

# Influence of the void ratio and the confining on the static liquefaction in slopes in changi sand

*Influencia de la relación de vacíos y el confinamiento en la licuación estática en taludes de arena de Changi*

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Fecha de recepción: 21 de febrero del 2014

Clasificación del artículo: investigación

Fecha de aceptación: 15 de agosto del 2014

Financiamiento: Pontificia Universidad Javeriana

DOI: <http://dx.doi.org/10.14483/udistrital.jour.tecnura.2015.1.a04>

**Keywords:** instabilities in sands, static liquefaction, stress anisotropy.

**Palabras clave:** anisotropía de esfuerzos, inestabilidades en arenas, licuación estática.

## ABSTRACT

A numerical study on the onset of static liquefaction in slopes under undrained conditions of loading was developed based on a general liquefaction flow instability criterion for elastoplastic soils based on the concept of loss of controllability. The criterion is applied to the case of axisymmetric loading to detect the onset of static liquefaction. The criterion is used in conjunction with an elastoplastic model for sands and is tested by means of numerical simulations of element tests. The numerical results are compared with experimental evidence obtaining good agreement. A quantitative study of the influence of the mean pressure, void ratio and the anisotropy of stress on the onset of static liquefaction is presented for the Changi sand. From the analysis of the numerical results, it can be concluded that: a. the mobilized friction angle at the onset of liquefaction is not an intrinsic property of the material, but is a state variable b. Despite of the multiple variables

involved in the process of generation of undrained instability, the state of stresses at the onset of static liquefaction can be conveniently represented by a linear relation between  $\Delta q/p_0$  and  $\eta_0$ . This graphical representation can be used in the practice of geotechnical engineering to quantify the margin of security against the static liquefaction of a sandy slope.

## RESUMEN

Se presenta un estudio numérico del inicio de la licuación estática en taludes, bajo condiciones no drenadas de carga, basado en un criterio de inestabilidad general para suelos elastoplásticos, fundamentado en el concepto de pérdida de controlabilidad. Se aplica el criterio al caso de carga axisimétrica, para detectar el punto de inicio de licuación con un modelo elastoplástico para arenas. Se comparan los resultados numéricos con

evidencia experimental, encontrando un buen nivel de concordancia. Se presenta un estudio cuantitativo de la influencia de la presión media, relación de vacíos y la anisotropía inicial de esfuerzos sobre el inicio de la licuación en la arena de Changi. Se concluye que: a) el ángulo de fricción movilizado al inicio de la licuación no es una propiedad del material, sino que es una variable de estado; b) a pesar de las múlti-

ples variables involucradas en el proceso de generación de inestabilidad no drenada, el estado de esfuerzos en el inicio de la licuación estática se puede representar convenientemente por una relación lineal entre  $\Delta q/p_0$  y  $\eta_0$ . Esta representación gráfica se puede usar en la práctica de la ingeniería geotécnica para cuantificar un margen de seguridad contra licuación estática de un talud arenoso.

\* \* \*

## INTRODUCTION

Failure of slopes under monotonic undrained conditions of loading is an instability process which occurs in loose sands. It is well known that the behavior of sands is greatly influenced by their initial conditions, such as the initial density and confining pressure (Hyodo, Tanimizu, Yasufuku, & Murata, 1994). A common approach used to tackle issues associated with liquefaction is based on phenomenology. Moreover, the most used methodologies for evaluating the potential of liquefaction are based on field test such as SPT and CPT (Youd *et al.*, 2001). These methodologies disregard important aspects like the anisotropy of stresses in the evaluation of susceptibility of liquefaction. An alternative and more rational way to understand the static liquefaction is based on sets of laboratory tests, and based on these results it's proposed a generalization of the behaviour. For example Wanatowski and Chu (2007) depict a relation between the stress ratio  $\eta_L = q_L/p_L$  and void ratio for undrained triaxial tests isotropically and anisotropically compressed ( $q_L$  and  $p_L$  are the main pressure and the deviatoric stress in the onset of static liquefaction, respectively). On other hand, Chu and Wanatowski (2008) propose a mathematical equation between  $\eta_L$  and the Been-Jefferies critical state parameter ( $\psi = e - e_c$ ). This equation is obtained by assum-

