

Methodological Proposal to Correct and Adjust a Geodesic Leveling Data

Propuesta Metodológica para Corregir y Ajustar un Circuito de Nivelación Geodésica

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Abstract

The objective of the work is to standardize the method by which the corrections are calculated and applied to make the adjustment to the geometric leveling data obtained in ground. For this purpose, was consulted related bibliography and two methodologies were obtained: one for the corrections, taking into account the National Oceanic and Atmospheric Administration (NOAA) technical norm of the United States, and one for the adjustment, based on the minimum method Squares correlated, exposed by the Cartographic Agency of the United States Department of Defense. The proposed methodology, generated by the geodesic leveling seedlings of the Geodesy Internal Geodesy Working Group of the Instituto Geográfico Agustín Codazzi (IGAC), was evaluated by its application to the ground data obtained in a leveling circuit located in the department of Cesar, Colombia, and satisfactory results were obtained.

Keywords: Altimetry, Leveling, Geodesy, Adjustment, Correction.

Resumen

El objetivo del trabajo fue estandarizar el método con el cual se calculan y se aplican las correcciones para realizar el ajuste a los datos de nivelación geométrica obtenidos en campo. Para ello se consultó bibliografía relacionada y se obtuvieron dos metodologías: una para las correcciones, teniendo en cuenta la norma técnica de la National Oceanic and Atmospheric Administration (NOAA) de los Estados Unidos, y una para el ajuste, fundamentada en el método de mínimos cuadrados correlativos expuesto por la Agencia Cartográfica del Departamento de Defensa de los Estados Unidos. La metodología propuesta, generada por el semillero de nivelación geodésica del Grupo Interno de Trabajo de Geodesia del Instituto Geográfico Agustín Codazzi (IGAC), fue evaluada mediante su aplicación a los datos de campo obtenidos en un circuito de nivelación localizado en el departamento del Cesar, Colombia, y se obtuvieron resultados satisfactorios.

Keywords: Altimetría, Nivelación, Geodesia, Ajuste, Corrección.

1. Introduction

With the objective/aim of contributing to the establishment for a vertical global reference system that unifies altimetric data from South American countries, initiative led by Sigas Workgroup III, arises the necessity to evaluate, identify and diagnose the state of each component for the vertical reference system at a local level. The components are: Leveling network, Densification of gravimetric network, Geoidal model and Generation of geopotential numbers.

Geodetic leveling as a basic component for the adjustment of vertical networks, must have a standard methodology for the treatment of leveling data, with respect to the correction of height differences obtained in the field and their corresponding geodetic adjust; given that the development of the local vertical reference system is one of the responsibilities of the Inner Workgroup in Geodesy at (IGAC).

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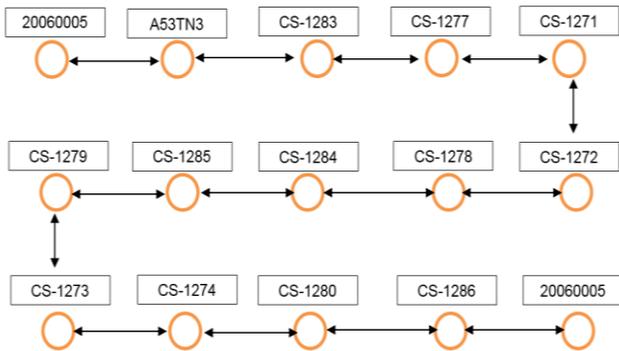


Figure 1 Leveling schematic.

The procedure for geodetic leveling has not presented considerable modifications in a long time, as to methodology of field surveying is referred, since geodetic equipment hasn't evolved significantly. However, data processing at the office has presented certain computing difficulties, due to the inexistence of a general method in the pertinent procedures for captured data. Hence, the determination of corrections and leveling data adjustment are proposed and explained step by step, and the correlative least-squares method, the structure of condition equations and the contents of the matrixes are presented.

With the result of geodetic adjustment the degree of precision and accuracy of field surveyed points is determined, such that one can determine the order of the geodetic network at which the points can be integrated, besides identify for which project types the national framework can be used, according to their specifications and/or accuracy. The article is structured in the following way: first we approach the procurement of the digital leveling field surveyed data; next, the determination and computation of corrections according to technical specs of the levels used. Then, we tackle the computation of adjustment with the correlative least-squares method for the corrected height differences; and last, we present and analyze the results obtained.

2. Materials and Measurement Methods

The field measurement method is realized through routines where the raw data is obtained with the digital levels. Leveling is executed in double path sections, round travels, where the length of the sections can be established between 0.8 and 1.5 kilometers, with first order closing errors [1]. The data provided by the Geographic Institute Agustín Codazzi belong to a leveling path of the urban area in Bosconia, Colombia, and follow the schematic presented in Figure 1. The field information is presented in Table 1.

Table 1 Leveling schematic.

