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# Optimization Model for Collective Energy Demand Management in Smart Homes

## Modelo de optimización para la gestión colectiva de la demanda de energía en hogares inteligentes

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## Abstract

Power systems are evolving towards smart grids to improve their efficiency and reliability through demand response and management strategies. This study presents the Multi-User Model of Controllable Electric Loads (MUMCEL), an optimization model developed to collectively manage the residential demand of multiple users, through Controllable Electric Load Scheduling (CELS). The objective of the model was to minimize the cost of energy and achieve a more uniform distribution of the electric load, taking into account dynamic pricing rates and specific constraints. The methodology was based on classical optimization techniques in two stages. The first stage focused on the single user level using the exhaustive search method to select solutions that minimize the cost of each user's bill. The second stage employed the local search method for multi-user optimization to find a flatter demand curve. For this purpose, an algorithm was designed in MATLAB® that simulated a scenario with 60 users for 24 hours, scheduling the most appropriate on/off periods of controllable loads. Two scenarios were compared: one where users manage their loads at their convenience and the other where the proposed model was applied. The results indicated a decrease in peak demand, with an average savings of 4.94 % on the electricity bill for all users and up to 12.34 % individually. The simulation achieved this optimal solution in 25 minutes, despite the computational complexity involved in managing the demand of 60 users. Therefore, the model used simple methods to optimize multiple variables, providing better performance compared to processing that would require a more complex algorithm.



### *Keywords*

Demand management, response to energy demand, electric load scheduling, energy consumption profile, mathematical optimization methods.



## Resumen

Los sistemas eléctricos están evolucionando hacia redes inteligentes para mejorar su eficiencia y confiabilidad mediante estrategias de gestión y respuesta a la demanda. Este estudio presenta el Modelo Multiusuario de Cargas Eléctricas Controlables (MMCEC), un modelo de optimización desarrollado para gestionar colectivamente la demanda residencial de múltiples usuarios mediante la Programación de Cargas Eléctricas Controlables (PCEC). El objetivo del modelo fue minimizar el costo de la energía y lograr una distribución más uniforme de la carga eléctrica, teniendo en cuenta tarifas dinámicas de precios y restricciones específicas. La metodología se basó en técnicas clásicas de optimización en dos etapas. La primera se enfocó a nivel de único usuario utilizando el método de búsqueda exhaustiva para seleccionar soluciones que minimicen el costo de la factura de cada usuario. La segunda etapa empleó el método de búsqueda local para la optimización multiusuario, para encontrar una curva de demanda más plana. Para ello, se diseñó un algoritmo en MATLAB® que simuló un escenario con 60 usuarios durante 24 horas, programando los periodos más adecuados de encendido/apagado de las cargas controlables. Se compararon dos escenarios: uno donde los usuarios administran sus cargas a su conveniencia y otro donde se aplicó el modelo propuesto. Los resultados indicaron una disminución de los picos de demanda, con un ahorro promedio del 4.94 % en la factura eléctrica para el conjunto de usuarios y hasta el 12.34 % individualmente. La simulación logró esta solución óptima en 25 minutos a pesar de la complejidad computacional que implica gestionar la demanda de 60 usuarios. Por tal motivo, el modelo planteado utilizó métodos simples para optimizar múltiples variables, proporcionando un mejor rendimiento en comparación con el procesamiento requerido por algoritmos más complejos.



### *Palabras clave*

Gestión de la demanda, respuesta a la demanda energética, programación de cargas eléctricas, perfil de consumo energético, métodos de optimización matemática.

# 1. INTRODUCTION

In recent decades, electrical systems have increasingly struggled with overexertion due to factors such as the continuous rise in energy demand, frequent line interruptions during peak hours, aging infrastructure, and the integration of distributed generation resources. These challenges have led to significant reliability issues, decreased efficiency, energy losses, and escalating electrical costs [1]-[5]. Moreover, the heavy reliance on fossil fuels has exacerbated environmental concerns, particularly due to the substantial greenhouse gas emissions associated with their use [6], [7].

To address these challenges, power grids are being transformed into smart grids [8], [9]. This evolution entails technological, financial, and social changes aimed at enhancing the quality, sustainability, safety, and reliability of electricity supply. Smart grids facilitate a bidirectional flow of electricity and data between distributors and end users by integrating information and communication technologies with automation techniques for the control, monitoring, and maintenance of electrical systems [10]-[12]. The transition toward smart grids, nonetheless, requires adopting Home Energy Management Systems (HEMSs), especially since residential installations account for more than one-third of total electricity consumption [2], [9], [11], [13], [14].

Smart grids encompass several fundamental concepts, including Demand Side Management (DSM) and Demand Response (DR). DSM involves planning and implementing strategies to modify electricity consumption patterns, with the primary objectives of reducing peak demand and $CO_2$ emissions [15]-[17]. This is achieved through the automatic switching on/off of loads in residential settings using home automation technologies and optimization models [18]. The primary goal is to minimize electricity costs and stabilize the demand curve [11], [19]. DR, another critical aspect of smart grids, refers to the use of dynamic pricing to incentivize users to reduce electricity consumption during specific tariff periods [6], [8], [9], [17]. Prices are adjusted based on demand, with higher rates applied during peak hours and lower rates during off-peak periods.

In the 1980s and 1990s, various DSM methods, such as peak shaving, strategic conservation, valley filling, strategic load growth, load shifting, and load flexibilization, were developed. However, it was not until the 2000s that distributed generation technologies were implemented, focusing on the control, monitoring, and metering of electrical systems using intelligent electronic devices [20]. DSM strategies are typically categorized into two approaches: indirect and direct control. Indirect control leverages price incentives and social interactions to influence consumption behavior through the use of optimization algorithms. Common tariff schemes under this approach include time-of-use, critical peak pricing, peak load pricing, and dynamic or real-time pricing [5]. Conversely, direct control implies marketers directly managing loads, thereby imposing stricter restrictions on users' ability to independently control their electricity usage [21].

End users often face challenges in manually programming the on/off schedules of electrical appliances due to limited technical knowledge, time constraints, and a lack of motivation to engage in DR activities at home [1], [22]. As a result, Controllable Electrical Load Scheduling (CELS) emerges as a crucial component of HEMSs. CELS is responsible for monitoring and controlling loads according to user preferences and specifications [23]. Furthermore, external variables, including electricity prices, environmental concerns, personal well-being, energy efficiency education, and users' sense of responsibility as active participants in DSM, influence the habits and behaviors of residential consumers when using their appliances. These factors, in turn, play a key role in developing HEMSs [9].

Importantly, scheduling household appliances at times convenient for users does not necessarily lead to a flattened load curve in distribution grids. The key challenge lies in effectively coordinating the timing of appliance use across different households. By staggering these loads throughout the day, it becomes possible to achieve a more balanced and optimized electricity consumption. In light of this, this study aims to develop a CELS strategy in the context of collective demand management in distribution grids. Specifically, it proposes a mathematical model to flatten the consumption curve and reduce electricity costs.

Several optimization algorithms have been developed to address challenges associated with demand management. These algorithms serve as critical mathematical tools for efficiently solving complex engineering and scientific problems within reduced processing times [24]. The techniques employed include heuristic and metaheuristic approaches, alongside exact methods based on gradient and interior point strategies. Additionally, when combined with branching and probing techniques, these algorithms are capable of addressing a wide variety of problems, ranging from linear programming to Mixed-Integer Nonlinear Programming (MINLP) models [22].

In the field of CELS, numerous algorithms have been designed and are well-documented in the literature. These solutions employ linear, nonlinear, heuristic, and metaheuristic methods, which consider parameters like user requirements, comfort constraints, and environmental and social factors. For instance, the authors of [1] presented an algorithm that focuses on home energy management by ensuring that household consumption remains below a specified demand limit while also taking into account user preferences and enhancing load flexibility. Similarly, in [10], a cooperative control scheme for a smart grid of residential buildings was proposed. This scheme utilizes a predictive control model to coordinate energy usage among buildings, optimizing the use of renewable resources and leveraging the flexibility of thermal loads.

In [23], a heuristic approach using the Greedy Randomized Adaptive Search Procedure (GRASP) was applied to maximize energy utilization from distributed resources in a smart home while minimizing reliance on the distribution grid. The authors highlighted that the primary advantage of this approach is its simplified model for CELS. However, they also acknowledged a drawback: the increased computation time required.

In [18] and [22], a mathematical model for CELS in smart buildings and homes, referred to as the Model for Controllable Electrical Load Scheduling (MCELS), was introduced. This model employs a classical linear optimization method, and its simplicity allows for better performance compared to the heuristic approach presented in [23], as it achieves greater cost reductions and lower dependence on the power grid. Furthermore, in the case of smart buildings, MCELS not only reduces electricity costs for individual users but also facilitates fair payment calculations among different users.

Other approaches include Mixed-Integer Nonlinear Programming (MINLP) and Mixed Integer Linear Programming (MILP). In [8], for example, the authors developed a multi-objective MINLP model that considers both electricity costs and user comfort. The results indicate that although improvements in user comfort can be achieved, they often result in higher electricity costs. In [25], a HEMS was evaluated under three different scenarios: normal, economic, and smart. This model, which uses a multi-objective MINLP framework, considers energy costs, user comfort, and the Peak to Average Ratio (PAR). According to the findings, in the smart scenario, the model significantly reduced energy costs with only minimal effects on user comfort and PAR. However, it requires users to manually adjust multiple parameters and lacks real-time adaptability to external changes.

Studies such as [2], [3], [6], and [7] emphasize the use of MILP. In particular, [3] and [6] employed a MILP model to reduce peak load and flatten the load curve in electrical grids. Nonetheless, the authors of [3] note that while increasing constraints can prevent new peaks after load adjustments, the model tends to prioritize cost-effectiveness, which is identified as a drawback. The authors of [2] also used MILP to minimize total production costs and reduce individual electricity costs. Nevertheless, unlike [3] and [6], this study takes into account the preferences of participating households, reaching a conclusion similar to that of [8]. Finally, the authors of [7] combined MILP with an exact solution method to lower electricity costs in a smart home and found that the exact method is more efficient than MILP.

Stochastic algorithms, such as those discussed in [9], [26], and [27], have also gained increasing attention. In [9], a prototype system was introduced that enables comprehensive energy measurement and management at homes, facilitating supervision, monitoring, and control through decision-making algorithms. Furthermore, the authors of [26] proposed an algorithm aimed at reducing the cost of each charging cycle for residential electric vehicle chargers while also flattening users' charging curve. To validate the model, the algorithm was embedded on a charger hardware. In [27], a bi-objective stochastic optimization approach was presented for scheduling controllable appliances in smart homes. This model, formulated as a mixed-integer programming problem, aims to minimize electricity costs while maximizing user satisfaction. To achieve these objectives, the authors employed both a simulation-optimization approach and a greedy heuristic. The findings suggest that although the simulation-optimization method produces better solutions, the heuristic technique is faster. Additionally, it is noted that scenarios involving multiple users are more difficult to solve than single-user cases. This is because of the increased complexity of coordinating appliance usage among different users, which ultimately results in longer processing times.

In the specialized literature, there are also software tools that incorporate optimization algorithms to improve demand management. For example, in [28], a mathematical model was developed for scheduling flexible loads and energy storage systems, specifically considering the changes in demand curves due to the COVID-19 pandemic. This model has been implemented in Python using the Gurobi optimizer.