ing that the increase of plastic volumetrics strain is equal to the increase of total volumetric strain and both of them are equal to zero in the peak of deviatoric stress. It means that the dilatancy is zero in the peak of undrained effective stress path. However, Lade (1994) demonstrates that sands subjected to undrained loading show a behavior that must be modelled by a flow rule highly non associative. A similar approximation for detecting the onset of static liquefaction is to characterize the instability line for a specific type of sand. The instability line can be defined as the boundary in which large strains are rapidly generated due to the inability of a soil element to sustain a given stress or load (Chu, Leroueil, & Leong, 2003). The instability line was firstly proposed by Vaid and Chern (1985) defining it as the locus of points at which flow liquefaction is initiated for the same initial void ratio under monotonic undrained triaxial tests. Many researchers have analysed the instability line (Lade, 1994; Chu & Wanatowski, 2008; Chu, Leroueil, & Leong, 2003; Wanatowski & Chu, 2007; Hyodo, Tanimizu, Yasufuku, & Murata, 1994; Andrade, 2009; Andrade, Ramos, & Lizcano, 2013). Andrade (2009) and Ramos, Andrade and Lizcano (2011) using two different elastoplastic constitutive models concluded that the slope of the instability line is not a constant of the material but it is a state parameter.

In this work, a criterion for detecting the onset of static liquefaction derived by Andrade, Ramos and Lizcano (2013) and Ramos, Andrade and Lizcano (2011) was extended to an isotropic elastoplastic constitutive model with kinematic hardening and bounding surface theory (Manzari-Dafalias model). The application of this criterion matches very well with experiments reported in the literature, allowing to generate numerical simulation for different conditions with a high level of confidence. By making use of numerical simulations previously validated with experiments, it is possible to supply the deficiency of the experimental approximation and to have a greater spectrum of the behavior of the sandy slopes subjected to static liquefaction. As result, the influence of void ratio, mean pressure and initial anisotropy on the onset of static liquefaction is investigated. Finally, despite of the multiple variables involved in the process of generation of static liquefaction, the behavior of the sand can be condensed in a linear relation between  $\Delta q/p_0$  and  $\eta_0$ . This kind of relation can be used in the practice of geotechnical engineering to quantify the degree of security of a slope given the initial conditions of stress and void ratio.

The paper is organized as follow. The first section of this paper shows the instability criterion for detecting the onset of static liquefaction using the Manzari-Dafalias elastoplastic constitutive model. The ‘Validation’ section presents numerical simulations for prediction of static liquefaction and compares them with laboratory experiments. Finally, the influence of the void ratio, mean pressure and the initial stress anisotropy on the onset of static liquefaction is presented based on several simulations conducted with the constitutive model and the instability criterion. Findings from the application of the criterion are highlighted in the ‘Conclusions’ section.

## STATIC LIQUEFACTION CRITERIONS AND CONSTITUTIVE MODEL

For the sake of simplicity, we limit the following derivation to infinitesimal strains and axisymmetric undrained conditions. Assuming a rate form for the constitutive model equation (1), the relation among the effective stress rate and the total strain rate can be written as

$$\begin{Bmatrix} \dot{p} \\ \dot{q} \end{Bmatrix} = \begin{bmatrix} C_{pp} & C_{pq} \\ C_{qp} & C_{qq} \end{bmatrix} \begin{Bmatrix} \dot{\epsilon}_v \\ \dot{\epsilon}_B \end{Bmatrix} \quad (1)$$

where  $\dot{\epsilon}_s = 2/3(\dot{\epsilon}_a + \dot{\epsilon}_r)$  is the volumetric strain rate, and  $\dot{\epsilon}_s = 2/3(\dot{\epsilon}_a + \dot{\epsilon}_r)$  is the deviatoric component of the strain rate. Also,  $\dot{p} = 1/3(\dot{\sigma}_a + 2\dot{\sigma}_r)$  is the effective pressure rate and  $\dot{q} = \dot{\sigma}_a - \dot{\sigma}_r$  is the deviatoric stress rate. We note the usage of Cambridge  $p - q$  stress invariants to describe triaxial conditions. Finally,  $\dot{\sigma}_a$  is the axial total stress rate and  $\dot{\sigma}_r$  is its radial counterpart.