Section	Direction	Distance (km)	Observed Height Difference (m)
A53TN3	I	0.26657	-0.04157
CS-1283	R	0.26657	0.04168
CS-1283	I	0.27972	4.19054
CS-1277	R	0.27972	-4.19116
CS-1277	I	0.22912	0.49222
CS-1271	R	0.22912	-0.49214
CS-1271	I	0.2334	0.68002
CS-1272	R	0.2334	-0.68058
CS-1272	I	0.47568	-2.78881
CS-1278	R	0.47568	2.78857
CS-1278	I	0.25306	-0.95892
CS-1284	R	0.25306	0.95921
CS-1284	I	0.39857	0.48059
CS-1285	R	0.39857	-0.48069
CS-1285	I	0.33683	1.55059
CS-1279	R	0.33683	-1.55097
CS-1279	I	0.20997	0.43499
CS-1273	R	0.20997	-0.43462
CS-1273	I	0.35439	1.44825
CS-1274	R	0.35439	-1.44783
CS-1274	I	0.42018	-1.49098
CS-1280	R	0.42018	1.49119
CS-1280	I	0.4197	-1.80528
CS-1286	R	0.4197	1.80463
CS-1286	I	0.44963	-0.17688
20060005	R	0.44963	0.17721
20060005	I	1.03887	-2.01811
A53TN3	R	1.03887	2.01677

3. Corrections to leveling data

Considering that no field observation is exempt of measurement errors, either generated by natural conditions or by observer's flaws, among other systematic errors that can affect the process, we have identified such aspects that lead to an inaccurate value in the leveling data. Six factors have been identified that can have an influence in the measurement true value [2]:

- Calibration scale in the level staff.
- Temperature of the level staff
- Atmospheric refraction
- Inexistence of horizontality in the leveling instrument line

- Effects of Moon and Sun on the Earth's equipotential surfaces
- Non-parallelism of equipotential surfaces

Here, we briefly describe each one of the corrections.

3.1. Correction to the Level Staff scale

The objective of this correction is to verify if the length of the level staff scale is within the guidelines of the standard longitudinal norm. In general, such correction is given by the manufacturer though a calibration, however, if this information is not available, the correction process is resumed in a comparison of the length measurements in the level staff scale with respect to a standard meter. The correction of the level staff scale is calculated with equation 1 [3], and is added with the resulting algebraic sign to the observed height difference:

$$C_r = De \quad (1)$$

where:

- C_r = Correction to the level staff scale, in millimeters.
- D = Observed height difference for the section, in meters
- e = Excess of average length of the level staff pair, in mm/m.

3.2 Temperature Correction

The temperature correction of the level staff is applied to the height difference between reference points using the mean of observed temperatures at the beginning and end of a section. This is calculated with equation 2 [3] and is added with the resulting algebraic sign of the observed height difference in the field:

$$C_t = (tm - ts) D CE \quad (2)$$

Where:

- C_t = Temperature correction of the level staff.
- tm = Average temperature observed in the level staff.
- ts = Standard temperature in the level staff.
- D = Observed height difference between reference points.
- CE = Mean thermal expansion coefficient per unit length, per temperature degree in the level staff pair.

With respect to these variables, it should be noted that:

- The average temperature is determined by the average between the temperature measurements at the beginning and the end of the section (section is the interval between materialized adjacent points)
- The units of D , tm and ts must coincide with the units of CE [2].

3.3 Refraction Correction

The atmospheric refraction phenomenon is presented when "the ray incoming from the target point does not follow a rectilinear trajectory but suffers successive refractions when travelling through a variable density atmosphere" [4]. While the refraction phenomenon varies depending on the conditions in the data acquisition instant, in case one can assume that conditions such as temperature and atmospheric pressure are homogenous along the section, this correction can be omitted.

To minimize the effect of refraction, equation 3 is suggested. It is noted that this is a reconsideration of the simplified version of the model developed by Professor T.J. Kukkamäki, of the Finnish Geodetic Institute [5] [6].

$$R = -(10^5) * \gamma(S/50)^2 * \delta(D/2) \quad (3)$$

Where:

- R = Refraction correction for the section, in millimeters.
- S = Section length, in meters.
- δ = Temperature difference between temperatures at 1,3m and 0,3m above the ground, in degrees Celsius.
- D = Height differences, in cm.

3.4 Collimation Correction

This correction comes to be the simplest of all, given that it refers to the coincidence in measurements in back-view and forward-view for a point must be exact; when the comparison between these two views differs from zero, this value is known as collimation error. This correction is added with the resulting algebraic sign of the observed height difference with equation 4 [2].

$$C_c = -(e SDS) \quad (4)$$

where:

- C_c = Level collimation correction, in millimeters
- e = Collimation error, in radians x 1,000 or in mm/m
- SDS = Accumulated differences of the longitudinal views for the section, in meters (forward-view and back-view).

3.5 Astronomical Correction

"Astronomical correction is applied to consider the effect of tide acceleration due to the Earth's equipotential surfaces on which the Sun and Moon are located" [2]. The main characteristic of this correction is that is suggested for leveling networks of great density, due that its incidence in leveling data for local networks is insignificant (this affirmation is linked to the context of the desired precision in the leveling network being developed).