Moreover, genetic and evolutionary algorithms are commonly used in the scientific community for demand management. These programming techniques, inspired by biological evolution, are employed to solve complex optimization problems. Table 1 provides an overview of several studies that have applied these algorithms for effective demand management.

Particle Swarm Optimization (PSO) is another widely recognized heuristic algorithm designed to find global minima or maxima. This method is inspired by the behavior of animals that move in groups, such as flocks or herds. Table 2 highlights several studies that have employed this optimization technique. Notably, references [15], [19], and [29] report superior performance results when compared to [13].

Bi-level optimization algorithms, for their part, are used to solve optimization problems that involve two hierarchical levels: a leader and $N$ followers. In these problems, the decisions made at one level directly impact those at the other, and vice versa [36]. Table 3 provides an overview of various studies that have employed these algorithms.

**Table 1.** Studies that have employed genetic and evolutionary algorithms for demand management. Source: own elaboration.

| Ref. | Description |
|---|---|
| [5] | This study explores the use of a genetic algorithm in two two-level optimization methods (suppliers and retailers). The first level focuses on technical requirements, such as flattening the load curve, while the second addresses economic factors like retailer profit. Although both approaches were found to improve the demand profile and increase retailer profit, the study concludes that the technical approach is preferable, as it ensures uniform load distribution without negatively affecting electricity cost reductions. |
| [11] | This paper designs an evolutionary algorithm for three types of customers: residential, commercial, and industrial. The proposed algorithm aims to optimize energy costs by exchanging load use schedules planned for the next day. |
| [17] | This article develops a hybrid (Genetic–Taguchi) algorithm for a HEMS that includes batteries and photovoltaic generators. This proposed algorithm shows greater efficiency than traditional genetic algorithms, as it achieves optimal results with fewer generations. |
| [21] | This study utilizes a genetic algorithm because of its inherent randomness, which helps prevent load concentration and allows the distributor to achieve optimization through indirect control without compromising user comfort. |
| [30] | This paper employs a genetic algorithm to minimize total electricity costs within a dynamic tariff structure while adhering to various constraints. The results indicate that the proposed method achieved approximately 48 % savings in electricity costs compared to a scenario with no optimized scheduling over a simulated day. |
| [31] | This article implements a genetic algorithm to manually complement demand control, a procedure commonly referred to as load shifting. This approach effectively reduces consumption during peak hours, thereby lowering peak demand and ensuring the continuity of the load profile. |
| [32] | This study uses a genetic algorithm combined with load-shifting techniques for DSM in residential settings. This algorithm calculates the average hourly load profile of consumer devices to reduce electricity costs and peak demand. |
| [33] | This paper proposes an evolutionary algorithm for DR between aggregators and consumers using renewable energy sources. This approach aims to maximize user comfort and minimize the peak-to-average demand ratio. |
| [34] | This article presents a HEMS that integrates renewable energy sources (including wind and photovoltaic systems), along with energy storage devices and both electrical and thermal loads. The proposed system employs a genetic algorithm enhanced with the Pareto front technique, taking into account renewable energy availability and user activity, to minimize energy costs and consumption. The results show a 25 % reduction in energy costs and increased utilization of renewable sources. |

**Table 2.** Studies that have employed particle swarm optimization for demand management. Source: own elaboration.

| Ref. | Description |
|---|---|
| [13] | PSO is particularly valued for its swarm intelligence, which offers a distinct advantage over other methods, as it only requires the objective function values for each possible solution. This allows it to generate progressively better alternatives through an iterative process. However, the authors of this paper note that the execution time for simulations is less than ideal, often exceeding 20 minutes. |
| [15] | This study applies PSO to a set of loads with varying operating characteristics across three sectors (residential, commercial, and industrial), which adds complexity to the model. The most significant results were observed in the residential sector, with a peak demand reduction of approximately 23 %, while the commercial and industrial sectors saw reductions of around 17 %. |
| [19] | This article introduces an improved version of PSO that is capable of scheduling smart devices under discrete power levels within a quadratic pricing model. The findings indicate a 4.6 % reduction in total costs and a 94.5 % decrease in execution time compared to traditional dynamic scheduling methods. |
| [29] | This paper uses PSO to schedule portable appliances, aiming to reduce peak demand among 200 residential users and lower daily electricity costs. The results show a reduction of approximately 20 % in both peak demand and daily electricity costs. |
| [35] | This study proposes an economic dispatch approach for power systems that incorporates probability distributions to model renewable energy sources and electric vehicles. Additionally, criteria are established for considering consumption centers as controllable loads. Such dispatch is optimized using an evolutionary differential PSO algorithm, which demonstrates that managing controllable loads can smooth demand profiles, reduce losses, and lower overall generation costs. |

**Table 3.** Studies that have employed bi-level optimization algorithms for demand management.
Source: own elaboration.

| Ref. | Description |
|---|---|
| [37] | This article explains the bi-level optimization model and examines its application in several areas, including DR using Time-of-Use (ToU) tariffs, the integration of electric vehicles into the power scheme, microgrid operations, and the expansion of power grids and generation systems. |
| [38] | This paper introduces two hybrid population-based methods: a bi-level evolutionary algorithm and a bi-level PSO algorithm. At the upper level, the objective is to maximize distributors' profit, while at the lower level, the goal is to minimize users' electricity costs. The results indicate that the bi-level PSO method produced superior solutions in most of the simulated scenarios. The study also highlights that load scheduling is influenced by retailer pricing and user comfort requirements. |
| [39] | This study proposes a real-time pricing scheme for DR management. In this scheme, the retailer sets the price and informs customers, who then adjust their loads accordingly. The proposed approach uses a Stackelberg game with a leader and $N$ followers to optimize both retailer profit and customer satisfaction. |
| [40] | This article presents a two-level hybrid approach, which incorporates Karush–Kuhn–Tucker (KKT) conditions to optimize retailer profit and customer satisfaction. |
| [41] | This paper develops a multi-objective optimization model for energy management in buildings with ToU tariffs. This model integrates photovoltaic generation and user comfort to optimize the economy of a building. |

Building on the above literature review, this study aims to optimize electricity consumption in households by developing a mathematical model that manages collective demand among multiple residential users. The primary goals are to reduce electricity costs and flatten the energy demand curve, as compared to a baseline scenario where customers do not optimize the switching on/off of their loads. To achieve these goals, two methodologies—exhaustive search and local search—are employed. These two approaches produce models that are simpler than those typically found in the literature, which often involve complex techniques and focus on optimizing a single household. The proposed mathematical model will be simulated and tested using MATLAB® and considering a group of users, allowing the effectiveness of the algorithm to be validated in a practical and realistic setting.

The rest of this paper is organized as follows: Section 2 outlines the mathematical models for both single- and multi-user optimization, detailing the relevant sets, parameters, and variables. Section 3 describes the solution methodology employed in the simulations. Section 4 presents and discusses the results. Finally, Section 5 provides the main conclusions of the study and suggests potential directions for future work to develop more efficient and effective models.

## 2. MATHEMATICAL MODEL

The primary purpose of this study is to reduce the collective cost of electricity for multiple households while minimizing consumption peaks during specific hours, all without significantly impacting user comfort. To achieve this, CELS is used to enable automatic switching on/off of loads based on schedules set by both end users and the grid operator and thus efficiently manage demand across households. The proposed Multi-User Model of Controllable Electrical Loads (MUMCEL) is implemented in two stages: the first stage focuses on the single-user (individual) level, while the second stage addresses the multi-user (collective) level. Table 4 presents the nomenclature employed in the mathematical model, and Tables 5, 6, and 7 provide details on the sets, parameters, and variables included in the model.

**Table 4.** Nomenclature used in the mathematical model. Source: own elaboration.

| Abbreviation | Description |
|---|---|
| CEL | Controllable electrical load. |
| NCEL | Non-controllable electrical load. |
| UFEL | Uninterruptible flexible electrical load. |
| IFEL | Interruptible flexible electrical load. |
| SEL | Subsequent electrical load. |

**Table 5.** Sets included in the mathematical. Source: own elaboration.

| Set | Description |
|---|---|
| $U_n = \{U_1, U_2, \dots, U_n\}$ | Set of users, each representing an evaluated household. |
| $T_k = \{t_1, t_2, \dots, t_k\}$ | Set of times for the analyzed period. |
| $CEC = \{CEC_1, CEC_2, \dots, CEC_n\}$ | Set of CELs. |

**Table 6.** Parameters included in the mathematical model. Source: own elaboration.

| Parameter | Description |
|---|---|
| $Ti_{CEL}$ | Start time of CEL $j$. |
| $Tf_{CEL}$ | End time of CEL $j$. |
| $Hload$ | Required time for the appliance to complete its operation. |
| $Ti_{SEL}$ | Start time of SEL. |
| $H_{SEL}$ | Operating time of SEL. |
| $Tf_{SEL}$ | End time of SEL. |
| $p_1(x_{jt}SEL)$ | Start time of SEL activity. |
| $p_{final}(x_{jt}UFEL)$ | Final time of UFEL activity. |
| $Power_{NCEL_t}$ | Power of NCELs. |
| $Power_{CEL_j}$ | Power of CEL $j$. |
| $P_t$ | Energy price during period $t$. |
| $ICF$ | Installed capacity factor. |
| $\mathrm{IC}_n$ | Installed capacity of each evaluated household. |
| $\varphi_t$ | Demand factor of the loads per hour. |
| $\theta_t$ | Factor of CELs operating per hour. |
| $numCELs$ | Total number of controllable electrical loads. |

**Table 7.** Variables included in the mathematical model. Source: own elaboration.

| Variable | Description |
|---|---|
| ${X_{jt}}^i$ | State of load $j$ at time $t$. If $x_{jt} = 1$, CEL $j$ is on at time $t$ ; if $x_{jt} = 0$, CEL $j$ is off. Superscript $i$ denotes the alternative options for CEL optimization for user $n$. |
| $Power_{Total_t}$ | Total power, including the power of the NCELs and the sum of the powers of CELs $j$ evaluated during period $t$. |
| $CLoads$ | Constraint on the number of loads allowed during period $t$. |
| $CPower$ | Constraint on the power allowed during period $t$. |
| $b_k$ | Possible combinations of the different solutions obtained in the single-user optimization for each user. |
| $C_{b,n,t}$ | Total power consumed by each user based on combination $b$, optimal option $i$, and time horizon $t$. |
| $\bar{C}$ | Mean of the data stored in $C_{b,n,t}$. |
| $xstd$ | Standard deviation of the possible combinations $b$ in the multi-user optimization. |

### 2.1 Single-user optimization model

At the single-user level, the MUMCEL aims to determine an optimal schedule for switching on/off loads to minimize each user's total daily electricity costs. This model, detailed in (1) to (21), provides a comprehensive framework for optimizing load scheduling.

