Self-Tuning Control for a Class of Bilinear Systems

Controladores auto-ajustables para una clase de sistemas bilineales

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PALABRAS CLAVES
Sistemas bilineales, controlador auto-ajustable, varianza mínima generalizada.

RESUMEN

En este trabajo, se propone un algoritmo auto-ajustable implícito basado en el criterio de varianza mínima generalizada para la estabilización de una clase de sistemas bilineales. La estabilidad del algoritmo propuesto es demostrada usando una función de Lyapunov y la estrategia de control por superficie deslizante. El algoritmo auto-ajustable propuesto es aplicado a una planta piloto térmica para evaluar su desempeño.

KEY WORDS
Bilinear systems, generalized minimum variance, self-tuning control.

ABSTRACT

In this paper, a self-tuning algorithm based on the generalized minimum variance criterion is proposed for the stabilization of a class of bilinear systems. Using a Lyapunov function and the sliding mode control approach, the stability of the proposed algorithm is proven. The proposed self-tuning algorithm is applied to a simulated example to evaluate its performance.

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INTRODUCTION

Bilinear systems comprise perhaps the simplest class of nonlinear systems which has a lot of applications in various fields, e.g. [1] and reference therein. Several control approaches have been proposed to treat the stabilization problem of bilinear systems (see [2, 3] and the references therein). However, only a few papers (e.g. [4, 5]) have focused on the stabilization problem of bilinear systems with time delay. So far, stability of implicit self-tuning control, based on generalized minimum variance criterion for minimum and a class of non-minimum phase linear systems has been demonstrated by the use of a Lyapunov function in [6], and for those systems, it suffices to use linear functions of the data to predict the system output response. However, in general, it may be desirable, or even necessary, to consider the use of nonlinear functions to get good predictions and hence good control performance.

By following the idea of Goodwin [7, 8], Sun [5] gave proof of the explicit self-tuning controller of bilinear systems. However, the proof relies on assuring parameter convergence in the close-loop system, when the projection algorithm is used.

In this paper, stability of the implicit self-tuning controllers for a special class of discrete-time bilinear systems, represented by the input-output relation with unknown parameters, is proven. The treated bilinear class is the class where only bilinearity exists between the measured and the control signals; additionally control signals must appear in the system structure in linear form. The proposed algorithm consists in the combination of the generalized minimum variance control and recursive identification of the control parameters. The control objective is to minimize the variance of a sliding mode surface proposed for this class of bilinear systems. The discrete-time bilinear model could be given directly in the discrete-time form or derived by discretization from the continuous-time (minimum or non-minimum phase) system. Stability of the algorithm is proven by using a Lyapunov function. The proposed self-tuning algorithm is applied to a simulated example to verify and to show the performance of the algorithm. Part of this work was presented in the CLCA 2008 [9].

The paper is organized as follows: firstly, the generalized minimum variance criterion for bilinear systems is given. Secondly, the recursive self-tuning controller parameter estimation, based on the generalized minimum variance criterion for the class of bilinear systems, is studied and the main results are given by the theorem which assures overall system stability. Then, the proposed algorithm is applied to a simulated example and digital simulations are shown. The paper concludes with some additional remarks.

GENERALIZED MINIMUM VARIANCE CONTROL FOR BILINEAR SYSTEMS

Bilinear systems are a special class of non-linear systems that are linear in input and linear in state but not jointly linear in state and input. Specifically, a time invariant single-input and single-output (SISO) bilinear system has a discrete-time form as follows:

$$A(z^{-1})y_k = z^{-d} B(z^{-1})u_k + z^{-d} N(z^{-1})\, y_k u_k, \quad (1)$$

where there are no common factors in $(A(z^{-1}), N(z^{-1}))$, or in $(A(z^{-1}), B(z^{-1}))$ and the time delay d is known. z denotes the time shift operator $z^{-1} y_k = y_{k-1}$. In the Laplace transformation, $z = e^{sT_o}$ where T_o is the sampling period (for simplicity, and without loss of generality, $T_o = 1$ is assumed).

