mixed-integer quadratic models

The problem of the effective load balancing in terminals of the substations assumes that in a three-phase distribution network, all the loads can be summarized as an equivalent load per phase, as presented in Figure 1.

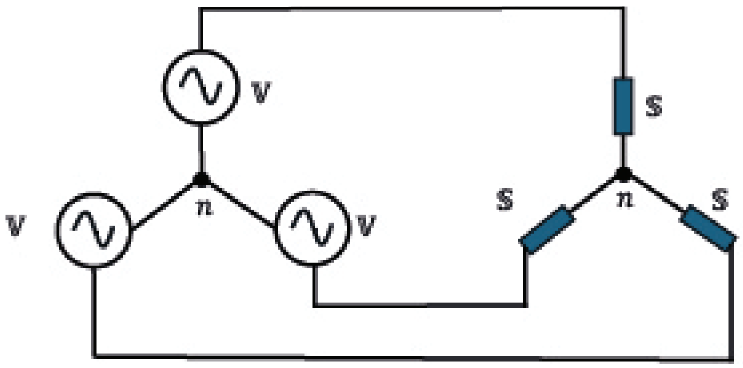

Figure 1. Equivalent load connections in terminals of the substation.

In Figure 1 it is observed that the voltage profiles in the substation are measured with respect to the neutral point n, which are defined as $\mathbb{V}_{an}$, $\mathbb{V}_{bn}$, and $\mathbb{V}_{cn}$, respectively, in addition, $\mathbb{S}_{an}$, $\mathbb{S}_{bn}$ and $\mathbb{S}_{cn}$ represent the equivalent apparent power load connections in terminals of the substation (i.e., complex values that can be separated in their real and imaginary parts called active and reactive powers). Note that for a three-phase, load, there exists six possible connections in terminals of a particular k node. Table 1 summarizes these possibilities, where M_{kj} represents the possible load rotation at node k, which have six options, i.e., $j = 1,2, \ldots ,6$, being $\mathbb{S}_{k,0}^{abc}$ the initial load connection and $\mathbb{S}_{k,f}^{abc}$ the final load connection at node k.

Table 1. Possible load redistribution in a particular k node

Initial Connection	Final connection $\mathbb{S}_{k,f}^{abc}$	Equivalent load connection
$\mathbb{S}_{k,0}^{abc} = \{\mathbb{S}_{kan} \quad \mathbb{S}_{kbn} \quad \mathbb{S}_{kcn}\}$	$\{\mathbb{S}_{kan} \quad \mathbb{S}_{kbn} \quad \mathbb{S}_{kcn}\}$	$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	$\{\mathbb{S}_{kcn} \quad \mathbb{S}_{kan} \quad \mathbb{S}_{kbn}\}$	$M_2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
	$\{\mathbb{S}_{kbn} \quad \mathbb{S}_{kcn} \quad \mathbb{S}_{kan}\}$	$M_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
	$\{\mathbb{S}_{kan} \quad \mathbb{S}_{kcn} \quad \mathbb{S}_{kbn}\}$	$M_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$
	$\{\mathbb{S}_{kbn} \quad \mathbb{S}_{kan} \quad \mathbb{S}_{kcn}\}$	$M_5 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	$\{\mathbb{S}_{kcn} \quad \mathbb{S}_{kbn} \quad \mathbb{S}_{kan}\}$	$M_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

From Table 1 it is observed for any k node it is possible to obtain a load rotation considering the possible load rotation M_j related with the binary variable x_{kj}, using a vector representation, as defined in Equation

$$\mathbb{S}_{k,f}^{abc} = \sum_{j=1}^{6} x_{kj} M_{kj} \mathbb{S}_{k,0}^{abc}, \{\forall k \in \boldsymbol{N}\} \quad (1).$$

where $\boldsymbol{N}$ is the set containing all the nodes of the network, in addition, from Equation (1) the real and imaginary parts of the apparent power (i.e., $P_{k,f}^{abc}$ and $Q_{k,f}^{abc}$) can be split into Equations (2) and (3).

$$P_{k,f}^{abc} = \sum_{j=1}^{6} x_{kj} M_{kj} P_{k,0}^{abc}, \{\forall k \in \boldsymbol{N}\} \quad (2)$$

$$Q_{k,f}^{abc} = \sum_{j=1}^{6} x_{kj} M_{kj} Q_{k,0}^{abc}, \{\forall k \in \boldsymbol{N}\} \quad (3)$$