One of Several Corrections Applied to Precise Leveling

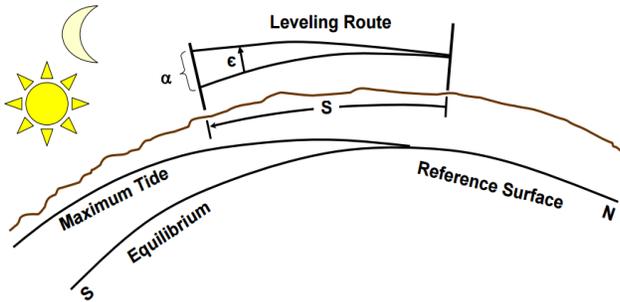


Figure 2 Effect (α) and tide deflection (ϵ) in a section of length and direction $s \rightarrow$ Source: [7].

The calculation is determined by equations 5 and 6 [7]:

$$C_a = 0.7K S \quad (5)$$

$$K = \tan \epsilon_m \cos(A_m - \alpha) + \tan \epsilon_s \cos(A_s - \alpha) \quad (6)$$

where:

- C_a = Astronomic correction
- S = Section length
- ϵ_m = Deflection due to the Moon
- ϵ_s = Deflection due to the Sun
- A_s = Azimut for the Sun
- A_m = Azimut for the Moon
- α = Azimuth for the section

The 0.7 is a constant based on Earth's elasticity [8].

3.6 Orthometric Correction

Considering that each height difference taken in the field is subject to a level surface related to topography and not to a constant physical surface, orthometric correction compensates the lack of parallelism of the geometric level surfaces, taking into account the range of latitude in the leveling line, direction of the line, latitude differences and average line height [9]. Equation 7 calculates approximate orthometric correction (based on normal gravity) for the observed height difference in a section [2].

$$C_0 = -2h\alpha \sin(2\rho) * [1 + (\alpha - \frac{2b}{\alpha}) * \cos(2\rho)] * \sin(d\rho) \quad (7)$$

where:

- C_0 = Orthometric correction

h = Average height for the section

$\alpha = 0.002644$

$b = 0.000007$

ρ = Average latitudes for the section

$d\rho$ = latitude difference between initial and final points of the section ($d\rho$ is positive when the final point of the section is located northwards of the initial point; this considering the direction of the leveling).

In the case of having real values for the gravity in each leveling point (not normal gravity), the recently proposed orthometric correction must be replaced by the formulation of geopotential numbers, which will determine a more accurate orthometric bound.

As to the orthometric bound is referred, is estimated that it can vary from very few millimeters to some decimeters with respect to the geometric bound, since its value depend not only of the ground topography, but also of the material lying underground [10]. After correcting the field leveling values, the next step is to adjust the leveling network in aim to compensate the closing error.

3.7 Practical Case

The corrections previously mentioned are all necessary in the leveling process. However, according to the type of equipment and the field survey methodology managed by the Geographical Institute Agustín Codazzi, one must consider the following annotations:

Correction of the level staff scale must not be conducted in height differences, since the staff's material is Invar and satisfy the standard calibration scale guaranteed by the manufacturer [1].

Refraction correction is programmed in the digital level, which automatically computes the correction as follows [11]:

$$k_2 = (r_k E^2) / (2R) \quad (8)$$

where:

- r_k = Refraction coefficient of the equipment.
- E = Distance of leveling change
- R = Earth's radius (6380000 m).

Collimation correction or collimation error is corrected in field by the appointed commission, employing the Förstner method, as specified in the procedure manual [1].

Astronomical correction must not be conducted in height differences, since the leveling circuit is too small, meaning that the survey is local, and its application is not required.

Table 2 Temperature Correction.

Section	Direction*	Measured Temperature	Standard Temperature	Observed Height Difference (m)	Temperature Correction (m)
A53TN3	I		25	-0.04157	-0.0000015
CS-1283	R	32	25	0.04168	0.0000026
CS-1283	I	29	25	4.19054	0.0001509
CS-1277	R	32	25	-4.19116	-0.0002640
CS-1277	I	31	25	0.49222	0.0000266
CS-1271	R	32	25	-0.49214	-0.0000310
CS-1271	I	32	25	0.68002	0.0000428
CS-1272	R	32	25	-0.68058	-0.0000429
CS-1272	I	34	25	-2.78881	-0.0002259
CS-1278	R		25		
CS-1278	I	35	25	-0.95892	-0.0000863
CS-1284	R	32	25	0.95921	0.0000604
CS-1284	I	36	25	0.48059	0.0000476
CS-1285	R	32	25	-0.48069	-0.0000303
CS-1285	I	37	25	1.55059	0.0001675
CS-1279	R	32	25	-1.55097	-0.0000977
CS-1279	I	38	25	0.43499	0.0000509
CS-1273	R	32	25	-0.43462	-0.0000274
CS-1273	I	39	25	1.44825	0.0001825
CS-1274	R	32	25	-1.44783	-0.0000912
CS-1274	I	40	25	-1.49098	-0.0002013
CS-1280	R	32	25	1.49119	0.0000939
CS-1280	I	42	25	-1.80528	-0.0002762
CS-1286	R	32	25	1.80463	0.0001137
CS-1286	I	43	25	-0.17688	-0.0000287
20060005	R	32	25	0.17721	0.0000112
20060005	I	28	25	-2.01811	-0.0000545
53TN3	R	32	25	2.01677	0.0001271

- Orthometric correction must be performed for each height difference obtained in the field. Initial data available in table 3 and table 4.