Equation (4) defines the total power consumption during each period $t$ as the sum of the powers of the NCELs and the CELs for each optimal alternative $i$ of the CELS, represented by ${X_{jt}}^{i}$. Building on this, (1) y (2) outline the objective function of the model. Specifically, (2) calculates the total electricity cost for option $i$, where $i$ denotes each alternative available to user $n$ for CEL optimization. Such cost is computed as the sum of the product of the energy price ($P_t$) and the total power consumed in each period $t$. Finally, (1) aims to minimize function $fi({X_{jt}}^{i})$, selecting the alternative(s) that yield the lowest cost for user $n$, considering all possible CELs for that user.

$$fobj = \min \{f1({X_{jt}}^{1}), f2({X_{jt}}^{2}), \dots, fi({X_{jt}}^{i})\} \tag{1}$$

$$fi({X_{jt}}^{i}) = \sum_{t \in T} P_t * {Power_{Total}}_t({X_{jt}}^{i}) \tag{2}$$

$$P_t = [P_1, P_2, \dots, P_T] \tag{3}$$

$$Power_{Total_t}({X_{jt}}^{i}) = Power_{NCEL_t} + \sum_{t \in T} Power_{CEL_j} * {X_{jt}}^{i} \tag{4}$$

To ensure the proper functioning of the model, several constraints are imposed, including constraints on installed capacity, allowable power, number of loads, time of use, and load operation.

### 2.2 Constraint on installed capacity

To prevent overloads and potential electrical safety risks, (5) ensures that the total power consumption, as determined by load scheduling decisions, does not exceed the maximum capacity of the household's electrical installation. To maintain a safety margin and prevent potential electrical issues, a Consumption Factor (CF), expressed as a percentage of the Installed Capacity (IC), should be established.

$$0 \leq {Power_{Total}}_t({X_{jt}}^{i}) \leq \mathrm{CF} * \mathrm{IC} \tag{5}$$

### 2.3 Constraint on allowable power during period *t*

Following an approach similar to that outlined in [18] and [22], a strategy is implemented to mitigate the impact of demand peaks occurring at specific times of the day. To this end, a constraint is imposed to limit the total power consumption for each hour, represented by variable $CPower$ and detailed in (6). This variable contains vector $\varphi_t$, whose elements represent the demand factor of the loads per hour, which are then multiplied by the daily demand of user $n$. The proposed constraint takes into account both the power of NCELs and that of CELs, offering a comprehensive view of the household's total energy demand and enabling more precise and efficient energy management.

Equation (7), for its part, ensures that the total power calculated for each alternative $i$ remains below $CPower$ across all periods $t$. This regulation is crucial for managing energy consumption and optimizing load distribution in residential settings.

$$CPower = [\varphi_{1,}\varphi_{2,} \dots, \varphi_T] * \left[\sum_j Hload_j * Power_{CEL\,j} + \sum_{t=1}^{T} Power_{NCEL\,t}\right] \tag{6}$$

$$Power_{Total\,t}({X_{jt}}^i) < CPower \ \ \forall t \tag{7}$$

## 2.4 Constraint on the number of loads allowed during period *t*

To reduce demand peaks when multiple CELs are operating simultaneously, variable $CLoads$ is introduced to represent the maximum number of loads that can be active during period $t$. This is formulated in (8), where vector $\theta_t$ accounts for the factor of CELs operating per hour, which is used to limit the total number of controllable electrical loads ($numCELs$) for user $n$.

$$CLoads = [\theta_1, \theta_2, \dots, \theta_T] * numCELs \tag{8}$$

Equation (9) further restricts the total number of CELs that can be active during period $t$ to the value determined by $CLoads$.

$$\sum_{j \,\epsilon\, numCELs} {X_{jt}}^i < CLoads \ \ \forall t \tag{9}$$

## 2.5 Constraint on time-of-use

This constraint requires that the sum of the powers of the CELs be greater than zero during the interval between $Ti_{CEL}$ and $Tf_{CEL}$, while outside this interval, the sum must be zero. As expressed in (10), this ensures that CELs operate exclusively within the specified time frame.

$$\begin{gathered} Power_{TotalCEL} = \textstyle\sum_j Power_{CEL\,j} * {X_{jt}}^i, where \\ \begin{cases} Power_{TotalCEL} > 0, & for\ t \in [Ti_{CEL}, Tf_{CEL}] \\ Power_{TotalCEL} = 0, & for\ t \notin [TiE_{CEL}, Tf_{CEL}] \end{cases} \end{gathered} \tag{10}$$

## 2.6 Type of loads and their constraints

This subsection presents the CELS model, a system developed to optimize the switching on/off of CELs to minimize total electricity costs and mitigate demand peaks. The constraints established in this model enable the effective configuration and management of loads within the HEMS using mathematical modeling, as discussed in [5], [18], and [22]. In this study, loads are classified into two categories based on their consumption characteristics: controllable loads and non-controllable loads.

#### 2.6.1 Controllable electrical loads

The CEL model aims to determine the optimal timing for switching appliances on/off in a smart home, considering both demand management and user comfort requirements. This entails finding a balance between direct and indirect control within DSM objectives. The proposed MUMCEL introduces a decision variable $x$ for each CEL at time $t$. Specifically, $x_{jt}$ indicates the state of load $j$ at time $t$: if $x_{jt}=1$, CEL $j$ is on; if $x_{jt}=0$, load $j$ is off. Using binary data, a result matrix $X_{jt}$—detailed in (11)—is constructed to represent the states of all loads $j$ across different time intervals $t$ for user $n$.

$$X_{jt}=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1t} \\ x_{21} & x_{22} & \cdots & x_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ x_{j1} & x_{j2} & \dots & x_{jt} \end{bmatrix}=\begin{bmatrix} 0 & 1 & \dots & 0 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \tag{11}$$

Equations (12) through (15) outline the constraints for CELs. Equations (12) and (13) ensure that the operational intervals of the loads fall within the scheduling horizon and that the operating time ($Hload$) of each appliance does not exceed its usage interval. Equation (14) stipulates that the sum of the binary states for the CEL must equal the number of hours the load is in operation. Equation (15), for its part, guarantees that the value of the state variable is zero outside the scheduled interval ($Ti_{CEL}$to $Tf_{CEL}$). Thus, if $Ti_{CEL}\leq t\leq Tf_{CEL}$, variable $x_{jt}$ can be either 0 or 1; otherwise, it is 0. The application of (14) and (15) is analogous to that of (10), ensuring that CELS remains consistent, efficient, and optimal within the allowed time limits.

$$Ti_{CEL}\leq Tf_{CEL}\leq T \tag{12}$$

$$Tf_{CEL}-Ti_{CEL}\geq Hload \tag{13}$$

$$\sum_{t=Ti_{CEL}}^{t=Tf_{CEL}} x_{jt}=Hload,\quad \forall j \tag{14}$$

$$\sum_{t<Ti_{CEL}\ o\ t>Tf_{CEL}} x_{jt}=0,\quad \forall j \tag{15}$$

CELs are categorized into three main types: Uninterruptible Flexible Electrical Loads (UFELs), Interruptible Flexible Electrical Loads (IFELs), and Subsequent Electrical Loads (SELs). UFELs must operate continuously without interruption until their operating cycle is complete. The model for this type of loads is provided in (12) to (15) along with (16), which ensures that the load remains uninterrupted during the scheduled hours. Equation (16) considers both the previous and current states of $x$ to verify uninterrupted operation. A load that operates continuously is classified as a UFEL; otherwise, it is categorized differently.

$$\sum_{t=Ti_{CEL}}^{Tf_{CEL}} x_{jt}*x_{j(t-1)}\geq Hload-1 \tag{16}$$

IFELs, for their part, can be controlled and may be temporarily switched off during their operating cycle, allowing them to operate intermittently within the predefined time interval. The model for this type of loads is given by (17), which specifies that within the interval between $Ti_{CEL}$ and $Tf_{CEL}$, variable $x_{jt}$ can be 0 or 1. This equation, in conjunction with the constraints outlined in (12) through (15), ensures proper management of IFELs throughout the entire time range.

$$x_{jt}(t) = \{0,1\}, t \in [Ti_{CEL}, Tf_{CEL}] \tag{17}$$

Finally, SELs are activated only after certain UFELs have completed their operation. For instance, a clothes dryer may start only after a washing machine has finished its cycle. SELs must operate as uninterruptible appliances. Their start times are dependent on the completion of UFELs' cycles, either immediately or later, and their end times are constrained by their operating time ($H_{SEL}$), with the last possible switch-off occurring at the end of the day. Equations (18) and (19) define the start ($Ti_{SEL}$) and end ($Tf_{SEL}$) times of SELs. Moreover, (20) guarantees that the operating cycle of a SEL aligns with its start and end times.

$$Ticl_{UFEL} + Hload_{UFEL} \leq Ti_{SEL} \leq T - H_{SEL} \tag{18}$$

$$Ti_{SEL} + H_{SEL} \leq Tf_{SEL} \leq T \tag{19}$$

$$Tf_{SEL} - Ti_{SEL} = H_{SEL} \tag{20}$$

Importantly, the number of loads of any type cannot be negative, as this would contradict the principles of the mathematical model (21).

$$UFELs, IFELs, SELs \geq 0 \;\forall j \tag{21}$$

#### *2.6.2 Non-controllable electrical loads*

NCELs operate based on pre-defined usage patterns and cannot be adjusted or managed during the optimization process. These loads are crucial, as they represent essential services that must be provided immediately upon user request to ensure the residents' well-being [8]. Refrigerators, for example, fall under this category, as do low-power devices such as household lighting. In the proposed mathematical model, no specific equations are formulated for NCELs; instead, hourly consumption data for these loads are obtained from established theoretical frameworks found in the literature.

### 2.7 Multi-user optimization model

At the multi-user level, the proposed MUMCEL evaluates the set of solutions generated in the single-user optimization, assessing various combinations to identify the optimal configuration. The goal is to achieve a flatter consumption curve, ensuring a more stable load distribution and reduced electricity costs for all users. Ideally, energy consumption across multiple users would produce a completely flat demand curve, thereby avoiding periods of over-demand or underutilization of the electrical system at specific times of the day.

To approximate this ideal scenario, the first step is to calculate the average power consumption of the households over a given time interval. The standard deviation is then used as a metric to evaluate how closely the actual consumption curve aligns with this average. This approach makes it possible to find a solution that is both optimal for individual users and stable and consistent across multiple users. Finally, the combination of alternatives that yields the lowest standard deviation is selected, resulting in a consumption curve that is closer to the ideal average and a more balanced load distribution for all customers.

Equations (22) through (26) below define the MUMCEL at the multi-user level. Equation (22) introduces variable $C_{b,n,t}$, which represents the total power consumed by each user as a function of combination $b$ in optimal option $i$. Here, $b$ denotes the various possible combinations in the model. Importantly, if multiple optimal solutions exist for a user, $C_{b,n,t}$ will have multiple values corresponding to these different combinations.