To ensure that at each k node only and only one possible load distribution (as shown in Table 1), the following constraint is imposed to the binary variables, as defined in Equation (2).

$$\sum_{j=1}^{6} x_{kj} = 1, \{\forall k \in \boldsymbol{N}\} \quad (4)$$

Regarding the objective function, the problem of the optimal load redistribution in terminals of the substation can be done with respect to the active, reactive power, apparent powers. The proposed three quadratic objective functions are defined from (5) to (7).

$$\min F_1 = \left(\frac{100\%}{3P_{av}^2}\right) \sum_{p \in \{a,b,c\}} \left(P_{k,f}^{p} - P_{av}\right)^2 \quad (5)$$

$$\min F_2 = \left(\frac{100\%}{3Q_{av}^2}\right) \sum_{p \in \{a,b,c\}} \left(Q_{k,f}^{p} - Q_{av}\right)^2 \quad (6)$$

$$\min F_3 = \frac{1}{2}(F_1 + F_2) \quad (7)$$

where F_1 is the average active power unbalance, F_2 is the average reactive power unbalance and F_3 represents the average apparent power unbalance, being P_{av} and Q_{av} the ideal active and reactive power consumption per phase.

B. The Broyden's three-phase power flow method

Broyden's power flow method for three-phase unbalanced distribution grids is a recently developed algorithm for dealing with the voltage variables in asymmetric grids proposed by authors of (Riaño-Enciso *et al*., 2023). The main advantages of Broyden's power flow method are (i) solving the power flow problem for radial and meshed topologies, (ii) its equivalence with classical graph-based power flow approaches (Marini *et al*., 2019; Shen *et al*., 2018), and (iii) its derivative-free modeling.

The general structure of Broyden's numerical method is defined in Equations (8) and (9), where the recursive formula is defined using an approximation of the Jacobian matrix.

$$x_{i+1} = x_i - [A_i]^{-1} F(x_i), \quad (8)$$

$$A_{i+1} = A_i + \frac{(y_{i+1} - [A_i]s_{i+1})s_{i+1}^{T}}{(s_{i+1}^{T})s_{i+1}^{T}}, \quad (9)$$

where A_i is the approximation of the Jacobian matrix at the iteration i, being x_i the initial value of the vector for variables x, while the vector $F(x_i)$ corresponds to the gradient of a nonlinear function evaluated at the current solution x_i. In addition, the vectors s_{i+1} and y_{i+1} are auxiliary vectors calculated as follows:

$$s_{i+1} = -[A_i]^{-1} F(x_i), \quad (10)$$

$$y_{i+1} = F(x_{i+1}) - F(x_i) \quad (11)$$

It is worth mentioning that the application of Broyden's method defined in Equations (8)-(11) is general for minimizing nonlinear functions that produce a set of equations with the structure of $F(x) = 0$, i.e., a simultaneous set of equations where the aim is to find a feasible solution with an acceptable convergence error. The convergence criterion is defined in (12).

$$max\{||x_{i+1}| - |x_i||\} \leq \varepsilon \quad (12)$$

where ε is the convergence error, which is typically defined as 1×10^{-10} (Montoya-Giraldo, Grisales-Noreña, Chamorro, & Alvarado-Barrios, 2021).

In the case of power flow studies, as proposed by authors of (Riaño-Enciso, Montoya-Giraldo, & Gil-González, 2023), the gradient vector $F(x)$ can be written in the complex domain using voltage variables as x. In this context, the gradient vector is defined in Equation (13).

$$F(V_{d3\varphi i}) = Y_{ds3\varphi} V_{s3\varphi} + Y_{dd3\varphi} V_{d3\varphi i} + I_{d3\varphi i} = 0 \quad (13)$$

where $V_{d3\varphi i}$ is the vector of voltage variables at the iteration i and $V_{s3\varphi}$ is the vector containing the three-phase output voltage in terminals of the substation. being $I_{d3\varphi i}$ the vector containing the demanded currents associated with the constant power terminals. Note that $Y_{dd3\varphi}$ and $Y_{ds3\varphi}$ are constant matrices related to the electrical connection among nodes in the three-phase network, where $Y_{dd3\varphi}$ is invertible (Riaño-Enciso, Montoya-Giraldo, & Gil-González, 2023). For complete details regarding the calculations required to implement Equation (12) consult the successive approximation power flow method reported by authors of (Cortés-Caicedo, Avellaneda-Gómez, Montoya, Alvarado-Barrios, & Chamorro, 2021).