Temperature correction must be performed for each of the obtained height differences in the field. For example, for the first height difference, the value of thermal expansion in the Invar level staffs employed at IGAC is 0.000009 [12], the standard temperature of the staff is 25°C [13]. Then, the temperature correction is obtained as follows:

$$C_t = (32 - 25) - 0.04157 * 0,000009$$

$$C_t = -0.0000015$$

From this calculation the temperature correction is obtained for each height difference, as presented on Table 2. As an example for the height difference that corresponds to the section between A53-TN-3 and 20060005, with the following data:

Latitude of 20060005 is 9.9757, latitude of A53-TN-3 is 9.9830, with a latitude average of 9.9794. The average height of the section is equal to 1.83856 m, α is equal to 0.002644 and β is equal to 0.000007, latitude difference is 0.0073278. Then, the orthometric correction is equal to:

$$C_o = -2*(1.83856*0.002644)*(sin (2*9.9794))* [1+(0.002644-((2*0.000007) / 0.002644))* (cos (2*9.9794))]*(sin (0.0073278))$$

Therefore,

$$C_o = 0.0000004.$$

From this calculation, orthometric correction is obtained as shown on Table 5

Table 3 Height average per section.

ID	Section	Section's Height Average
1	A53TN3 a CS-1283	0.28798
2	CS-1283 a A53TN3	-0.07174
3	CS-1283 a CS-1277	2.7838
4	CS-1277 a CS-1283	-2.9853
5	CS-1277 a CS-1271	-0.92077
6	CS-1271 a CS-1277	-1.7466
7	CS-1271 a CS-1272	0.44898
8	CS-1272 a CS-1271	-0.56947
9	CS-1272 a CS-1278	-2.22194
10	CS-1278 a CS-1272	0.84496
11	CS-1278 a CS-1284	-0.53011
12	CS-1284 a CS-1278	0.73779
13	CS-1284 a CS-1285	0.54373
14	CS-1285 a CS-1284	0.00952
15	CS-1285 a CS-1279	0.44087
16	CS-1279 a CS-1285	-1.39955
17	CS-1279 a CS-1273	-0.14682
18	CS-1273 a CS-1279	-0.93832
19	CS-1273 a CS-1274	1.13997
20	CS-1274 a CS-1273	-0.71349
21	CS-1274 a CS-1280	-0.58609
22	CS-1280 a CS-1274	1.1205
23	CS-1280 a CS-1286	-1.15171
24	CS-1286 a CS-1280	1.29662
25	CS-1286 a 20060005	-0.14926
26	20060005 a CS-1286	0.16224
27	20060005 a A53TN3	-0.50717
28	A53TN3 a 20060005	1.83856

Table 4 Latitude and Longitude for the vertices.

Point	Latitude	Longitude
A53-TN-3	9.9830	-73.8928
CS-1271	9.9835	-73.8873
CS-1272	9.9995	-73.8857
CS-1273	9.9797	-73.8838
CS-1274	9.9778	-73.8814
CS-1277	9.9835	-73.8893
CS-1278	9.9810	-73.8878
CS-1279	9.9785	-73.8853
CS-1280	9.9764	-73.8837
CS-1283	9.9821	-73.8913
CS-1284	9.9798	-73.8897
CS-1285	9.9768	-73.8878
CS-1286	9.9745	-73.8862
CS-1275	9.9763	-73.8804
CS-1276	9.9746	-73.8790
CS-1281	9.9749	-73.8823
CS-1282	9.9727	-73.8809
20060005	9.9757	-73.888

4. Network adjustment

For the elimination of closing errors according to the corresponding mathematical requirements, the principle of least-squares is applied. This principle is based on searching that the sum of the squared residues from a set of observations is minimal through the most probable value (MPV) for a quantity obtained through repeated observations of equal weight [14]. To find this value that minimizes the sum of the squared residues, first one should identify:

- The values of the known vertices or bounds (in case the circuit is closed; is not necessary)
- The unknowns or height differences (dh) to be adjusted
- The bounds to be corrected as product of the height differences adjustment
- The distances between sections
- The direction of the circuit or leveling line

As a basis, we should provide a main system of equations or *general equations*. These equations will define the standard or general relationships, will depend on the context of the network and they will be the basis of comparison and a-posteriori correction of observation errors.

$$BX = \left\{ \begin{array}{l} b_{11}x_1 + bx_2 + \dots + b_{1n}x_n = h_1 \\ b_{21}x_1 + bx_2 + \dots + b_{2n}x_n = h_2 \\ \dots \quad \dots \quad \dots \quad \dots \\ b_{m1}x_1 + b_{m2}x_2 + \dots + b_{mn}x_n = h_m \end{array} \right\} \quad (9)$$

Now, for the practical case we add the height differences to a bound, which makes the final value equal to that of the considered bound, both in the forward and backward direction. Additionally, height differences in forward and backward directions for a section should add up to zero.

$$BX = \left\{ \begin{array}{l} \text{Bound} + dh_1 + dh_3 + \dots + dh_{27} = \text{Bound} \\ \text{Bound} + dh_2 + dh_4 + \dots + dh_{28} = \text{Bound} \\ dh_1 + dh_2 = 0 \\ dh_3 + dh_4 = 0 \\ \dots \\ dh_{27} + dh_{28} = 0 \end{array} \right\} \quad (10)$$

Table 5 Orthometric Correction.