In this section, to derive the nominal control law, the polynomials $A(z^{-1})$, $B(z^{-1})$ and $N(z^{-1})$ are assumed to be known, and represented as:

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} \ldots + a_n z^{-n}$$

$$B(z^{-1}) = b_o + b_1 z^{-1} + b_2 z^{-2} \ldots + b_m z^{-m},\ b_o \neq 0$$

$$N(z^{-1}) = n_o + n_1 z^{-1} + \ldots + n_\varsigma z^{-\varsigma},\ n_o \neq 0$$

Remark 1: The special class of discrete-time bilinear systems to be considered in this paper is the class where the discrete-time system can be described as in (1).

This means that polynomials $A(z^{-1}) \neq 0$, $B(z^{-1}) \neq 0$, $N(z^{-1}) \neq 0$, $d > 0$, and the bilinearity is considered only between the output (measured state) and the input variable.

The following notations are introduced:

$$z^{-t}u_k = u_{k-t},\ (z^{-t}u_k)v_k = u_{k-t}\,v_k,\ z^{-t}u_kv_k = u_{k-t}\,v_{k-t},$$

$$z^{t}z^{-t}u_kv_k = z^{t}z^{-t}(u_kv_k) = u_kv_k.$$

The control objective is to minimize the variance of the controlled sliding mode variable s_{k+d}, which is defined in the deterministic case as:

$$s_{k+d} = C(z^{-1})(y_{k+d} - r_{k+d}) + Q(z^{-1})u_k + H(z^{-1})y_ku_k \quad (2)$$

where $H(z^{-1}) = E(z^{-1})N(z^{-1})$, and polynomial $E(z^{-1})$ will be defined later. The polynomials:
$C(z^{-1}) = 1 + c_1z^{-1} + c_2z^{-2} + \dots c_nz^{-n}$, and
$Q(z^{-1}) = q_o(1 - z^{-1})$ are to be designed, so that the specification given below should be satisfied. The error signal e_k is defined as $e_k = y_k - r_k$, where r_k is the reference signal. The proposed idea is similar to that of the discrete time sliding mode control in [10, 11].

Multiplying (1) by $E(z^{-1})$, the following is obtained:

$$E(z^{-1})A(z^{-1})y_k = z^{-d}E(z^{-1})B(z^{-1})u_k + z^{-d}E(z^{-1})N(z^{-1})\,y_k\,u_k. \quad (3)$$

Using the Diophantine equation:

$$C(z^{-1}) = A(z^{-1})E(z^{-1}) + z^{-d}F(z^{-1}), \quad (4)$$

where,

$$E(z^{-1}) = e_o + e_1z^{-1} + \dots + e_{d-1}z^{-d+1},$$

$$F(z^{-1}) = f_o + f_1z^{-1} + \dots + f_{n-1}z^{-n+1},$$

equation (3) is rewritten as:

$$C(z^{-1})y_k - z^{-d}F(z^{-1})y_k = z^{-d}E(z^{-1})B(z^{-1})u_k + z^{-d}E(z^{-1})N(z^{-1})y_ku_k, \quad (5)$$

and rewriting (5) in the time $k+d$, then

$$C(z^{-1})y_{k+d} = E(z^{-1})N(z^{-1})y_ku_k + F(z^{-1})y_k + E(z^{-1})B(z^{-1})u_k \quad (6)$$

Combining (6) and (2), the sliding mode variable results in:

$$s_{k+d} = G(z^{-1})u_k + F(z^{-1})y_k - C(z^{-1})r_{k+d} \quad (7)$$

where the polynomial $G(z^{-1})$ is defined as:

$$G(z^{-1}) = E(z^{-1})B(z^{-1}) + Q(z^{-1}). \quad (8)$$

Then the generalized minimum variance control input required to vanish s_{k+d} in (2) is given by:

$$u_k = -\frac{F(z^{-1})y_k - C(z^{-1})r_{k+d}}{G(z^{-1})}. \quad (9)$$

In closed-loop, the characteristic polynomial from the output signal y_k to the reference signal r_k is given by:

$$T(z^{-1}) = B(z^{-1})C(z^{-1}) + A(z^{-1})Q(z^{-1}) \quad (10)$$

For the closed-loop design, polynomial $C(z^{-1})$ must be chosen Schur (all roots of $C(z^{-1})$ must be inside the unit disk) and the gain q_0 in $Q(z^{-1})$ is designed as any $q_0>0$ that makes the nominal control system stable, the root-locus technique may be used to choose q_0 [6].