The implementation of Broyden's power flow method to three-phase asymmetric networks considers, as recommended by (Riaño-Enciso, Montoya-Giraldo, & Gil-González, 2023), the selection of the initial A_i as $Y_{dd3\varphi}$, which permits to solve the vector of voltage profiles ensuring linear convergence. Once the voltage profiles, i.e., $V_{d3\varphi}$, are obtained, the expected level of power losses is reached for the analyzed load connections. The calculation of the power losses is made using Equation (14).

$$P_{loss} = \begin{pmatrix} V_{s3\varphi} & V_{d3\varphi} \end{pmatrix}^T Y_{bus3\varphi} \begin{pmatrix} V_{s3\varphi} \\ V_{d3\varphi} \end{pmatrix}, \quad (14)$$

being $Y_{bus3\varphi}$ the three-phase nodal admittance matrix.

C. Summary of the solution methodology

The proposed solution methodology to reduce the level of power losses in distribution networks via the load balancing in terminals of the substation considering a mixed-integer quadratic formulation considers tree main steps. These steps are listed below.

i) The solution of the power flow problem considering the initial load connections, i.e., setting the variable $x_{kj} = 1$ for each $j = 1$ at each node, using Broyden's numerical method. With the solution of the voltage variables is determined the initial level of power losses using formula (14).

ii) Solve the optimization models given by objective functions (5)-(7) subject to the set of constraints (1)-(4).

iii) For each solution reported above, solve the power flow problem using Broyden's method to obtain the level of power losses of each potential solution.

Note that once all the expected power losses value is calculated for each optimization model, i.e., the minimization of functions F_1, F_2, or F_3, the solution that will be implemented regarding the load redistribution at each node will be the solution that the lower power losses produce after evaluating formula (14).

3. Results and discussion

In this section is presented the test feeders under analysis, which consist to the 15-bus and 35-bus unbalanced distribution grids, and the numerical validations of the proposed optimization models for optimal load balancing the total power consumptions in terminals of the substation and their effect of the expected power losses level.

3.1 Test feeders

The medium-voltage distribution networks under analysis correspond to the 15- and 35-bus grids, which are unbalanced distribution systems with radial topology.

The 15-bus grid

The 15-bus grid is a distribution network operating with a line-to-line voltage of 13.2 kV composed of 14 distribution lines with 6 type of branches impedances. The total constant power load consumption in phases a, b, and c is $9605 + j5226$ kVA, $6480 + j4940$ kVA, and $11977 + j8778$ kVA, respectively. Table 2 and 3 defines the power consumption per node and the nodal connection (branches information including the impedance type).

Table 2. Load information for each phase in the 15-bus grid

Node	$\mathbb{S}_{da}$ (kVA)	$\mathbb{S}_{db}$ (kVA)	$\mathbb{S}_{dc}$ (kVA)
1	$0 + 0i$	$0 + j0$	$0 + j0$
2	$0 + j0$	$725 + j300$	$1100 + j600$
3	$480 + j220$	$720 + j600$	$1040 + j558$
4	$2250 + j1610i$	$0 + j0$	$0 + j0$
5	$700 + j225$	$0 + j0$	$996 + j765$
6	$0 + j0$	$820 + j700$	$1220 + j1050$
7	$2500 + j1200$	$0 + j0$	$0 + j0$
8	$0 + j0$	$960 + j540$	$0 + j0$
9	$0 + j0$	$0 + j0$	$2035 + j1104$
10	$1519 + j1250$	$1259 + j1200$	$0 + j0$
11	$0 + j0$	$259 + j126$	$1486 + j1235$
12	$0 + j0$	$0 + j0$	$1924 + j1857$
13	$1670 + j486$	$0 + j0$	$726 + j509$
14	$0 + j0$	$850 + j752$	$1450 + j1100$
15	$486 + j\,235$	$887 + j722$	$0 + j0$
Total	$\mathbf{9605 + j5226}$	$\mathbf{6480 + j4940}$	$\mathbf{11977 + j8778}$

Table 3. Information of nodal connection and impedance type for the 15-bus grid.

Send	Receive	Type	Length (ft)	Send	Receive	Type	Length (ft)
1	2	1	603	8	9	6	225
2	3	2	776	9	10	6	1050
3	3	4	825	3	11	3	837
4	5	3	1182	11	12	4	414
5	6	4	350	12 13	13	5	925
2	7	5	691	6	14	4	386
7	8	6	539	14	15	2	401

The set of impedances associated with each type of conductor listed in Table 3 is presented in Table 4.