Id	Section's Height Average	α	β	Section's Latitude Average	Latitude Difference	Orthometric Correction
1	0.28798	0.002644	0.000007	9.982609	0.0008813	-0.00000001
2	-0.07174	0.002644	0.000007	9.982609	-0.0008813	0.00000000
3	27.838	0.002644	0.000007	9.982863	-0.0013884	0.00000010
4	-29.853	0.002644	0.000007	9.982863	0.0013884	0.00000010
5	-0.92077	0.002644	0.000007	9.983539	0.0000366	0.00000000
6	-174.660	0.002644	0.000007	9.983539	-0.0000366	0.00000000
7	0.44898	0.002644	0.000007	9.991527	-0.0160124	0.00000020
8	-0.56947	0.002644	0.000007	9.991527	0.0160124	0.00000030
9	-222.194	0.002644	0.000007	9.990298	0.0184700	0.00000130
10	0.84496	0.002644	0.000007	9.990298	-0.0184700	0.00000050
11	-0.53011	0.002644	0.000007	9.980467	0.0011923	0.00000002
12	0.73779	0.002644	0.000007	9.980467	-0.0011923	0.00000003
13	0.54373	0.002644	0.000007	9.978346	0.0030492	-0.00000010
14	0.00952	0.002644	0.000007	9.978346	-0.0030492	0.00000000
15	0.44087	0.002644	0.000007	9.977697	-0.0017522	0.00000002
16	-139.955	0.002644	0.000007	9.977697	0.0017522	0.00000010
17	-0.14682	0.002644	0.000007	9.979161	-0.0011754	-0.00000001
18	-0.93832	0.002644	0.000007	9.979161	0.0011754	0.00000003
19	113.997	0.002644	0.000007	9.978805	0.0018881	-0.00000010
20	-0.71349	0.002644	0.000007	9.978805	-0.0018881	-0.00000004
21	-0.58609	0.002644	0.000007	9.977174	0.0013740	0.00000003
22	112.05	0.002644	0.000007	9.977174	-0.0013740	0.00000005
23	-115.171	0.002644	0.000007	9.975532	0.0019100	0.00000010
24	129.662	0.002644	0.000007	9.975532	-0.0019100	0.00000010
25	-0.14926	0.002644	0.000007	9.975149	-0.0011454	-0.00000001
26	0.16224	0.002644	0.000007	9.975149	0.0011454	-0.00000001
27	183.856	0.002644	0.000007	9.979386	0.0073278	-0.00000040
28	-0.50717	0.002644	0.000007	9.979386	-0.0073278	-0.00000010

For the practical case one should also provide a system of observation equations that will have the unknowns as a function of observations, with the following characteristics:

- The number of observations should be greater or equal than the number of unknowns
- Equations should hold no mathematical relation between them (should be linearly independent)
- All measured magnitudes should be expressed through the selected equations
- The number of observation equations is given by $n - k$:
 - n = Number of measured observations

- k = number of unknowns

$$AX \left\{ \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = k_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = k_2 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = k_m \end{array} \right\} (11)$$

The coefficients in the condition equations (Table 6) will define the first matrix, Matrix A, which is a $m \times n$ matrix (m = number of equations, n = number of observations). Roman numerals point to the employed equations.

To specify matrix notation of the Gauss normal equation system, we will design the transpose matrix of Matrix A as AT, and inverse matrix of matrix P as P-1. The fourth matrix is matrix N, square matrix and symmetric with respect to the main quadratic diagonal that contains the coefficients of the normal equations. This matrix is of size m x m (m = number of equations).

$$N = A * P^{-1} * A^T = \begin{matrix} \mathbf{N}_{11} & \mathbf{N}_{12} & \dots & \mathbf{N}_{1m} \\ \mathbf{N}_{12} & \mathbf{N}_{22} & \mathbf{N}_{21} & \mathbf{N}_{2m} \\ \dots & \mathbf{N}_{21} & \dots & \dots \\ \mathbf{N}_{1m} & \mathbf{N}_{2m} & \dots & \mathbf{N}_{mm} \end{matrix} \quad (\text{IV})$$

The next step is the computation of the correlative factors, auxiliary factors that will be introduced in the condition equations, looking to find the minimum for the sum of the squared remainders using the LaGrange method. This methodology is known as compensation by correlative method. The values of these factors shall define the fifth matrix, matrix K, of size m x 1 (m = number of equations).

$$K = N * W = \begin{matrix} K_1 \\ K_2 \\ \dots \\ K_m \end{matrix} \quad (\text{V})$$

Last, the minimum quadratic solution of the observations X is obtained from solving the correlative equation system, and from these equalities one can find the desired corrections Vi. Obtaining such corrections Vi over the observed values allows to eliminate the closing errors W in the equations.