SELF-TUNING CONTROL OF BILINEAR SYSTEMS BASED ON GENERALIZED MINIMUM VARIANCE CRITERION

In this section, the system in (1) is considered as a system with the same structure having parametric uncertainties.

The overall stability of the self-tuning control based on generalized minimum variance criterion for SISO linear systems has been proved in [6], when the system constant parameters are not accurately known, by recursive estimation of the controller parameter $F(z^{-1})$ and $G(z^{-1})$, i.e. $\hat{F}(z^{-1})$ and $\hat{G}(z^{-1})$ are estimates of $F(z^{-1})$ and $G(z^{-1})$, under the following assumptions [6].

Assumptions 1 [6]: 1. The order of the system (1) is known. 2. The delay step d is known. 3. Polynomial $C(z^{-1})$ is Schur. 4. The considered system (1) with parametric uncertainties is in the class of systems which can be stabilized by the polynomials $Q(z^{-1})$ and $C(z^{-1})$ designed for the nominal system model. 5. The reference signal r_k is bounded, i.e. $|r_k|<m_r$ for all k, where m_r is a positive constant.

Assumption 4 comes from the algorithm´s robust stability analysis explained in [6], and the robust stability is achieved by designing $Q(z^{-1})$ and $C(z^{-1})$ as explained before.

In this paper, for the bilinear case, the overall stability of self-tuning control for bilinear systems based on generalized minimum variance criterion is given by the following recursive estimation equations:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \frac{\Gamma_{k-1}\phi_{k-d}}{1+\phi_{k-d}^{T}\Gamma_{k-1}\phi_{k-d}}\,[s_k + C(z^{-1})r_k + H(z^{-1})y_{k-d}u_{k-d} - \phi_{k-d}^{T}\hat{\theta}_{k-1}] \quad (11)$$

$$\Gamma_k = \Gamma_{k-1} - \frac{\Gamma_{k-1}\phi_{k-d}\phi_{k-d}^{T}\Gamma_{k-1}}{1+\phi_{k-d}^{T}\Gamma_{k-1}\phi_{k-d}} \quad (12)$$

where

$$\phi_k^{T} = [y_k, \ldots, y_{k-n+1}, u_k, \ldots, u_{k-m-d+1}, \ldots, y_k u_k, y_{k-1}u_{k-1}, \ldots, y_{k-\varsigma(d-1)}u_{k-\varsigma(d-1)}] \quad (13)$$

is the vector containing measured output and control signal data,

$$\theta^{T} = [f_o, \ldots, f_{n-1}, g_0, \ldots, g_{m+d-1}, \ldots, h_0, h_1, \ldots, h_{\varsigma(d-1)}] \quad (14)$$

is the vector containing the controller parameters, and

$$\hat{\theta}^{T} = [\hat{f}_o, \ldots, \hat{f}_{n-1}, \hat{g}_0, \ldots, \hat{g}_{m+d-1}, \ldots, h_0, h_1, \ldots, h_{\varsigma(d-1)}] \quad (15)$$

is the estimate of θ. Note that the parameters of $H(z^{-1})$ do not need to be estimated.

The controller uses identified parameters as follows:

$$u_k = -\frac{\hat{F}(z^{-1})y_k - C(z^{-1})r_{k+d}}{\hat{G}(z^{-1})} \quad (16)$$

Theorem 1: (Recursive estimates of controller parameters based on generalized minimum variance criterion for bilinear systems.) Given a positive definite matrix Γ_0 and the initial parameters vector $\hat{\theta}_0$, if the estimate $\hat{\theta}_k$ of the controller (15) satisfies the recursive equations (11) and (12), under the set of Assumptions 1, then the self-tuning controller combining (16), (11) and (12) for the bilinear system (1) with parametric uncertainties is stable.