Table 4. Impedance assignable to each type of conductor listed in Table 3.

Type	Impedance (Ω/mil)
1	$\begin{bmatrix} 0.093654+j0.040293 & 0.031218+j0.013431 & 0.031218+j0.013431 \\ 0.031218+j0.013431 & 0.093654+j0.040293 & 0.031218+j0.013431 \\ 0.031218+j0.013431 & 0.031218+j0.013431 & 0.093654+j0.040293 \end{bmatrix}$
2	$\begin{bmatrix} 0.15609+j0.067155 & 0.05203+j0.022385 & 0.05203+j0.022385 \\ 0.05203+j0.022385 & 0.15609+j0.067155 & 0.05203+j0.022385 \\ 0.05203+j0.022385 & 0.05203+j0.022385 & 0.15609+j0.067155 \end{bmatrix}$
3	$\begin{bmatrix} 0.046827+j0.0201465 & 0.015609+j0.0067155 & 0.015609+j0.0067155 \\ 0.015609+j0.0067155 & 0.046827+j0.0201465 & 0.015609+j0.0067155 \\ 0.015609+j0.0067155 & 0.015609+j0.0067155 & 0.046827+j0.0201465 \end{bmatrix}$
4	$\begin{bmatrix} 0.031218+j0.013431 & 0.010406+j0.004477 & 0.010406+j0.004477 \\ 0.010406+j0.004477 & 0.031218+j0.013431 & 0.010406+j0.004477 \\ 0.010406+j0.004477 & 0.010406+j0.004477 & 0.031218+j0.013431 \end{bmatrix}$
5	$\begin{bmatrix} 0.062436+j0.026862 & 0.020812+j0.008954 & 0.020812+j0.008954 \\ 0.020812+j0.008954 & 0.062436+j0.026862 & 0.020812+j0.008954 \\ 0.020812+j0.008954 & 0.020812+j0.008954 & 0.062436+j0.026862 \end{bmatrix}$
6	$\begin{bmatrix} 0.078045+j0.0335775 & 0.026015+j0.0111925 & 0.026015+j0.0111925 \\ 0.026015+j0.0111925 & 0.078045+j0.0335775 & 0.026015+j0.0111925 \\ 0.026015+j0.0111925 & 0.026015+j0.0111925 & 0.078045+j0.0335775 \end{bmatrix}$

The 35-bus system

The 35-bus grid corresponds to a radial distribution network composed of 35 nodes and 34 distribution lines. The line-to-ground voltage in terminals of the substation corresponds to 15 kV. The total constant power load consumption in phases a, b, and c $4435 + j1940$ kVA, $2868 + j2059$ kVA, and $2925 + j1370$ kVA, respectively. Table 5 and 6 defines the power consumption per node and the nodal connection (branches information including the impedance type).

Table 5. Load information for each phase in the 35-bus system.

Node	$\mathbb{S}_{da}$ (kVA)	$\mathbb{S}_{db}$ (kVA)	$\mathbb{S}_{dc}$ (kVA)
1	$0 + j0$	$0 + j0$	$0 + j0$
2	$100 + j50$	$100 + j60$	$50 + j50$
3	$50 + j0$	$70 + j40$	$50 + j40$
4	$120+ j75$	$100 + j80$	$150 + j90$
5	$60 + j20$	$60 + j30$	$30 + j30$
6	$400 + j180$	$0 + j0$	$300 + j150$
7	$200 + j150$	$110 + j70$	$100 + j100$
8	$200 + j0$	$100 + j100$	$150 + j150$
9	$120 + j75$	$0 + j0$	$0 + j0$
10	$0 + j0$	$600 + j400$	$0 + j0$
11	$130 + j100$	$0 + j0$	$0 + j0$
12	$125 + j75$	$60 + j35$	$155 + j100$
13	$60 + j110$	$60 + j35$	$60 + j35$
14	$120 + j80$	$190 + j80$	$0 - j400$
15	$60 + j10$	$0 + j0$	$0 + j0$
16	$60 + j20$	$110 + j80$	$60 + j20$
17	$60 + j20$	$150 + j95$	$0 + j0$
18	$90 + j40$	$100 + j0$	$90 + j40$
19	$300 + j150$	$0 + j0$	$90 + j40$
20	$210 + j50$	$85 + j40$	$70 + j75$
21	$90 + j40$	$110 + j40$	$110 + j20$
22	$300 + j400$	$0 + j0$	$90 + j40$
23	$90 + j50$	$70 + j0$	$0 + j0$
24	$300 + j200$	$0 - j600$	$250 + j100$
25	$120 + j75$	$0 + j0$	$150 + j100$
26	$60 + j25$	$80 + j25$	$0 + j0$
27	$210 + j145$	$80 + j25$	$0 + j0$
28	$60 + j20$	$48 + j24$	$60 + j20$
29	$120 + j70$	$185 + j75$	$220 + j90$