The relationship between the correlative values and the observations matrix, with their respective weights is given by the sixth matrix, matrix V, of size n x 1 (n = number of observations).

$$V = P^{-1} * A * K = \begin{matrix} V_1 \\ V_2 \\ \dots \\ V_n \end{matrix} \quad (\text{VI})$$

So, the compensated values for the measured observations are computed using equation 15:

$$X = X_i + V_i \quad (16)$$

where:

X = Adjusted or compensated value
 X_i = Observation value
 V_i = Correction

5. Errors

With the purpose of evaluating the degree of functionality for the cited adjustment, the different errors are defined below.

5.1 Observation Error

The definition starts from the concept of variance. Considering that the square root of the variance of an observation represents its standard deviation or quadratic mean error, equation 16 is cited, since it's the equation that better defines said quadratic mean error [15].

$$s = e_{mc} = \pm \sqrt{\frac{\sum_{i=1}^m e_i^2}{m}} \quad (17)$$

In order to express the quadratic mean error as a function of the compensations, based in the equation $P_{-1} W_{-1} + P_{-2} W_{-2} + \dots + P_{-n} W_{-n} = \sum P_{-i} W_{-i}$, one can reach the following inequality [15]:

$$\sqrt{\frac{\sum_{i=1}^m e_i^2}{m}} \geq \sqrt{\frac{\sum_{i=1}^m v_i^2}{m}} \quad (18)$$

Following an intermediate mathematical development is concluded that "the mean error for an isolated observation in a set of m observation relations with a lower number of unknowns n is obtained through the following expression" [16].

$$S = e_{mc} = \pm \sqrt{\frac{\sum_{i=1}^m v_i^2}{m-n}} \quad (19)$$

The sum of the squared compensations within the radical can be rewritten as follows:

$$\sum_{i=1}^m v_i^2 = V^t * V \quad (20)$$

Now, considering that the compensations made are weighted, the quadratic mean error of the observation is defined as:

$$S = e_{mc} = \pm \sqrt{\frac{V^t * P * V}{m-n}} \quad (21)$$

With the purpose of weighting the mean error for each of the observations, we use the following equation

$$m_i = \frac{S}{\sqrt{P_i}} \quad (22)$$

Here mi is the mean error of the original observations.



Figure 3 Leveling line

5.2 Adjusted Value Error

Given the intervention of field observations for the computation of adjusted or compensated values, these results are also characterized by a mean error. The errors in the compensated bounds are in function of the initial measurement errors, which is known as propagation of error

For the case of altimetric leveling, a propagation of error is generated, since the initial field measurement error is intrinsic in the compensated bound error. Given that the function that relates compensated height with field observation (height difference) is linear, the propagation of error is linear in a circuit or leveling line (Figure 3).

$$H_B = H_A + dh_{AB} \quad (23)$$

Here:

H_B =Bound for B

H_A =Bound for A

dh_{AB} =Height difference from A to B

6. Precision of determined quantities

Though adjustment, the unknowns are determined indirectly, i.e., they are determined from indirect observations related to the expected unknowns through linear functions, such as observation equations; then, the standard deviation for the unit weight is expressed as [15]:

$$S_0 = \sqrt{\frac{\sum pv^2}{m-n}} \quad (24)$$

Here m is the number of observations, n is the number of unknowns. Thus, $m - n$ are the degrees of freedom.

The precision of height differences is determined from the covariance matrix formed from the coefficients of the unknowns in the observation equations.

This is used to calculate the standard deviations of the adjusted values that are indirectly determined and are functions of observed values; the covariance matrix is equal to the inverse matrix of the normal equation coefficient matrix multiplied by the unit variation

$$S_x^2 = S_0^2 (A^T P A)^{-1} = S_0^2 Q \quad (25)$$

The calculated standard deviation S_i , for any given unknown, having been computed from an observation equation system, can be expressed as

$$S_{x_i} = S_0 \sqrt{Q_{x_i x_j}} \quad (26)$$

where Q_{ij} is the diagonal element (row i , column j) from the covariance matrix [15].

7. Results and discussion

The proportionality of corrections inference in height differences is presented in figure 4. Orthometric corrections were less considerable in all height differences, with respect to the temperature correction, whence is concluded that the leveling zone can present a constant gravity model.

However, as previously mentioned, the way of precisely stating this affirmation is through geopotential numbers. Thereby, the respective corrections for the circuit data obtained in field by the Geographic Institute Agustín Codazzi are summarized in Table 7.

Regarding the least-squares adjustment, the calculated mean squared error was 0.05478 centimeters. The observation equations were selected with the criteria of conditioning the adjustment towards a bound closing, both in forward and backward directions.

Also, the remaining equations are related taking as a rule that both the forward and backward direction height differences must be equal, thus their sum should be equal to zero. Obtained results are presented in Table 8, where:

n = Height difference.

dh = Height difference Initial value (without adjustment corrections).

V_i (m) = Obtained corrections.

d_{hii} (m) = Corrected height difference value.

P_i = Weight for each height difference.

m_i (m) = Mean error for original observations. See equation 21.

Q_{xx} = Correlation factor.