Following the concept of loss of controllability for elemental test conditions (Nova, 1994), the volumetric strain rate and the deviatoric stress rate under triaxial conditions can be controlled giving pressure increments and shear strain increments, equation (2), such that

$$\begin{Bmatrix} \dot{\epsilon}_v \\ \dot{q} \end{Bmatrix} = \begin{bmatrix} C_{pp}^{-1} & C_{pq} C_{pp}^{-1} \\ C_{qp} C_{pp}^{-1} & C_{qq} - C_{pq} C_{qp} C_{pp}^{-1} \end{bmatrix} \begin{Bmatrix} \dot{p} \\ \dot{\epsilon}_s \end{Bmatrix} \quad (2)$$

Here, we look for the vanishing of the determinant of the matrix relating the controlled variables on the left to the emerging or responding variables on the right. The requirement of singularity implies  $C_{qq} / C_{pp} = 0$  which requires

$$C_{qq} = 0 \quad (3)$$

This condition, equation (3), will furnish a general criterion for detecting static liquefaction instability in terms of loss of controllability. Andrade *et al* (2012) demonstrated the similarity of the loss of controllability with the concept of loss of uniqueness and the second order work (Darve & Laouafa, 2000). We will adapt this general criterion to the case of the Dafalias and Manzari (2004) constitutive model. Now, the Dafalias and Manzari (2004) model will be briefly described. For a complete description of the model, interested readers are referred (Manzari & Dafalias, 1997; Dafalias & Manzari, 2004). The constitutive model is framed in the critical state soil mechanics concept (Schofield, 1968), and the elastic response is hypoelastic. The shear and bulk moduli are given by equation (4).

$$G = G_0 p_{at} \frac{(2.97 - e)^2}{1 + e} \left( \frac{p}{p_{at}} \right)^{1/2} \text{ and}$$

$$K = \frac{2(1 + \nu)}{3(1 - 2\nu)} G \quad (4)$$

where  $G_0$  is a constant,  $\nu$  is Poisson's ratio,  $e$  is the current void ratio, and  $p_{at}$  is the atmospheric pressure. The elastic region is enclosed by a yield surface in effective stress space which defines a wedge, equation (5).

$$F(\eta, \alpha) = |\eta - \alpha| - m \quad (5)$$

with  $\eta = q/p$  as the stress ratio,  $\alpha$  as the back stress, and  $m$  as a constant defining the width of the wedge so that in  $p - q$  space, the wedge has an opening of  $2mp$  at any value  $p$ . The inclination of the wedge defining the elastic region is given by the back stress, equation (6) whose evolution is governed by a kinematic hardening law

$$\dot{\alpha} = H \dot{\epsilon}_s^p \quad (6)$$

where  $H$  is the hardening modulus. To complete the description of the constitutive model, evolution of the hardening modulus  $H$  and dilatancy  $\beta$  must still be explained. The hardening modulus, equation (7), is a function of the state of the material whose sign is controlled by its relative distance to the bounding stress, i.e.,

$$H = h(M^b - \eta) \text{ with}$$

$$h = \frac{G_0 h_0 (1 - c_h e)}{|\eta - \eta_{in}|} \left( \frac{p}{p_{at}} \right)^{-1/2} \quad (7)$$

where  $h$  is a positive function,  $M^b$  is the bounding stress ratio, and  $h_0$  and  $c_h$  are positive constants. The evolution of the dilatancy, equation (8), is given by a function similar to that of the hardening modulus, with the sign of the function dictated by its distance to the dilatancy stress so that

$$\beta = A_d (M^d - \eta) \quad (8)$$

with  $M^d$  as the dilatancy stress ratio. When the value of  $h$  is less than the value of  $M^d$ , response is contractive. For all other cases the model predicts dilation. The positive scaling function for dilatancy  $A_d$ , equation (9), is affected by changes in fabric such that