Equation (23) calculates the average of the data stored in variable $C_{b,n,t}$, taking into account both the number of users and the time horizon. Equation (24) then determines the standard deviation of users' consumption for each possible combination $b$. Following this, (25) aggregates all the standard deviations from these combinations. Finally, (26) defines the objective function: to minimize the standard deviation across all possible combinations, thereby identifying the combination $b$ with the lowest variability in power consumption. This ultimately results in the flattest demand curve for the group of users.

$$C_{b,n,t} = \{Power_{Total_t}{}^{1}(X_{jt}{}^{i}); Power_{Total_t}{}^{2}(X_{jt}{}^{i}); \dots; Power_{Total_t}{}^{n}(X_{jt}{}^{i})\}, \quad \text{where } n = 1,2,\dots, n \text{ users} \tag{22}$$

$$\bar{C} = \frac{1}{T}\sum_{t=1}^{T}\sum_{n=1}^{N} C_{b,n,t} \tag{23}$$

$$xstd(C_b) = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left|\sum_{n=1}^{N} C_{b,n,t} - \bar{C}\right|^2} \tag{24}$$

$$xstd(C) = [xstd(C_{b1}), xstd(C_{b2}), \dots., xstd(C_{bk})] \tag{25}$$

$$Objective\ function\ = min_b(xstd(C)) \tag{26}$$

## 3. SOLUTION METHODOLOGY

This section details the implementation of the proposed MUMCEL, which focuses on CELS in smart homes to efficiently manage the energy demand of multiple residential users under a dynamic hourly pricing scheme. The model will be executed in MATLAB®, and additional procedures and considerations beyond those outlined in Section 2 will be incorporated to achieve optimal load management across multiple users. Additionally, Subsection 3.3 will provide further details on the initial simulation parameters, including load behavior, hourly energy prices, household installed capacity, the number of users, and the types of loads.

The methodology employed in this study involves two interconnected stages: single-user optimization and multi-user optimization. Single-user optimization employs exhaustive search, while multi-user optimization utilizes local search. The exhaustive search method, a brute-force approach, systematically evaluates all possible solutions within a defined search space to find the optimal global solution. The local search method, for its part, is an iterative heuristic technique that starts with an initial solution and then explores and compares alternatives within a local environment. If an improved solution is identified, it replaces the current one, and the search continues until no further improvement is found [18], [36], [42].

### 3.1 Single-user optimization

To implement the proposed MUMCEL at the single-user level, the exhaustive search method is here used, which systematically explores all potential solutions for each user to find the optimal configuration. This process is divided into two distinct phases to effectively minimize electricity costs for the evaluated user.

In the first phase, the method examines all possible scenarios for switching loads on/off within the search space. Binary matrices are employed to represent the state (on or off) of each load per hour. Then, the method verifies that each load is in operation within its specified time intervals ($Ti_{CEL}$ to $Tf_{CEL}$) and adheres to its total operating time ($Hload$). Subsequently, all feasible schedule combinations are generated and assessed based on the types of CELs, discarding those failing to meet the constraints outlined in Subsection 2.6.1.

In the second phase, the simulation code evaluates all viable schedule combinations identified in the first phase against the constraints detailed in Subsections 2.2 to 2.5. Combinations that do not satisfy these criteria are discarded. The remaining combinations are then validated, and those that successfully minimize electricity costs for the evaluated user are selected.

#### *3.1.1 Considerations for the first phase of exhaustive search*

In this phase, the scheduling horizon ($T$) is set to 24 hours and divided into one-hour periods (each denoted as $t$), where $t \in T$. In addition, parameters $Ti_{CEL}, Tf_{CEL}$, and $Hload$ are defined for each user based on their preferred schedule for switching loads on/off, allowing for some comfort. Then, as outlined in Subsection 2.6.1, decision variable $x$ is employed to generate various combinations of load states (on or off) using binary matrices. Once the input variables and matrices are initialized, these combinations are processed to determine possible load states and organize the state matrices according to the constraints defined in (12) through (15). Importantly, these equations are applied to all types of CELs to make sure that loads are not switched on outside the schedule defined by users.

Next, combinations that do not satisfy constraints (16) to (21)—which correspond to the criteria for UFELs, IFELs, and SELs—are discarded. During the simulations of switching IFELs on/off, it is crucial to exclude options that show uninterruptible behaviors. Similarly, SELs require special considerations: last switch-off must occur precisely at 23:59, and switch-on schedules may occur before or overlap with IFELs' operation. Equations (27) through (29) are used to address this and eliminate invalid options.

To validate SELs' switch-on times, the start time of the SEL ($p_1$) is compared with the last activity time of the corresponding UFEL ($p_{\text{last}}$). If $p_1$ minus $p_{\text{last}}$ is positive, the switch-on time is considered valid; otherwise, it is deemed invalid (see (29)).

$$p_1(x_{jt}SEL) = min\{i|x_{jt}SEL_i = 1\} \tag{27}$$

$$p_{last}(x_{jt}UFEL) = \max\{i|x_{jt}UFEL_i = 1\} \tag{28}$$

$$\begin{cases} If\ p_1 - p_{last} > 0, & valid\ x_{jt}SL\ option; \\ otherwise, & invalid\ x_{jt}SL\ option \end{cases} \tag{29}$$

Finally, valid state options that meet the scheduling criteria for switch-on and switch-off are selected for the evaluated user. The resulting state matrices are then forwarded to the next scheduling phase.

#### *3.1.2 Considerations for the second phase of exhaustive search*

In the second phase, the exhaustive search method is employed to find the optimal configuration that minimizes electricity costs across multiple options. The process begins by determining the number of CELs ($numCELs$) to be evaluated for a given user, which establishes the $Cload$ limit for each time period. The total daily demand of the household is then calculated by summing the power of both CELs and NCELs, as will be detailed in Subsection 3.3. This calculation defines the $CPower$ matrix for the hourly power limits (see (6)). To ensure a safety margin and prevent potential electrical issues, the consumption limit at any given time $t$ must not exceed 90 % of the Installed Capacity (IC).

The next step involves incorporating all state matrices of the CELs ($X_{jt}$) from the previous phase into the search space for user $n$, as described in (30).

$$X_{jt_n} = \{X_{jt}{}^1, X_{jt}{}^2, \dots, X_{jt}{}^i\}\ where\ i = 1,2, \dots, k \tag{30}$$

In this equation, superscript $i$ denotes each individual alternative of $X_{jt}$, and $k$ represents the total number of alternatives for user $n$. For each alternative $k$ of $X_{jt_n}$ for the CELs, the hourly energy consumption is calculated by summing the power of the CELs and NCELs, as outlined in (4) of the proposed MUMCEL.

The exhaustive search then evaluates each alternative $k$ for user $n$ using constraints (5), (7), (9), and (10). Alternatives that do not meet these constraints are discarded. The goal is to find the optimal configuration(s) that minimize electricity costs by applying (1) through (3).

Finally, the configuration(s) that minimize daily electricity costs are selected, ensuring that the final solution aligns with the system's capabilities and constraints. The implementation of (1) through (21) and (27) through (30), combined with the exhaustive search method, defines the behavior of the proposed MUMCEL at the single-user level. This process yields one or more optimal solutions that effectively reduce electricity costs. The simulation results in MATLAB® include the total power for each period $t$ ($Power_{Total_t}(X_{jt}{}^i)$) and the scheduling of controllable loads ($X_{jt}{}^i$) of each valid solution, which are then used as inputs for the multi-user optimization model.

### 3.2 Multi-user optimization

At the multi-user level, the MUMCEL employs the local search method, building on the options selected in the single-user optimization, while adhering to the design criteria and simulation specifications. This heuristic approach sequentially combines the potential solutions for each user, following a local path to identify the most favorable configuration.

The simulation process starts by selecting and combining the demand curves of the first two users (Users 1 and 2). Importantly, each user may have multiple alternatives with low electricity costs, which may result in various possible combinations. The best combination is determined by calculating the standard deviation (which serves as an evaluation metric), while the remaining options are discarded. Subsequently, the demand curve(s) for User 3 are added to the optimal combination of Users 1 and 2, and the standard deviation is recalculated to identify the best configuration for all three users. This iterative process continues, adding users one at a time and always selecting the combination with the lowest standard deviation until the final user is incorporated.

The model identifies a local solution by processing users sequentially, rather than exploring all possible combinations across the entire search space of $n$ users to find a global optimum. As a result, the final outcome depends on the starting point, and varying the order in which user solutions are combined may lead to different local solutions. Although this approach does not guarantee a globally optimal solution, it provides satisfactory results, as will be discussed in Section 4. This methodology was chosen taking into account the processing time required to solve the problem, as well as the number of variables and combinations that would arise from considering all possible user solutions.

Given these considerations, the implementation of the MUMCEL at the multi-user level is structured into three distinct phases. In Phase 1, variables such as $Power_{Total_t}\left({X_{jt}}^i\right), {X_{jt}}^i$, and the electricity costs for the first user, obtained in the single-user optimization, are stored. The same variables are then selected and stored for the second user. In Phase 2, an independent vector $a$ is created for each user to represent the number of solutions. Possible combinations are then determined using this vector and stored in variable $b_k$. Subsequently, variable $C_{b,n,t}$ is defined, which contains the hourly consumption behavior of each option according to $b_i$. In Phase 3, during the cycle of $b_i$ and the creation of $C_{n,t}$, the standard deviation across iterations is calculated, and the smallest value is selected as the metric for choosing the best combination. Following this, variables $Power_{Total_t}\left({X_{jt}}^i\right)$, ${X_{jt}}^i$, and the electricity costs for the two users in the selected combination are stored. This process is then repeated by combining the variables generated in the previous phase with the solutions for the third user. The three-phase process is executed again for each additional user until the final result is achieved.

In (31), the third phase is defined as the search for the optimal combination $b_i$ that minimizes the standard deviation. This is accomplished by iterating over neighboring options and evaluating a combination where the current standard deviation ($xstd_{current}$) is lower than the minimum standard deviation ($xstd_{min}$) per cycle, as outlined in (24) through (26).

$$xstd_{current} < xstd_{min} \tag{31}$$

In summary, the consumption behavior of residential users often leads to demand peaks at certain times of the day due to their load usage decisions and preferences. The proposed MUMCEL addresses this by coordinating and scheduling the switching on/off of appliances for multiple users, resulting in a flatter load profile and reduced daily electricity costs. The standard deviation serves as a key metric for evaluating the quality of power consumption combinations, enabling a stable and uniform distribution in the scheduling of loads within a defined time interval. This, in turn, promotes more efficient resource management and contributes to enhanced stability in the power supply.

## 3.3 Initial parameters

This subsection outlines the input parameters necessary for simulating the proposed MUMCEL, which include energy prices, load characteristics, the number of users, and household installed capacity. Importantly, the simulation is scheduled to run over a full 24-hour period, i.e., from 00:00 to 23:59.

### *3.3.1 Energy prices across different periods ($P_t$)*

In residential electricity markets like those in Colombia, a flat-rate pricing system is commonly used. Under this system, a uniform energy cost is applied throughout the day, regardless of the time. Although straightforward, this tariff structure does not encourage users to adjust or optimize their energy consumption, thereby limiting the potential benefits of effective demand management [16]. In contrast, countries such as Spain, Brazil, and Uruguay have adopted dynamic pricing schemes, where energy costs vary according to peak, flat, and off-peak hours. The effectiveness of these tariff systems, however, depends on each country's specific conditions and energy infrastructure [43].

In Colombia, the electricity market is gradually evolving, facing key challenges such as adapting the tariff structure for regulated users, integrating new market participants, and progressively incorporating smart grid infrastructure [16]. To address these challenges, strategies have been proposed for implementing dynamic pricing schemes for end users in the country [16]. Based on these strategies, tariffs will be here determined for each time interval considered in the simulation of the proposed mathematical model (see Table 8). Furthermore, (3) introduces the dynamic tariff system by defining vector $Pt$, which contains energy prices for specific time intervals.