30	$200 - j500$	$0 + j400$	$300 + j350$
31	$150 + j70$	$120 + j90$	$150 + j70$
32	$210 + j100$	$120 + j35$	$180 + j50$
33	$60 + j40$	$100 + j350$	$0 + j0$
34	$0 + j0$	$0 + j0$	$60 + j10$
35	$0 + j0$	$60 + j450$	$0 + j0$
Total	$\mathbf{4435 + j1940}$	$\mathbf{2868 + j2059}$	$\mathbf{2925 + j1370}$

Table 6. Impedance information for each branch in the 35-bus system

k	m	R_{km} (Ω)	X_{km} (Ω)	k	m	R_{km} (Ω)	X_{km} (Ω)
1	2	0.0922	0.0477	2	19	0.1640	0.1565
2	3	0.4930	0.2511	19	20	1.5042	1.3554
3	4	0.3660	0.1864	20	21	0.4095	0.4784
4	5	0.3811	0.1941	21	22	0.7089	0.9373
5	6	0.8190	0.7070	3	23	0.4512	0.3083
6	7	0.1872	0.6188	23	24	0.8980	0.7091
7	8	1.7114	1.2351	24	25	0.8960	0.7011
8	9	1.0300	0.7400	6	26	0.2030	0.1034
9	10	1.0400	0.7400	26	27	0.2842	0.1447
10	11	0.1966	0.0650	27	28	1.0590	0.9337
11	12	0.3744	0.1238	28	29	0.8042	0.7006
12	13	1.4680	1.1550	29	30	0.5075	0.2585
13	14	0.5416	0.7129	30	31	0.9744	0.9630
14	15	0.5910	0.5260	31	32	0.3105	0.3619
15	16	0.7463	0.5450	32	33	0.3410	0.5302
16	17	1.2860	1.7210	15	34	0.2819	0.4012
17	18	0.7320	0.5740	34	35	0.1958	0.2714

It is worth mentioning that in the case of the impedance information of the 35-bus system, the impedance assignable to each branch is diagonal, i.e., this presents the structure of defined in Equation (15).

$$Z_{km} = \begin{bmatrix} R_{km} + jX_{km} & 0 & 0 \\ 0 & R_{km} + jX_{km} & 0 \\ 0 & 0 & R_{km} + jX_{km} \end{bmatrix} \quad (15)$$

3.2 Results

In this subsection, the numerical analysis of both test feeders is reported. The benchmark case and the solution of each of the three-proposed optimization models are also reported, added with the final value of the power losses. Note that all the numerical results reported in this section were reached with the usage of the Gurobi solver in the Julia programming language by its combinations with the HiGHS solver in the JuMP optimization environment (Bezanson *et al.*, 2017; Lubin *et al.*, 2022).

Numerical results for the 15-bus grid

Table 7 reports the solution reached with the solution of each objective function defined in the optimization model (1)-(7) by comparing the initial and final values of each objective function and their corresponding power losses for the 15-bus system simulation case.

Table 7. Numerical results in the 15-bus grid.

Objective function	Initial value (%)	P_{loss} (kW)	Final value (%)	P_{loss} (kW)
F_1	5.7918		0	111.0828
F_2	7.6430	134.2472	5.5730x10^{-07}	114.5305
F_3	6.7174		8.9209x10^{-05}	116.8456

Numerical results in Table 7 shows that:

i) Each optimization model using the quadratic formulation allows minimizing the total expected power unbalanced aggregated in terminals of the substation since function F_1, F_2, and F_3 is minimized as much as possible, owing the theoretical global optimal solution of each one is zero.

ii) The expected level of active power losses, as defined by formula (14), is reduced with respect the benchmark case. When F_1 is minimized, the reduction is about 23.1644 kW, i.e., 17.2550 %. In the case of minimizing F_2, the total reduction in power losses is about 19.7167 kW, i.e., 14.6869 %. Finally, in the case of minimizing function F_3, the total reduction in power losses is about 12.9624 %, i.e., 17.4016 kW.