Proof: s_{k+d} is written as:

$$s_{k+d} = \hat{G}(z^{-1})u_k + \hat{F}(z^{-1})y_k - C(z^{-1})r_{k+d} + \phi_k^{T}\tilde{\theta}_{k+d} \quad (17)$$

where $\tilde{\theta}_k = \theta - \hat{\theta}_k$. (18)

Using the control law (16), equation (17) is rewritten as:

$$s_{k+d} = \phi_k^{T}\tilde{\theta}_{k+d}. \quad (19)$$

Consider the candidate Lyapunov function:

$$V_k = \frac{1}{2}s_k^2 + \frac{1}{2}\tilde{\theta}_k^{T}\Gamma_k^{-1}\tilde{\theta}_k. \quad (20)$$

The time difference of (20) is:

$$\Delta V_k = V_k - V_{k-1}, \quad (21)$$

$$\Delta V_k = \frac{1}{2}s_k^2 - \frac{1}{2}s_{k-1}^2 + \frac{1}{2}\tilde{\theta}_k^{T}\Gamma_k^{-1}\tilde{\theta}_k - \frac{1}{2}\tilde{\theta}_{k-1}^{T}\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}, \quad (22)$$

$$\Delta V_k = -\frac{1}{2}s_{k-1}^2 - \frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^{T}\Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1}) + \frac{1}{2}s_k^2 + \frac{1}{2}\tilde{\theta}_k^{T}(\Gamma_k^{-1} + \Gamma_{k-1}^{-1})\tilde{\theta}_k - \tilde{\theta}_k^{T}\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1}, \quad (23)$$

$$\Delta V_k = -\frac{1}{2}s_{k-1}^2 - \frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^{T}\Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1}) - \frac{1}{2}s_k^2 + s_k^2 + \frac{1}{2}\tilde{\theta}_k^{T}(\Gamma_k^{-1} - \Gamma_{k-1}^{-1})\tilde{\theta}_k + \tilde{\theta}_k^{T}\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1} - \tilde{\theta}_k^{T}\Gamma_{k-1}^{-1}\tilde{\theta}_{k-1} \quad (24)$$

From (19), s_k is:

$$s_k^2 = \tilde{\theta}_k^{T}\phi_{k-d} + \phi_{k-d}^{T}\tilde{\theta}_k. \quad (25)$$

Substituting (25) into (24), the following relation is derived:

$$\Delta V_k = -\frac{1}{2}s_{k-1}^2 - \frac{1}{2}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^{T}\Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1}) + \frac{1}{2}\tilde{\theta}_k^{T}(\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^{T})\tilde{\theta}_k + \tilde{\theta}_k^{T}\Gamma_{k-1}^{-1}(\theta_k - \theta_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^{T}\tilde{\theta}_k). \quad (26)$$

The term:

$$\frac{1}{2}\tilde{\theta}_k^{T}(\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^{T})\tilde{\theta}_k$$

in (24) can be made equal to zero as follows:

$$\Gamma_k^{-1} - \Gamma_{k-1}^{-1} - \phi_{k-d}\phi_{k-d}^{T} = 0,$$

$$\Gamma_k = (\Gamma_{k-1}^{-1} + \phi_{k-d}\phi_{k-d}^{T})^{-1},$$

$$\Gamma_k = \Gamma_{k-1} - \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^{T}(\Gamma_{k-1} + \phi_{k-d}\phi_{k-d}^{T})^{-1},$$

And this yields (12) by the matrix inversion lemma.

The term:

$$\tilde{\theta}_k^{T}\Gamma_{k-1}^{-1}(\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\phi_{k-d}\phi_{k-d}^{T}\tilde{\theta}_k)$$

in (26) also can be made equal to zero as described below:

$$\tilde{\theta}_k - \tilde{\theta}_{k-1} + \Gamma_{k-1}\emptyset_{k-d}\,\emptyset_{k-d}\,\tilde{\theta}_k = 0, \tag{27}$$

$$\tilde{\theta}_k + \Gamma_{k-1}\emptyset_{k-d}\,\emptyset^{\mathrm{T}}_{k-d}\tilde{\theta}_k = \tilde{\theta}_{k-1}, \tag{28}$$