**Table 8.** Hourly rates. Source: data obtained from the analysis of tariffs for end users in demand response programs in Colombia [43]. COP: Colombian peso (2020).

| Intervalo de tiempo (h) | 0-3 | 4-8 | 9-11 | 12-17 | 18-20 | 21-22 | 23 |
|---|---|---|---|---|---|---|---|
| Precio (COP $) | 381.34 | 563.75 | 646.60 | 563.75 | 646.60 | 563.75 | 381.34 |

### *3.3.2 Parameters for controllable electrical loads*

Tables 9 and 10 present the parameters that users can configure for the proposed MUMCEL, such as the operation intervals of the CELs ($Ti_{CEL}$ to $Tf_{CEL}$) and the characteristics of the loads ($Hload_j$, $H_{SEL}$, $Power_{CEL\,j}$). The set of appliances includes 14 UFELs, 2 SELs, and 4 IFELs, each offering multiple options for switching on/off based on user preferences. In the developed model, these appliances are considered to have various operational configurations. Consequently, the programmable devices provide a total of 67 possible configurations, including 41 UFELs, 15 IFELs, and 11 SELs.

**Table 9.** Parameters for UFELs and IFELs. Source: own elaboration. Note: Data presented in this table were taken from different sources, including [44]–[50], and were organized by the authors.

| # | Load | $Ti_{CL}$ (h) | $Tf_{CL}$ (h) | H_load (h) | Nominal power (kW) | Power (kW) | Type of load |
|---|---|---|---|---|---|---|---|
| 1 | Dishwasher (intensive mode) | 6 | 13 | 2 | 1.80 | 1.630 | UFEL |
| 2 | | 18 | 24 | 2 | 1.80 | 1.630 | |
| 3 | | 8 | 12 | 2 | 1.80 | 1.630 | |
| 4 | Dishwasher: (normal mode) | 0 | 5 | 2 | 1.80 | 1.020 | |
| 5 | | 19 | 23 | 2 | 1.80 | 1.020 | |
| 6 | | 13 | 18 | 2 | 1.80 | 1.020 | |
| 7 | Dishwasher (eco mode) | 4 | 10 | 3 | 1.80 | 0.840 | |
| 8 | | 14 | 19 | 3 | 1.80 | 0.840 | |
| 9 | | 18 | 23 | 3 | 1.80 | 0.840 | |
| 10 | | 20 | 24 | 3 | 1.80 | 0.840 | |
| 11 | Dishwasher (quick wash) | 5 | 7 | 1 | 1.80 | 0.640 | |
| 12 | | 13 | 16 | 1 | 1.80 | 0.640 | |
| 13 | | 20 | 23 | 1 | 1.80 | 0.640 | |
| 14 | | 9 | 12 | 1 | 1.80 | 0.640 | |
| 15 | Washing machine (automatic) | 0 | 6 | 2 | 1.70 | 0.182 | |
| 16 | | 7 | 11 | 2 | 1.70 | 0.182 | |
| 17 | | 17 | 21 | 1 | 1.70 | 0.182 | |
| 18 | | 20 | 24 | 1 | 1.70 | 0.182 | |
| 19 | Washing machine (auto. w/ heating) | 0 | 6 | 2 | 1.70 | 0.882 | |
| 20 | | 7 | 11 | 2 | 1.70 | 0.882 | |
| 21 | | 15 | 20 | 2 | 1.70 | 0.882 | |
| 22 | | 17 | 22 | 1 | 1.70 | 0.882 | |
| 23 | | 20 | 24 | 1 | 1.70 | 0.882 | |
| 24 | Oven | 16 | 20 | 1 | 1.20 | 1.000 | |
| 25 | | 11 | 13 | 1 | 1.20 | 1.000 | |
| 26 | Water heater | 4 | 7 | 2 | 1.50 | 1.500 | |
| 27 | | 6 | 10 | 2 | 1.50 | 1.500 | |
| 28 | | 19 | 24 | 1 | 1.50 | 1.500 | |
| 29 | Vacuum cleaner | 8 | 15 | 2 | 0.75 | 0.675 | |
| 30 | | 11 | 18 | 3 | 0.75 | 0.675 | |
| 31 | Iron | 19 | 23 | 2 | 1.00 | 0.600 | |
| 32 | | 18 | 22 | 1 | 1.00 | 0.600 | |
| 33 | Rice cooker | 11 | 14 | 1 | 0.70 | 0.700 | |
| 34 | | 10 | 13 | 1 | 0.70 | 0.700 | |
| 35 | Stove | 11 | 14 | 2 | 1.50 | 1.200 | |
| 36 | | 18 | 21 | 2 | 1.50 | 1.200 | |
| 37 | Electric shower | 4 | 7 | 1 | 1.20 | 1.200 | |
| 38 | | 7 | 10 | 1 | 1.20 | 1.200 | |
| 39 | | 20 | 23 | 1 | 1.20 | 1.200 | |
| 40 | Fryer | 12 | 15 | 1 | 1.00 | 1.000 | |
| 41 | | 19 | 21 | 1 | 1.00 | 1.000 | |

| # | Load | $Ti_{CL}$ (h) | $Tf_{CL}$ (h) | H_load (h) | Nominal power (kW) | Power (kW) | Type of load |
|---|---|---|---|---|---|---|---|
| 42 | Air conditioner | 12 | 24 | 8 | 1.35 | 1.013 | IFEL |
| 43 | | 8 | 18 | 8 | 1.35 | 1.013 | |
| 44 | | 0 | 3 | 1 | 1.35 | 1.013 | |
| 45 | | 8 | 11 | 2 | 1.35 | 1.013 | |
| 46 | | 13 | 16 | 2 | 1.35 | 1.013 | |
| 47 | Fan | 9 | 18 | 7 | 0.09 | 0.090 | |
| 48 | | 8 | 15 | 6 | 0.09 | 0.090 | |
| 49 | | 19 | 24 | 4 | 0.09 | 0.090 | |
| 50 | Coffee maker | 7 | 10 | 1 | 0.90 | 0.720 | |
| 51 | | 13 | 16 | 1 | 0.90 | 0.720 | |
| 52 | | 19 | 22 | 1 | 0.90 | 0.720 | |
| 53 | Heater | 7 | 11 | 3 | 1.50 | 1.500 | |
| 54 | | 6 | 9 | 2 | 1.50 | 1.500 | |
| 55 | | 13 | 16 | 2 | 1.50 | 1.500 | |
| 56 | | 18 | 22 | 3 | 1.50 | 1.500 | |

As observed, Tables 9 and 10 also present the nominal power values, which include both the power indicated on the appliance nameplates and the average effective power consumed during real-world operation [44]–[51]. The latter value will be used for load scheduling.

**Table 10.** Parameters for SELs. Source: own work. Note: Data presented in this table were taken from different sources, including [44]–[51], and were organized by the authors.

| # | Load | H_load (h) | Nominal power (kW) | Power (kW) | # of preceding load | Preceding load |
|---|---|---|---|---|---|---|
| 1 | Dryer | 2 | 2.2 | 0.874 | 15 | Washing machine (automatic) |
| 2 | | 2 | 2.2 | 0.874 | 16 | |
| 3 | | 1 | 2.2 | 0.874 | 17 | |
| 4 | | 1 | 2.2 | 0.874 | 18 | |
| 5 | | 2 | 2.2 | 0.874 | 19 | Washing machine (auto. w/heating) |
| 6 | | 2 | 2.2 | 0.874 | 20 | |
| 7 | | 2 | 2.2 | 0.874 | 21 | |
| 8 | | 1 | 2.2 | 0.874 | 22 | |
| 9 | | 1 | 2.2 | 0.874 | 23 | |
| 10 | Polisher | 2 | 0.5 | 0.500 | 29 | Vacuum cleaner |
| 11 | | 2 | 0.5 | 0.500 | 30 | |

### *3.3.3 Consumption of non-controllable loads (Power_CELs)*

As previously discussed, NCELs exhibit a fixed consumption pattern throughout the day. For the proposed scenario, the studies reported in [20], [51], and [52] serve as reference points, providing load profile curves for five residential users ($Power_{CL_t}$). These curves offer a realistic approximation of consumption patterns for the mathematical model. Moreover, the consumption profiles for all users are constructed based on these curves, with the relevant parameters presented in Table 11.

**Table 11.** Power of the NCELs. Source: own elaboration.

| User | U1 (kW) | U2 (kW) | U3 (kW) | U4 (kW) | U5 (kW) |
|---|---|---|---|---|---|
| Hour 0 | 0.066 | 0.085 | 0.168 | 0.156 | 0.119 |
| Hour 1 | 0.058 | 0.040 | 0.173 | 0.130 | 0.100 |
| Hour 2 | 0.054 | 0.045 | 0.167 | 0.118 | 0.096 |
| Hour 3 | 0.091 | 0.050 | 0.169 | 0.127 | 0.109 |
| Hour 4 | 0.214 | 0.107 | 0.217 | 0.165 | 0.176 |
| Hour 5 | 0.272 | 0.130 | 0.278 | 0.261 | 0.235 |
| Hour 6 | 0.329 | 0.170 | 0.293 | 0.276 | 0.267 |
| Hour 7 | 0.301 | 0.220 | 0.283 | 0.316 | 0.280 |
| Hour 8 | 0.290 | 0.175 | 0.276 | 0.270 | 0.253 |
| Hour 9 | 0.230 | 0.162 | 0.190 | 0.265 | 0.212 |
| Hour 10 | 0.228 | 0.150 | 0.198 | 0.275 | 0.213 |
| Hour 11 | 0.256 | 0.157 | 0.198 | 0.226 | 0.209 |
| Hour 12 | 0.212 | 0.190 | 0.171 | 0.249 | 0.206 |
| Hour 13 | 0.173 | 0.175 | 0.182 | 0.227 | 0.190 |
| Hour 14 | 0.173 | 0.155 | 0.183 | 0.219 | 0.182 |
| Hour 15 | 0.202 | 0.151 | 0.169 | 0.216 | 0.185 |
| Hour 16 | 0.208 | 0.178 | 0.174 | 0.216 | 0.194 |
| Hour 17 | 0.190 | 0.182 | 0.189 | 0.240 | 0.200 |
| Hour 18 | 0.198 | 0.225 | 0.364 | 0.311 | 0.274 |
| Hour 19 | 0.301 | 0.213 | 0.385 | 0.330 | 0.307 |
| Hour 20 | 0.348 | 0.217 | 0.389 | 0.305 | 0.315 |
| Hour 21 | 0.246 | 0.212 | 0.306 | 0.298 | 0.265 |
| Hour 22 | 0.137 | 0.200 | 0.240 | 0.246 | 0.206 |
| Hour 23 | 0.074 | 0.140 | 0.225 | 0.181 | 0.155 |
| Hour 24 | 0.066 | 0.085 | 0.168 | 0.156 | 0.119 |

### 3.3.4 Installed capacity

The next parameter to consider is the installed capacity ($IC$) per user. For this study, values were selected within the average range for single-family and two-family dwellings in Colombia [53], establishing an interval between 6 kW and 12 kW.