Note that in the case of the 15-bus grid, the proposed optimization model (1)-(7) presents a better result when the objective function regarding the active power balance is minimized; however, it is recommended that for each distribution network, all the performance indexes in Equations (5)-(7) must be tested, since the numerical performance regarding power losses can be vary, which make the optimization result for a particular objective function non-generalizable.

Numerical results for the 35-bus grid

Table 8 reports the solution reached with the solution of each objective function defined in the optimization model (1)-(7) by comparing the initial and final values of each objective function and their corresponding power losses for the 35-bus system simulation case.

Table 8. Numerical results in the 35-bus grid

Objective function	Initial value (%)	P_{loss} (kW)	Final value (%)	P_{loss} (kW)
F_1	4.5299		7.6473x10^{-06}	449.0975
F_2	2.8231	473.1181	6.9381x10^{-06}	436.5039
F_3	3.6765		7.2927x10^{-06}	446.4338

Numerical results in Table 8 shows that:

i) The values reached for each objective function are near to zero, i.e., with values lower than 1x10^{-05} %, which implies that the proposed quadratic convex models reach the optimal solution for each one of the objective functions analyzed. Note that for many practical applications the expected solution equal to zero is not reachable since it is highly probably that with the discrete movements in loads the ideal power consumption per phase is not practically obtained.

ii) The expected level of active power losses, as defined by formula (14), is reduced with respect the benchmark case. When F_1 is minimized, the reduction is about 24.0206 kW, i.e., 5.0771 %. In the case of minimizing F_2, the total reduction in power losses is about 36.6142 kW, i.e., 7.7389 %. Finally, in the case of minimizing function F_3, the total reduction in power losses is about 5.6401%, i.e., 26.6843 kW.

Results in the case of the 35-bus system shows that the proposed optimization model (1)-(7), allows reaching the lowest possible power losses when function F_2 is minimized. These results confirm that it is not possible to correlate the minimum power losses results with a particular objective function, which implies that for each distribution network under analysis, all the quadratic functions must be evaluated to obtain the higher reduction in power losses via Broyden's power flow approach.

4. Conclusions

This article has presented three mixed-integer quadratic programming models to reduce the expected active, reactive, or apparent power unbalance in terminals of the substations in three-phase asymmetric networks. The proposed optimization models belong to the family of mixed-integer convex models and allows ensuring the solution repeatability at each execution using the Gurobi solver in the Julia programming language with the help of the HiGHS solver in the JuMP optimization environment.

A two-stage optimization approach was developed to evaluate the initial and final expected power losses in the three-phase network via the power flow approach based on Broyden's numerical method. The numerical results in the 15- and 3 bus grids demonstrated that:

In the case of the 15-bus grid, the best optimization model corresponds to the minimization of objective function F_1 with an expected reduction in the power losses of about 17.2550 %. However, in the case of the 35-bus grid, the best objective function was F_2, since it allowed reduction in power losses of about 7.7389 %.

The aforementioned results showed that is not possible to relate the maximum power losses reduction calculated with a power flow approach (i.e., Broyden's numerical method) with a particular objective function (see Equations (5)-(7)), since for each distribution network is mandatory to evaluate the proposed solution methodology in order to identify which is the better objective function indicator when the final objective is minimizing the expected grid power losses.

To improve the numerical performance of the proposed solution methodology it is recommended to include in the mixed-integer quadratic programming formulation the effect of the distribution branches. This effect could be modeled using a linear approximation of the power flow equations, which can help to have better numerical values for the expected power losses when compared to the proposed two-stage optimization approach.

Possible future works derived from this research can be the following: (a) to extend the proposed two-stage optimization approach to bipolar direct current networks with asymmetric loading; (b) the addition in the mixed-integer quadratic formulation of the current flow effect in branches using a linear approximation for the three-phase power flow problem, and (c) the application of hybrid optimization algorithms based on the combination of metaheuristic optimizers with Broyden numerical method. The metaheuristic will be entrusted with determining the load connection per node, while the power flow method (Broyden's numerical method) will be encharged with determining the power losses level.

Authors' Contribution

Lina María Riaño-Enciso: formal analysis, investigation, Methodology, software, validation, visualization, writing - original draft.

Oscar Danilo Montoya-Giraldo: conceptualization, project administration, formal analysis, investigation, Methodology, software, validation, visualization, writing - review editing.

Walter Julián Gil-González: data curation, formal nálisis, investigation, methodology, validation, visualization, writing – original draft.

Ethical implications

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Conflicts of interest

There are no conflicts of interest from the authors in the writing or publication of this article.

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