$$(I + \Gamma_{k-1}\emptyset_{k-d}\emptyset^{\mathrm{T}}_{k-d})\tilde{\theta}_k = (I + \Gamma_{k-1}\emptyset_{k-d}\emptyset^{\mathrm{T}}_{k-d})\tilde{\theta}_{k-1} - \Gamma_{k-1}\emptyset_{k-d}\emptyset^{\mathrm{T}}_{k-d}\tilde{\theta}_{k-1}, \tag{29}$$

and using (18), then:

$$\tilde{\theta}_k = \tilde{\theta}_{k-1} + \frac{\Gamma_{k-1}\emptyset_{k-d}\emptyset^{\mathrm{T}}_{k-d}\,(\theta-\hat{\theta}_{k-1})}{(1 + \emptyset^{\mathrm{T}}_{k-d}\Gamma_{k-1}\emptyset_{k-d})} \tag{30}$$

From (8):

$$s_k = \emptyset^{\mathrm{T}}_{k-d}\,\theta - C(z^{-1})r_k - Hy_{k-d}\,u_{k-d} \tag{31}$$

thus (11) is derived.

Using the recursive equations (11) and (12) in (26), for $k = 1$, the following relation is obtained:

$$V_1 - V_0 = -\frac{1}{2}s_0^{\,2} - \frac{1}{2}(\tilde{\theta}_1 - \tilde{\theta}_0)^T\,\Gamma_0^{-1}(\tilde{\theta}_1 - \tilde{\theta}_0) \tag{32}$$

Initially $\tilde{\theta}_1 - \tilde{\theta}_0 \neq 0$, then $V_1 - V_0 < 0$ which means that $V_1<V_0$. For $k = 2$,

$$V_2 + \frac{1}{2}s_1^2 + \frac{1}{2}\,(\tilde{\theta}_2 - \tilde{\theta}_1)^T\,\Gamma_1^{-1}\,(\tilde{\theta}_2 - \tilde{\theta}_1) = V_1<V_0 \tag{33}$$

Then, for k=N, where N is large, the following relation is derived:

$$V_N + \frac{1}{2}\sum_{k=2}^{N}\Big[s^2_{k-1} + (\tilde{\theta}_k - \tilde{\theta}_{k-1})^T\,\Gamma^{-1}_{k-1}\,\,(\tilde{\theta}_k - \tilde{\theta}_{k-1})\Big] = V_1<V_0, \tag{34}$$

Equation (34) implies that s_N and $(\theta_N - \theta_{N-1})$ vanish as N approaches infinity, thus ΔV_k is negative semi-definite for all k and the generalized minimum variance is minimized, which proves the overall system stability.

Signal Boundedness: the actual signals y_k, u_k and e_k are shown to be bounded as follows, multiplying (2) by $B(z^{-1})$:

$$B(z^{-1})s_{k+d} = B(z^{-1})\,C(z^{-1})y_{k+d} - B(z^{-1})C(z^{-1})r_{k+d} + B(z^{-1})Q(z^{-1})u_k - B(z^{-1})H(z^{-1})y_k u_k\,, \tag{35}$$

$$B(z^{-1})s_k = B(z^{-1})C(z^{-1})y_k - B(z^{-1})C(z^{-1})r_k + z^{-d}\,B(z^{-1})Q(z^{-1})u_k - B(z^{-1})H(z^{-1})y_k u_k\,, \tag{36}$$

and using (1):

$$B(z^{-1})s_k = -z^{-d}\,B(z^{-1})E(z^{-1})N(z^{-1})y_k u_k - B(z^{-1})C(z^{-1})r_k - z^{-d}Q(z^{-1})N(z^{-1})y_k u_k + B(z^{-1})C(z^{-1})y_k + A(z^{-1})Q(z^{-1})y_k\,. \tag{37}$$

Then, from (8):

$$y_k = \frac{B(z^{-1})}{T(z^{-1})}s_k + \frac{B(z^{-1})C(z^{-1})}{T(z^{-1})}\,r_k + \frac{N(z^{-1})G(z^{-1})}{T(z^{-1})}\,y_{k-d}u_{k-d}\,, \tag{38}$$

where $T(z^{-1})$ is defined as in (10):

The signal s_k was proven to go to zero as $k\to\infty$. The signal r_k is assumed to be bounded for all k and the signal $y_{k-d}\,u_{k-d}$ was proven to be bounded from the boundeness of vector $\emptyset^T_k$. From the set of Assumptions 1, number 4 means that the closed-loop characteristic polynomial, considering the described plant with parametric uncertainties, in (1), $T(z^{-1})$, is Schur. Thus, y_k in closed-loop is proven to be bounded.