### 3.3.5 Number of users

Finally, it is essential to define the number of end users participating in the simulation process ($NumU$). In this study, the simulation includes a total of 60 residential users.

The choice of initial parameters and optimization strategies can significantly influence the simulation results. Therefore, sensitivity analyses should be performed to assess how changes in these parameters affect the outcomes. The computational methodology provides valuable insights for enhancing the efficiency and sustainability of energy demand management, particularly in the context of developing electricity markets that incorporate dynamic tariff schemes, such as those in Colombia.

### 3.4 Flow diagram of the proposed multi-user model of controllable electrical loads

The proposed MUMCEL aims to optimize both the demand curve and electricity costs for 60 residential users, while also considering their comfort levels. To achieve this, CELS will be applied using a series of initial parameters (see Section 3.3), in which each user has a specific behavior for various types of loads (see section 3.3.2), adjusted to a dynamic tariff, and considering a base power per user (non-controllable power).

The employed methodology, as outlined in Sections 3.1 and 3.2, involves a computational simulation carried out in MATLAB®, version R2022b. This software is widely recognized by the scientific community for its ability to handle and calculate matrix operations. The simulation was run on a computer equipped with an Intel(R) Core (TM) i5-8250U CPU @ 1.60 GHz - 1.80 GHz processor and 12 GB of RAM. The simulation algorithm comprises four stages, each of which is described in detail in Table 12.

**Table 12.** Stages of the MUMCEL simulation. Source: own elaboration.

| Stage | Description |
|---|---|
| 1. Definition of initial parameters | The input parameters for the case study are established, including energy prices, characteristics of the loads used in the algorithm, number of users, and installed capacity (see Section 3.3). |
| 2. Implementation of the single-user optimization model | The exhaustive search method is employed to find the optimal solutions that minimize electricity costs for each user (see Section 3.1). Each solution for user $n$ is identified by index $i$. In these matrices (${X_{jt}}^{i}$), the rows represent the scheduling of the controllable load ($j$), while the columns represent the time periods ($t$) within the scheduling horizon. |
| 3. Implementation of the multi-user optimization model | The multi-user optimization model is implemented, incorporating features of the local search method. The algorithm identifies the optimal combinations from the individual solutions of each user (see section 3.2). These combinations are evaluated based on the standard deviation of the users' total power consumption and electricity costs. Different combinations of multi-user solutions are compared to identify the one that achieves the lowest variability and the most uniform energy consumption over time. |
| 4. Model output and visualization | A document is generated containing the simulation results, including tables with load profile data, electricity costs, and matrices that facilitate the proper scheduling of controllable loads. |

Figure 1 shows the flow diagram of the implemented optimization algorithm, providing a visual representation of the key stages and decisions. The diagram begins with importing the number of users and initializing the iteration variable, $n$. For each user, the corresponding parameters are imported, state matrices for the loads are created, and the exhaustive search process is initiated. During this process, options that do not meet the specified constraints are eliminated, and the next option is evaluated. The optimal solutions identified at the single-user level are then stored, and an initial combination is generated to perform the local search, which iteratively evaluates different combinations. The standard deviation of power is subsequently calculated, and the best combinations are updated accordingly. This process is repeated by incrementing $n$ until all users have been evaluated. Finally, the results are exported, including the optimal solution and its details.

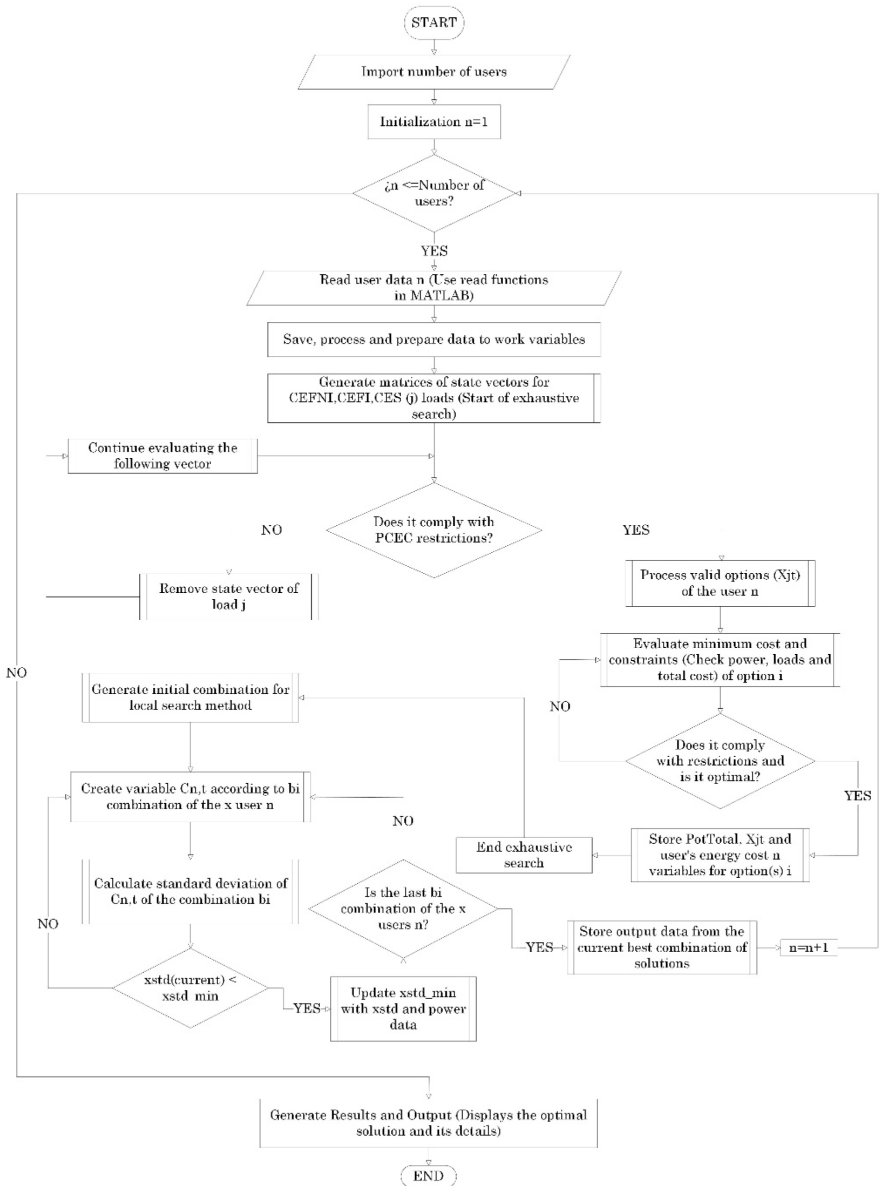


**Figure 1.** Flow diagram of the proposed optimization process. Source: own elaboration.

## 4. RESULTS AND DISCUSSION

This section presents a comparative analysis between two case studies: one without optimization, where each of the 60 users has a set of appliances with a fixed daily electricity consumption pattern, and another where demand is optimized using a CELS model.

Figure 2 illustrates this comparison for one of the 60 users, referred to as User (a). The bar chart depicts the operation times of the CELs and NCELs over the time horizon $T$, highlighting the loads activated during each time period $t$ when optimization is applied. In addition, the green line graph represents the unoptimized load profile for this user.

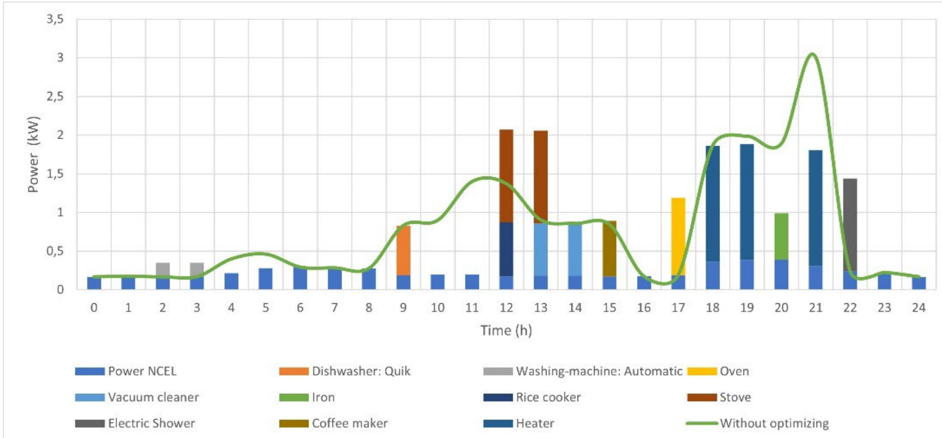


**Figure 2.** Unoptimized load profile and CEL schedule for User (a). Source: own work.

To analyze the specific consumption patterns following the implementation of the CELS model, the demand curves for four additional users are presented in Figure 3. The base scenario, depicted by a blue line graph, shows when the user operates the CELs at their discretion, while the optimized scenario is illustrated by an orange-filled area graph.

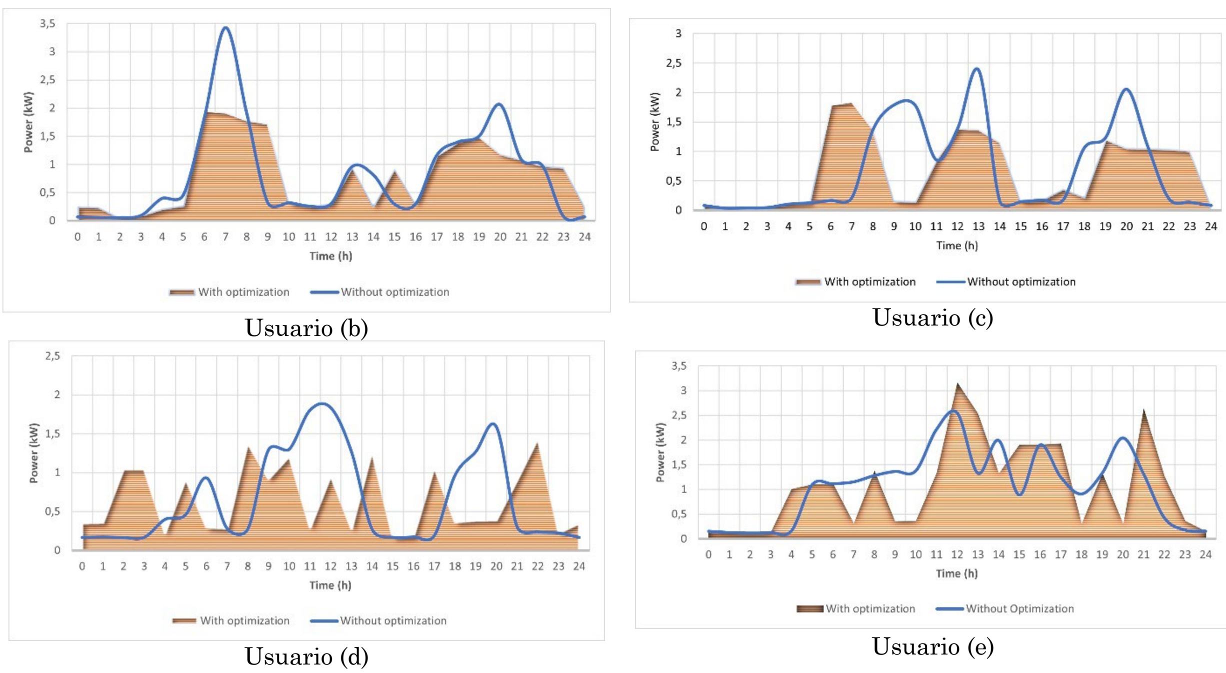


**Figure 3.** Unoptimized load profile vs. optimized load profile for four users. Source: own work.

Before discussing the results further, it is important to define the concept of Peak-to-Average Ratio (PAR), which is the ratio between the maximum (peak) value of a curve and its average value. For the four users analyzed, a reduction in peak demand during periods of higher energy prices is observed when implementing the CELS model. Table 13 presents the results for the peak power and PAR for both the optimized and unoptimized scenarios.