$$A_d = A_0 (1 + \langle sz \rangle) \text{ with}$$

$$Z = -Cz \langle -\dot{\epsilon}_v^p \rangle (SZ_{\max} + Z) \quad (9)$$

where  $A_0$  is a positive constant and  $s = \pm 1$  according to  $\eta = \alpha \pm m$ . The brackets  $\langle \rangle$  are Macaulay brackets representing as  $\langle \text{value} \rangle = \text{value}$  if  $\text{value} > 0$  or  $\langle \text{value} \rangle = 0$  if  $\text{value} \leq 0$ . In addition,  $Z_{\max}$  represents the maximum possible value of the state parameter  $z$ . The model is made to comply with critical state soil mechanics by postulating exponential evolution equations for the bounding and dilatancy stress ratios, equation (10). They are respectively,

$$M^b = M \exp(-n^b \psi) \text{ and}$$

$$M^d = M \exp(-n^d \psi) \quad (10)$$

with  $n^b$  and  $n^d$  as positive constants. Conceptually, the evolution equations shown above require  $M^b$  and  $M^d$  to coincide with  $M$  as  $y \rightarrow 0$ , requiring its state to tend to critical state. The state parameter  $\psi = e - e_c$  was defined by Been and Jefferies (1985) and measures the distance to the critical state from the current state in void ratio space. Finally, the critical state line is defined in void ratio space according to the relationship proposed by Li and Wang (1998), equation (11)

$$e_c = e_{c0} - \lambda_c \left( \frac{p_c}{p_{at}} \right)^\zeta \quad (11)$$

with  $e_{c0}$  as the void ratio at  $p_c = 0$  and  $\lambda_c$  and  $\zeta$  as constants.

In order to present a self-content paper, the derivation of the static liquefaction criterion will be summarized. However its original development was proposed by the same authors of this paper in Andrade, Ramos and Lizcano (2013).

The Dafalias and Manzari (2004) constitutive model can be seen in the matrix form of Equation (1) by re-sorting the additive decomposition of incremental stress-strain relation as follows, equation (12) and equation (13)

$$\dot{\epsilon}_s^e = \frac{\dot{q}}{3G} \quad \dot{\epsilon}_v^e = \frac{\dot{p}}{K} \quad (12)$$

$$\dot{\epsilon}_s^p = \frac{\dot{\eta}}{H} \quad \dot{\epsilon}_v^p = \beta \left| \dot{\epsilon}_s^p \right| \quad (13)$$

Superscripts  $e$  and  $p$  denote the elastic and plastic part of strains. The increment in stress ratio is calculated as  $\dot{\eta} = -\eta / p \dot{p} + \dot{q} / p$ , where  $\eta = q/p$ . Based on the rates of total volumetric and deviatoric strains, equation (1) especially adapted

to the Dafalias and Manzari (2004) constitutive model reads, equation (14)

$$\begin{Bmatrix} \dot{p} \\ \dot{q} \end{Bmatrix} = \frac{1}{x} \begin{bmatrix} 3KG + KHp & -3KG\beta \text{sgn}(\dot{\epsilon}_s^p) \\ 3KG\eta & 3GHp - 3KG\beta\eta \text{sgn}(\dot{\epsilon}_s^p) \end{bmatrix} \begin{Bmatrix} \dot{\epsilon}_v \\ \dot{\epsilon}_s \end{Bmatrix} \quad (14)$$

where,  $\chi = 3G + Hp - K\beta\eta \text{sgn}(\dot{\epsilon}_s^p)$ . Equation (3), which indicates loss of controllability and the onset of static liquefaction, when especially adapted to the Dafalias and Manzari model is, equation (15)

$$C_{qq} = \frac{3G(Hp - K\beta\eta \text{sgn}(\dot{\epsilon}_s^p))}{x} = 0 \quad (15)$$

which, to be true in general, requires the quantity inside the parenthesis to vanish. In elastoplastic models the hardening modulus  $H$  is an indicator of the soil state. Andrade (2009) deduced a critical hardening modulus as a predictor of static liquefaction for an elastoplastic constitutive model with two invariants. From equation (15) a closed form of the hardening modulus that is able to detect the onset of static liquefaction can be derived (Andrade, Ramos, & Lizcano, 2013), equation (16).

$$H_L = \frac{K\beta\eta}{p} \text{sgn} C_{qq} = \dot{\epsilon}_s^p \quad (16)$$

At the moment when the hardening modulus equals the critical hardening modulus ( $H - H_L = 0$ ) instability occurs in the form of static liquefaction.