Similarly for u_k, multiplying (2) by $A(z^{-1})$, it is obtained that:

$$A(z^{-1})s_{k+d} = A(z^{-1})C(z^{-1})y_{k+d} + A(z^{-1})Q(z^{-1})u_k - A(z^{-1})C(z^{-1})r_{k+d} - A(z^{-1})H(z^{-1})y_k u_k\,, \tag{39}$$

and by using (1):

$$A(z^{-1})s_{k+d} = A(z^{-1})C(z^{-1})y_{k+d} - A(z^{-1})C(z^{-1})r_{k+d} + A(z^{-1})Q(z^{-1})u_k - A(z^{-1})H(z^{-1})y_k u_k \tag{40}$$

$$A(z^{-1})s_{k+d} = -A(z^{-1})E(z^{-1})N(z^{-1})y_k u_k + C(z^{-1})N(z^{-1})y_k u_k - A(z^{-1})C(z^{-1})r_{k+d} + A(z^{-1})Q(z^{-1})u_k + B(z^{-1})C(z^{-1})u_k\,. \tag{41}$$

Then, from (4):

$$u_k = \frac{A(z^{-1})}{T(z^{-1})}s_{k+d} + \frac{A(z^{-1})C(z^{-1})}{T(z^{-1})}\,r_{k+d} - \frac{N(z^{-1})\hat{F}(z^{-1})}{T(z^{-1})}y_{k-d}u_{k-d} \tag{42}$$

Thus, u_k is proven to be bounded. Using (2), when $k\to\infty$ and $s_{k-d}\to 0$, the error is derived:

$$e_k = \frac{E(z^{-1})N(z^{-1})}{C(z^{-1})}y_{k-d}u_{k-d} - \frac{Q(z^{-1})}{C(z^{-1})}\,u_{k-d}\,. \tag{43}$$

Because $C(z^{-1})$ was designed as a Schur polynomial, as $k\to\infty$ the signal e_k is bounded for all k. Especially when the signals u_{k-d} and $y_{k-d}u_{k-d}$ become constant, the the error converges to zero to zero, i.e. $e_k\to 0$.

SIMULATED EXAMPLE

In this section, a simulated example is presented to support the obtained theoretical results. A discrete-ti-