**Table 13.** Peak active power analysis results for five users. Source: own elaboration.

| User | Peak power (KW) without optimization | Peak power (KW) with optimization | PAR without optimization | PAR with optimization | % of peak power reduction |
|---|---|---|---|---|---|
| (a) | 3.01 | 2.07 | 3.78 | 2.60 | 31.08 |
| (b) | 3.43 | 1.95 | 4.09 | 2.33 | 42.90 |
| (c) | 2.38 | 1.85 | 3.37 | 2.63 | 22.10 |
| (d) | 1.84 | 1.44 | 2.76 | 2.17 | 21.53 |
| (e) | 2.55 | 3.16 | 2.32 | 2.87 | -23.90 |

Figures 2 and 3 show that the five users employ different strategies to manage their demand. Additionally, Table 14 provides the electricity costs for each user, revealing savings ranging between 1 % and 8 % for the day evaluated in the simulation. A common strategy employed by all five users is load shifting, where consumption is shifted to periods with lower energy costs.

Particularly, the optimization algorithm for User (b) implements a peak shaving strategy with slight valley filling at certain times of the day. Similarly, User (a) adopts a combined approach that includes peak shaving and strategic conservation, allowing them to maintain peak demand periods while adjusting the magnitude of these peaks as necessary. Although Users (a) and (b) achieve significant reductions in peak demand, their bill savings are lower than those of other users. This outcome is likely due to their focus on reducing consumption peaks rather than shifting loads to lower-cost periods.

In the case of User (c), the algorithm shifts one demand peak to a lower-cost period while shaving the other two peaks and filling the valleys. For User (d), consumption is more evenly distributed throughout the day compared to the base case, where peaks are concentrated during expensive energy periods. Users (c) and (d), for their part, achieve better savings, with reductions of 5.74 % and 8.12 %, respectively.

However, while the peak values for Users (a), (b), (c), and (d) decrease by 21 % to 42 %, User (e) experiences a 23 % increase in peak demand after optimization. This increase results from the model accumulating loads during lower-cost periods and allowing user autonomy in load scheduling, which could be disadvantageous if there are no power limits or caps on the number of loads in operation. Despite this, User (e) still achieves a 3.01 % reduction in their electricity bill.

The strategies or actions described above can be observed in the optimization applied to the load profiles of all 60 users, leading to a reduction in their electricity bills. It is also important to note that the model performs cost optimization at the individual level and then coordinates a group of users to flatten the overall demand curve. This is done while ensuring user comfort and providing flexibility in scheduling loads.

**Table 14.** Electricity costs for the five users. Source: own work.

| User | Cost without optimization (COP) | Cost with optimization (COP) | Savings |
|---|---|---|---|
| (a) | 11314.26 | 11007.60 | 2.71 % |
| (b) | 11765.58 | 11584.16 | 1.54 % |
| (c) | 10169.90 | 9586.11 | 5.74 % |
| (d) | 9493.78 | 8722.53 | 8.12 % |
| (e) | 15550.05 | 15081.64 | 3.01 % |

In the preceding paragraphs, an individual analysis was provided to show how the model optimizes various aspects for each user. From this point onward, the results will be presented collectively for all evaluated users.

Figure 4 illustrates the load profile for the 60 users, with the green dotted line representing the proposed price in COP/kWh (see Table 8), and the orange and blue bars indicating the cases with and without optimization, respectively. The figure reveals that, in the optimized scenario, loads are shifted from the peak hours of the unoptimized scenario to periods with lower prices. In the base scenario, energy consumption occurs without any constraints or specific load scheduling, preventing users from benefiting from dynamic pricing and leading them to operate appliances during periods of highest energy prices. Conversely, the optimized scenario exhibits a load profile that takes advantage of the lowest prices while ensuring user comfort and avoiding significant demand peaks during other time intervals.

To prevent excessively high peaks during any period, constraints are applied to the mathematical models. Particularly, this study considers constraints associated with active power, such as installed capacity, to prevent overloads and potential electrical safety risks, as well as hourly power limits to minimize the impact of demand peaks at other times of the day. These two constraints depend on the household's electrical characteristics and consumption habits (see Section 2). However, adding more constraints, such as setting a low demand limit, could compromise user comfort and cause new demand peaks during off-peak periods, which could damage the household's electrical infrastructure or the grid. Therefore, the implemented model also includes a sensitivity analysis for these two constraints, prioritizing user preferences and flexibility in load scheduling.

An example of this approach is when users set the $Ti_{CEL}$ and $Tf_{CEL}$ times, granting them a degree of autonomy through indirect control over energy consumption, thus reducing costs. Conversely, if users aim for more significant cost reductions, they may need to sacrifice some level of comfort.

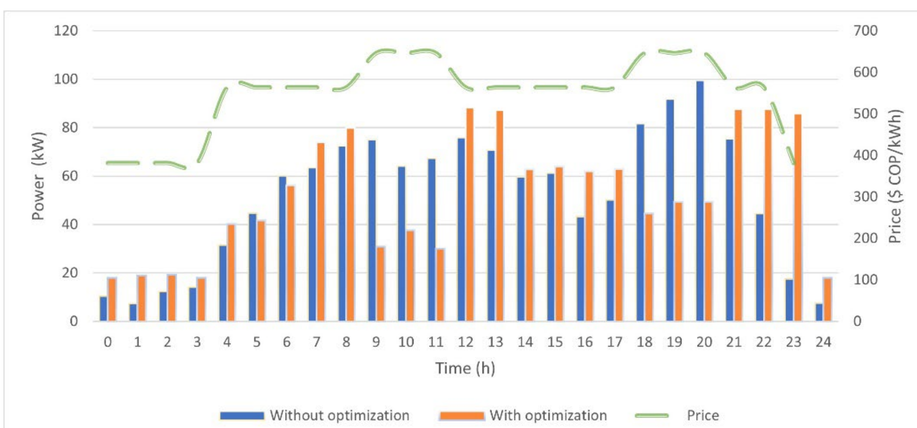


**Figure 4.** Load profile of the 60 users. Source: own work.

The analysis of peak demand yielded the following results: 35 users exhibited higher peak demand in the optimized scenario, 6 users experienced no change in peak demand between the two scenarios, and 19 users had lower peak demand in the optimized scenario. These outcomes were evaluated over the time horizon $T$ and were based on the level of flexibility available to each user. As is typical in real households, not all users share the same consumption habits. Moreover, to avoid negatively impacting user comfort, there are days when the optimized scenario displays higher demand peaks, as seen with User (e), who shifts loads to lower-cost periods, causing them to cluster within a specific time frame. However, appropriate constraints are also imposed to prevent significant demand peaks from being generated and causing problems at the multi-user level.

As illustrated in Figure 4, the involvement of the 60 users in CELS results in a 11.49 % reduction in maximum power, lowering peak power to 88.11 kW and the PAR to 1.63, compared to the 99.55 kW peak power and 1.84 PAR observed in the unoptimized scenario. These results demonstrate that CELS enables a more uniform load distribution among users and reduces peak consumption during high-price periods. This improvement is achieved using (24) through (26), where the standard deviation serves as a metric to evaluate data dispersion. The outcome is a solution characterized by less variability in each user's load scheduling, ultimately producing a flatter demand curve (PAR) compared to the unoptimized scenario.

Another objective of the implemented optimization model is to minimize electricity costs for a certain number of users. Figure 5 compares electricity costs between the unoptimized (blue bars) and optimized (orange bars) scenarios for the 60 users. The graph indicates that electricity costs are lower for all users when the methodology designed in this study is applied. This is because the demand profile flattens the curve during peak hours, when energy prices are at their highest. However, the extent of cost reduction varies depending on the characteristics of each user's loads, their consumption habits, and their individual needs. Consequently, some users may realize substantial savings, while others may experience more modest reductions. Figure 6 illustrates this variation, showing the highest reduction at 12.34 % and the lowest at 0.54 %. On average, the group of users achieves approximately 4.94 % savings on their electricity bills for the simulation day.

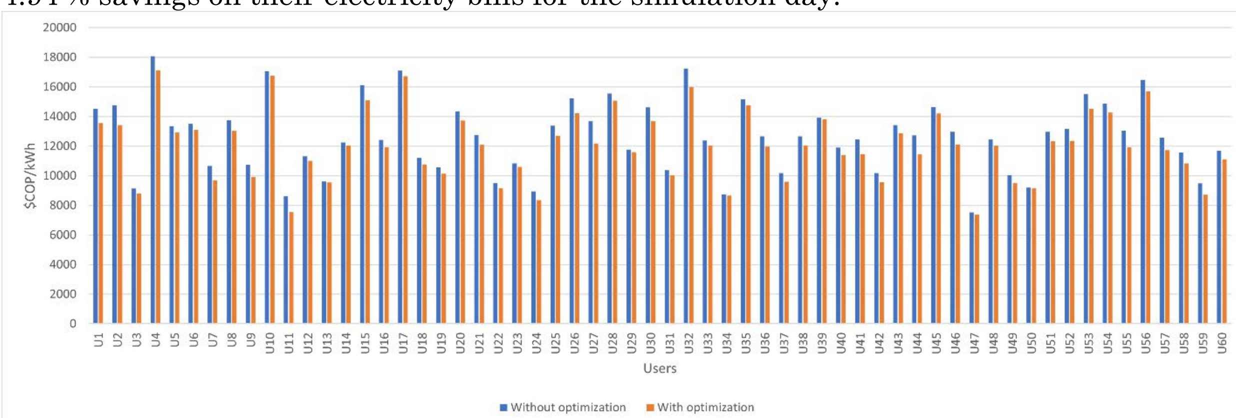


**Figure 5.** Comparison of electricity costs per user. Source: own work.

Table 15 presents the results for maximum power, PAR, and electricity costs, comparing the base case with the proposed optimized model. The optimized model shows a 11.49 % reduction in maximum active power, a decrease in PAR from 1.84 to 1.63 (flattening of the curve), and average savings of 4.94 % for the 60 users. In addition to the benefits for end users, demand management through curve flattening leads to significant savings in the costs associated with constructing new generation and transmission infrastructure to meet demand peaks. It also reduces the cost of purchasing electricity in the market during peak demand periods, thereby easing the burden on grid operators by enhancing system reliability and efficiency. This improvement is achieved by avoiding high consumption peaks, which in turn reduces the likelihood of forced generation outages and failures in transmission and distribution infrastructure. Furthermore, this approach mitigates the risk of blackouts due to sudden demand fluctuations, allowing more time for the integration of new plants to respond to natural demand growth. Additionally, the carbon footprint can be reduced by avoiding the construction and use of fossil fuel-fired power plants, which typically serve as reserves when demand exceeds current generation capacity.