S_x (m) = Mean error for adjusted observations. See equation 25

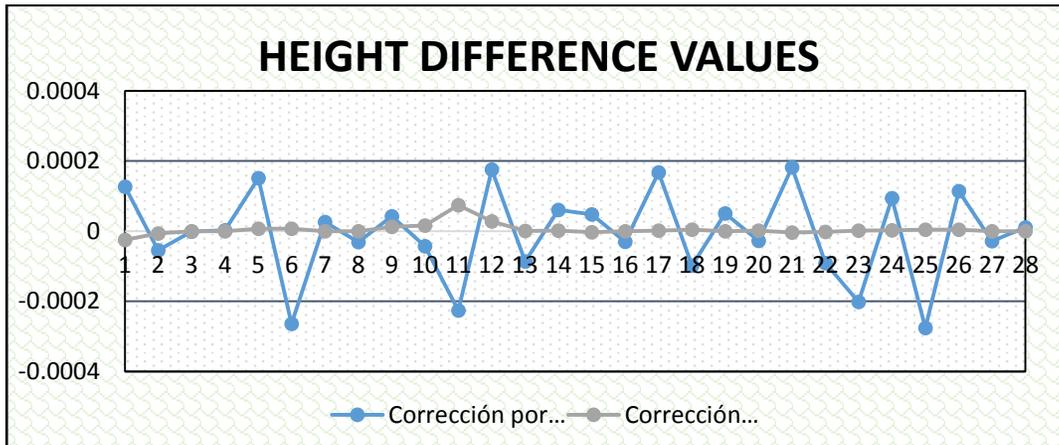


Figure 4 Value of Correction by height difference.

Table 8 Adjustment results. See text for key.

n	dh _i (m)	V _i (m)	dh _{ii} (m)	P _i	m _i (Cm)	Q _{xx}	S _x (Cm)
dh1	-0,0416	-0,00009	-0,0417	1,9368	0,0394	0,24238	0,02697
dh2	0,0417	0,00020	0,0419	1,9368	0,0394	0,24238	0,02697
dh3	4,1905	-0,00052	4,1900	1,8908	0,0398	0,24789	0,02727
dh4	-4,1912	-0,00022	-4,1914	1,8908	0,0398	0,24789	0,02727
dh5	0,4922	-0,00014	0,4921	2,0891	0,0379	0,36190	0,03295
dh6	-0,4921	0,00014	-0,4920	2,0891	0,0379	0,36190	0,03295
dh7	0,6800	-0,00042	0,6796	2,0699	0,0381	0,22775	0,02614
dh8	-0,6806	-0,00014	-0,6807	2,0699	0,0381	0,22775	0,02614
dh9	-2,7888	-0,00034	-2,7892	1,4499	0,0455	0,31670	0,03083
dh10	2,7886	0,00005	2,7886	1,4499	0,0455	0,31670	0,03083
dh11	-0,9589	-0,00001	-0,9589	1,9879	0,0389	0,23655	0,02664
dh12	0,9592	0,00028	0,9595	1,9879	0,0389	0,23655	0,02664
dh13	0,4806	-0,00018	0,4804	1,5840	0,0435	0,42820	0,03585
dh14	-0,4807	0,00018	-0,4805	1,5840	0,0435	0,42820	0,03585
dh15	1,5506	-0,00032	1,5503	1,7230	0,0417	0,27026	0,02848
dh16	-1,5510	0,00001	-1,5510	1,7230	0,0417	0,27026	0,02848
dh17	0,4350	0,00007	0,4351	2,1823	0,0371	0,21669	0,02550
dh18	-0,4346	0,00033	-0,4343	2,1823	0,0371	0,21669	0,02550
dh19	1,4483	0,00009	1,4483	1,6798	0,0423	0,27669	0,02881
dh20	-1,4478	0,00043	-1,4474	1,6798	0,0423	0,27669	0,02881
dh21	-1,4910	-0,00013	-1,4911	1,5427	0,0441	0,29925	0,02997
dh22	1,4912	0,00024	1,4914	1,5427	0,0441	0,29925	0,02997
dh23	-1,8053	-0,00059	-1,8059	1,5436	0,0441	0,29909	0,02996
dh24	1,8046	-0,00022	1,8044	1,5436	0,0441	0,29909	0,02996
dh25	-0,1769	-0,00004	-0,1769	1,4913	0,0449	0,30867	0,03043
dh26	0,1772	0,00035	0,1776	1,4913	0,0449	0,30867	0,03043
dh27	-2,0181	-0,00092	-2,0190	0,9811	0,0553	0,44816	0,03667
dh28	2,0168	-0,00034	2,0164	0,9811	0,0553	0,44816	0,03667

Table 9 Adjustment Bounds.

Section	Adjusted Levelled Bound	S _x Cm
CS-1283	80.729	0.0270
CS-1277	84.920	0.0273
CS-1271	85.412	0.0330
CS-1272	86.092	0.0261
CS-1278	83.303	0.0308
CS-1284	82.344	0.0266
CS-1285	82.824	0.0358
CS-1279	84.375	0.0285
CS-1273	84.810	0.0255
CS-1274	86.257	0.0288
CS-1280	84.766	0.0300
CS-1286	82.961	0.0300
20060005	82.784	0.0304

8. Conclusions

Corrections made to the obtained height differences in field did not exceed 0.1 mm on average, but in some sections the corrections come to 0.03 centimeters. Although the corrections do not modify considerable the height differences, these should not be disregarded, since geodetic leveling is based on high accuracy precisions, whereby a variation comparable to a centimeter is considerable for height differences that usually don't exceed 1 meter.