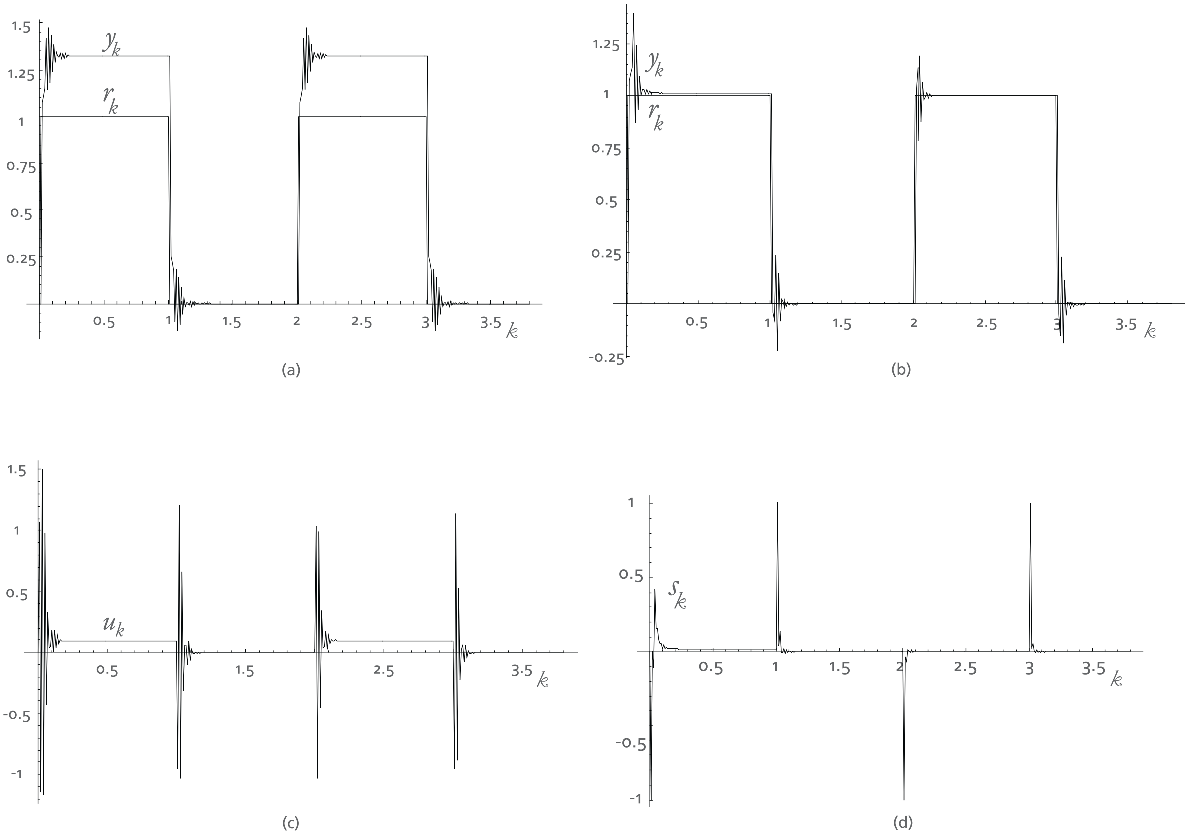

Figure 1. (a) Output response of (45), controlled by the nominal controller (9) designed for (44). (b) Output response of (45), controlled by the proposed self-tuning algorithm (11) and (12). (c) Control law dynamics when the self-tuning algorithm is used. (d) Sliding mode variable dynamics

me non-minimum phase plant with parametric uncertainties is considered. The plant model is known as:

$$(1-z^{-1} + 0.5z^{-2})y_k = z^{-1}(1 + 1.2z^{-1})u_k - z^{-1}0.7y_k u_k. \quad (44)$$

For the control design, the following polynomials are chosen: $C(z^{-1}) = 1+1.18z^{-1} + 0.0045z^{-2}$ and $Q(z^{-1}) = 0.1(1 - z^{-1})$.

Using (44), (4) and (8) the following polynomials for the nominal control law (9) are obtained: $F(z^{-1}) = 1.18 - 0.4955z^{-1}$ and $G(z^{-1}) = 1.1 + 1.1z^{-1}$.

Polynomials $F(z^{-1})$and $G(z^{-1})$ give the initial estimates $\hat{F}(z^{-1})$ and $\hat{G}(z^{-1})$ for the proposed self-tuning algorithm.

For the simulation, the perturbed plant is considered as:

$$(1 - z^{-1} + 0.2z^{-2})y_k = z^{-1}(1 + 1.1z^{-1})u_k - z^{-1}0.5y_k u_k. \quad (45)$$

Fig. 1 (a) shows the output response y_k of the system (45), when the nominal controller is used. In Fig. 1 (b) the proposed self-tuning algorithm is used, initial condition for Γ is set to the identity matrix. Fig. 1 (c) and Fig. 1 (d) show the control law u_k and the sliding mode variable s_k. The reference signal r_k is chosen as a sequence of unit-steps with a length of 100 samples.

The simulations show that the proposed self-tuning control algorithm is able to make the output signal follow the reference signal even though there are parametric uncertainties in the system.

CONCLUSIONS

This paper considered the self-tuning control of a class of bilinear systems with constant but unknown parameters. The analysis has been limited to single-input single-output systems. The overall stability of the proposed self-tuning control algorithm for a class of bilinear systems was proven. The validity of the proposed algorithm was demonstrated through a simulated example.

For future research, the structure of the system will be enhanced to a class of bilinear systems having the presence of a non homogeneous term.

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