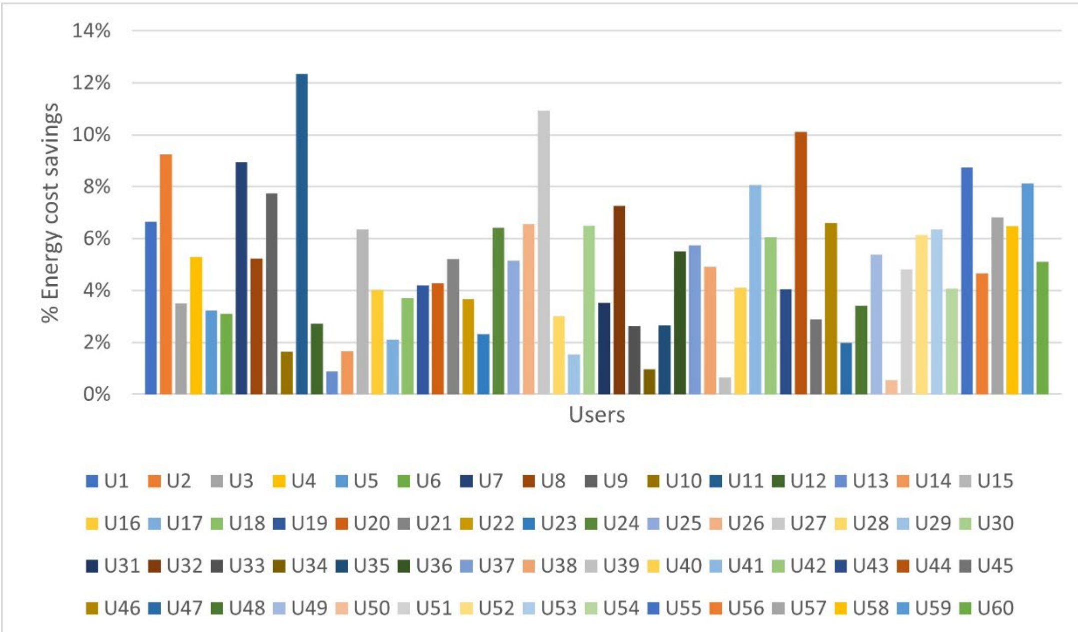

**Figure 6.** Percentage of electricity cost savings for the 60 users. Source: own elaboration.

Just as end users may face challenges in scheduling their loads—either due to a lack of proactive participation in demand management or technical ignorance—distributors must take on the challenge of implementing technologies that support DSM programs. These technologies can monitor energy consumption more effectively, provide real-time information, and automate consumption processes, thereby contributing to a robust smart grid infrastructure. Consequently, DSM requires strategic planning and operation of electricity systems, as well as programs that encourage users to manage their demand at home and use infrastructure such as smart meters, HEMS, and distributed generation resources.

**Table 15.** General results for the evaluated users. Source: own elaboration.

| Scenario | Maximum power (kW) | PAR | Cost (COP) |
|---|---|---|---|
| Without optimization | 99.55 | 1.84 | 757695.88 |
| With optimization | 88.11 | 1.63 | 720095.80 |

Considering the results from other studies, the authors of [6] proposed an optimization strategy for a single residential user with two CELs and an Electric Vehicle (EV) charging point. This strategy achieved a 43 % reduction in peak demand and a 6 % decrease in electricity costs. In comparison, the best case in this study achieved a 42 % decrease in peak demand and a 1.55 % reduction in electricity costs. This difference is attributed to the EV load shifting implemented in [6], which optimized the consumption schedule for lower energy prices, leading to a more substantial overall cost reduction.

Similarly, in [7] and [18], higher savings were achieved through load scheduling—36 % and 16.98 %, respectively—thanks to the integration of renewable energies and energy storage batteries. Although the present study did not focus on such technologies, significant benefits were still realized, including maximum savings of 12.34 % by considering multiple users and CELs, despite the complexity of managing numerous variables in the implementation of the proposed MUMCEL.

In the multi-user scenario described in [5], which simulates ten users with three CELs each, a peak reduction of 16 % and savings of 17.68 % were achieved through a so-called technical approach, which included optimization of distributor prices. In contrast, the present study involves a greater number of loads per user, resulting in a peak reduction of 11.49 % and average savings of 4.94 % among all users. The difference in these results can be attributed to the model addressed in this study considering energy price as a fixed parameter rather than a variable for optimization.

Moreover, in [21], two residential users with a single CEL each achieved savings of 1.51 %. This figure is lower than that obtained in this study, where the model optimized between nine and twelve CELs per user. This highlights the complexity of this model and its realistic consideration of consumer habits.

Furthermore, the study in [22] reported savings ranging from 18.53 % to 26 % for seven residential users, employing renewable energy sources, energy storage devices, and EVs. While the present model does not incorporate these technologies, it does manage a larger number of CELs per user.

Finally, the authors of [34] proposed an optimization for 200 users, with the number of CELs ranging from one to fourteen based on customer preference. They reported savings of up to 20 % and a peak reduction of 22.23 %. In a sample of five users, they reported savings between 9 % and 14 %, whereas the present model achieves individual savings of up to 12 % and an overall peak reduction of 11.94 %. This underlines the importance of using a robust algorithm, like that in [34], for managing multiple users, which could serve as a reference for future work, taking into account the characteristics of the present study and the contributions of the studies mentioned above.

The implementation of the model developed in this study has yielded positive results in terms of both cost reduction and demand curve management for all users. However, it is important to acknowledge the limitations of the proposed methodology. One limitation is the data processing requirement inherent in the exhaustive search approach, which arises from using a considerable number of IFELs over an extensive time range and the need to evaluate a large number of possibilities to reach the optimal solution. To address this, it is key that the number of CELs per user in the simulation does not exceed the specified limit. Despite this, satisfactory performance is achieved by assigning between nine and twelve CELs, considering the UFELs, SELs, and IFELs per user.

Another limitation lies in the variation of the local search approach implemented, which does not guarantee a global solution to the problem but instead focuses on finding a local optimal solution. However, for multi-user modeling, this methodology is usually more efficient in terms of time and computational resources compared to exhaustive search.

Finally, the use of straightforward methods or techniques has demonstrated greater efficiency in solving problems with multiple constraints. This is reflected in this study, where the simulation was conducted on a computer with an Intel(R) Core (TM) i5-8250U CPU @ 1.60 GHz - 1.80 GHz processor, 12GB of RAM, and the Microsoft Windows 11 Home Single Language operating system. The problem was solved for 60 users in approximately 25 minutes, despite the large number of variables, achieving adequate solutions for the defined objectives. It is worth noting that the time required to solve the problem may vary depending on the capacity of the computer used.

## 5. CONCLUSIONS

This study focused on modeling high-consumption residential loads, specifically CELs, which encompass UFELs, IFELs, and SELs. The model introduced here assigned a significantly higher number of CEL types compared to the studies referenced in Section 1. It also considered a wide variety of time intervals for each CEL, as shown in Tables 9 and 10. The methodology developed in this paper is based on classical optimization techniques and is divided into two stages. In the first stage, the exhaustive search method was implemented to select the most cost-efficient load scheduling options for each user. In the second stage, the local search method was employed to combine the solutions obtained in the first stage and find a joint solution with a flatter consumption curve and the lowest electricity cost.

Two scenarios were evaluated: the base case, where users manage their loads at their convenience without considering dynamic energy prices, and the optimized case, which uses an algorithm to reduce both peak demand and costs for users. By comparing the demand curves of the two scenarios, several response strategies were identified, such as peak shaving, valley filling, strategic conservation, and load shifting from higher-priced to lower-priced hours. It was also observed that, due to the autonomy given to users in selecting hourly ranges for their daily load schedules and the shifting of these loads to lower-priced hours, peak demand increased for 58.33 % of users in the optimized case compared to the base case. However, the model's constraints ensured that this increase did not cause issues at either the individual or collective level, leading to a moderate flattening of the load curve according to the PAR indicator, all without exceeding the established limits.

In addition, the proposed MUMCEL demonstrated optimal results, effectively reducing electricity costs and peak demand during higher-priced periods both at the individual and collective levels, which translates into a positive overall impact. The model successfully found a solution within 25 minutes, considering nine to twelve CELs for 60 users—a significantly large number of variables that has not been addressed in other studies. This performance underscores the importance of using a straightforward model like the one developed in this study, as it effectively manages multiple users and variables with lower computational effort compared to more complex algorithms.

It is also noteworthy that if users seek further cost minimization or if the implemented model exceeds the specified constraints, this could require a higher level of sacrifice in terms of customer comfort or potentially damage the home's electrical infrastructure. However, as emphasized in this paper, the model is designed to avoid negatively impacting these aspects, instead aiming to provide flexibility in load scheduling, allowing users to adjust their preferences according to their needs and comfort levels.

Nevertheless, some limitations were identified in the model, such as the restricted number of CELs per household and the selection of IFELs with extended usage times. These factors lead to an excess of data in the exhaustive search, which in turn reduces the speed of information processing during simulation. Consequently, in the multi-user stage, the local search method is employed instead of the exhaustive search, as it is more efficient, albeit at the expense of finding only local solutions.

In conclusion, this study demonstrated that the proposed MUMCEL, through CELS, can optimally manage power demand in smart homes. It successfully flattened the demand curve, reduced peak power by 11.49 %, and achieved average savings of 4.94 %, all without significantly affecting customers' habits. These results pave the way for further research in the field of residential energy management and highlight the importance of engaging users as active participants in demand optimization.

For future work in this field, the following considerations are suggested: (i) Incorporating a larger number of users (e.g., a neighborhood) while maintaining a balance between reducing electricity costs, flattening the load curve, and preserving customer comfort. This also includes adding other types of loads with different characteristics and variables and exploring the possibility of including industrial and commercial sectors in the CELS model. (ii) Integrating EV charging points, energy storage devices, and non-conventional energy sources into the algorithm. (iii) Investigating the application of other optimization algorithms for CELS in a multi-user context. (iv) Extending load scheduling to longer time periods (e.g., weeks, months) while taking the above considerations into account. Additionally, it is recommended to explore the Colombian regulatory framework related to the use of non-conventional energy sources, the benefits of selling surplus energy, and the adoption of smart grid technologies.

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## CONFLICTS OF INTEREST

The authors declare that they have no financial, professional, or personal conflicts of interest related to the publication of this article.

## AUTHOR CONTRIBUTIONS

Nelson Mauricio Bejarano-Bejarano: Research, methodology, design and programming of the optimization algorithms, writing, revision, and editing.

Francisco David Moya Chaves: Conceptualization, methodology, revision, writing, suggestions, and editing.

Óscar Danilo Montoya Giraldo: Conceptualization, writing, suggestions, editing, and final revision of the paper.

## STATEMENT ON IA ASSISTANCE

During the preparation of this manuscript, the authors used ChatGPT to assist with the writing process, including spelling, grammar, and style improvements. After employing this tool, the authors carefully reviewed and edited the content as necessary and take full responsibility for the final content of the publication.

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