Additionally, depending on the extension or location of the survey, physical factors documented in the corrections (temperature, height, latitude, etc.) may vary in greater extent, hence the corrections can be greater.

Accuracy of adjustment for the leveling circuit is reflected in the quadratic mean error equivalent to 0.06 centimeters. This error represents an accuracy confidence around half millimeter, which is remarkable considering the accuracy required in geodesy.

The reason that led to the high accuracy of the adjustment was, at first an optimal definition of the initial observation equations in which the basic principles for the practical leveling case were considered.

The precision range for the obtained adjusted heights varies between 0.0255 and 0.0358 centimeters, which represents a standard deviation around half a millimeter, assuring highly precise leveled height values.

Obtaining high precision values for each measurement enables the use of geodetic information separately, i.e. after the adjustment each height difference can be used in different projects depending their requirement of precision and accuracy; this provides different companies and entities more accurate and independent altimetric values.

For the Geographic institute Agustín Codazzi the determination of a local vertical reference system has become a very important aim, due to the desire of contributing to the establishment of the global vertical reference system. The computation of geopotential numbers and normal heights is a first approximation to the obtention of such systems, and since adjusted geometric height differences are one of the main inputs for said computation, this article supplies the need of standardizing leveling data treatment in the office.

References

- [1] J. Baeyens, Q. Kang, L. Appels, R. Dewil, Y. Lv, and T. Tan, "Challenges and opportunities in improving the production of bio-ethanol," *Prog. Energy Combust. Sci.*, vol. 47, pp. 60–88, 2015.
- [2] A. Gupta and J. P. Verma, "Sustainable bio-ethanol production from agro-residues: A review," *Renew. Sustain. Energy Rev.*, vol. 41, pp. 550–567, 2015.
- [3] M. Petitfrere and D. Vladimir, "Multiphase equilibrium calculations using a reduction method," *Fluid Phase Equilib.*, vol. 401, pp. 110–126, 2015.
- [4] A. F. Cristino, S. Rosa, P. Morgado, A. Galindo, E. J. M. Filipe, A. M. F. Palavra, and C. A. Nieto de Castro, "High-temperature vapour-liquid equilibrium for the water-alcohol systems and modeling with SAFT-VR: 1. Water-ethanol," *Fluid Phase Equilib.*, vol. 341, pp. 48–53, 2013.
- [5] E. C. Voutsas, C. Pamouktsis, D. Argyris, and G. D. Pappa, "Measurements and thermodynamic modeling of the ethanol-water system with emphasis to the azeotropic region," *Fluid Phase Equilib.*, vol. 308, no. 1–2, pp. 135–141, 2011.
- [6] A. Farajnezhad, O. A. Afshar, M. A. Khansary, S. Shirazian, and M. Ghadiri, "Correlation of interaction parameters in Wilson, NRTL and UNIQUAC models using theoretical methods," *Fluid Phase Equilib.*, vol. 417, pp. 181–186, 2016.
- [7] L. Axelsson, M. Franzén, M. Ostwald, G. Berndes, G. Lakshmi, and N. H. Ravindranath, "Perspective: Jatropha cultivation in southern India: Assessing farmers' experiences," *Biofuels, Bioprod. Biorefining*, vol. 6, no. 3, pp. 246–256, 2012.
- [8] Y. Dadmohammadi, S. Gebreyohannes, B. J. Neely, and K. A. M. Gasem, "Multicomponent phase behavior predictions using

- QSPR-generalized NRTL and UNIQUAC models,” *Fluid Phase Equilib.*, vol. 409, pp. 318–326, 2016.
- [9] J. A. Sandarasi, A. J. Kidnay, and V. F. Yesavage, “Compilation of parameters for a polar fluid Soave-Redlich-Kwong equation of state,” *Ind. Eng. Chem. Process Des. Dev.*, vol. 25, no. 4, pp. 957–963, 1986.
- [10] J. M. Smith, H. Van Ness, and M. M. Abbott, *Introduction to Chemical Engineering Thermodynamics*, 7th ed. 2005.
- [11] M. Llano-Restrepo and J. I. Carrero-Mantilla, “Futility or usefulness of common implementations of the area and slope consistency tests for partial molar properties in binary mixtures,” *Fluid Phase Equilib.*, vol. 398, pp. 72–79, 2015.
- [12] B. Daniel, M. Peters, D. Glasser, and D. Hildebrandt, *Understanding Distillation Using Column Profile Maps*, 1st ed. 2013.
- [13] Dortmund-Data-Bank, “Dortmund Data Bank,” 2016. [Online]. Available: <http://www.ddbst.com/ddb.html>.
- [14] Chemicals Enigeer’s Guide, “Chemicals Enigeer’s Guide,” 2016. [Online]. Available: <http://cheguide.com/>. [Accessed: 01-Jun-2016].