Validation of the criterion for detecting static liquefaction under anisotropic initial conditions of stress was given in Ramos, Andrade and Lizcano (2011). Ramos, Andrade and Lizcano (2011) developed numerical simulations using the generalized static liquefaction criterion introduced in equation (3) and adapted to the Manzari-Dafalias model utilizing the limiting hardening modulus encapsulated in equation (16). The anisotropic

initial conditions of stress is representative of the conditions of stress in a slope which can be represented in the  $q - p$  space of stress by mean of  $q \neq 0$  and  $p \neq 0$ . Wanatowski and Chu (2007) report a comprehensive set of triaxial and biaxial tests in Changi sand under both, isotropically and anisotropically compressed, and sheared under undrained conditions of loading. These sets of experiments generated a great opportunity to check the performance of the criterion (Eq.16) to detect the onset of static liquefaction. Parameters for the elastoplastic constitutive model were calibrated based on the sets of experiments reported by Leong, Chu and the (2000) (table 1).

**Table 1.** Material Parameters for the Manzari-Dafalias Model for Changi sand

|                  | Constant    | Changi Sand |
|------------------|-------------|-------------|
| Elasticity       | $G_0$       | 125         |
|                  | $\nu$       | 0.05        |
| Critical State   | $M$         | 1.35        |
|                  | $\lambda_c$ | 0.0919      |
|                  | $E_{c0}$    | 0.963       |
|                  | $\Xi$       | 0.4         |
| Yield Surface    | $m$         | 0.05        |
| Plastic Modulus  | $h_0$       | 5           |
|                  | $C_h$       | 0.8         |
|                  | $n^b$       | 1.1         |
| Dilatancy        | $A_0$       | 1.54        |
|                  | $n^d$       | 3.5         |
| Dilatancy-fabric | $Z_{max}$   | 4           |
|                  | $C_z$       | 600         |

Source: Leong et al. (2000).

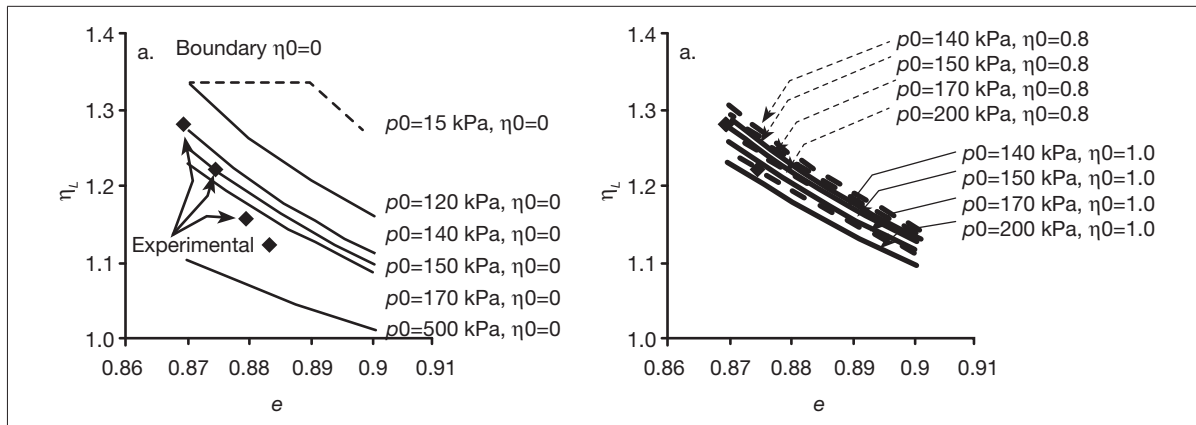
(Ramos, Andrade and Lizcano (2011) simulated two isotropically compressed triaxial tests CU ( $p_0 = 150$  kPa,  $q_0 = 0$  kPa,  $e_c = 0.916$ ); ( $p_0 = 150$  kPa,  $q_0 = 0$  kPa,  $e_c = 0.888$ ) and three triaxial compression test anisotropically compressed under a  $k_0$  stress path KOU ( $p_0 = 191.22$  kPa,  $q_0 = 152.83$  kPa,  $e_c = 0.899$ ; ( $p_0 = 199.57$  kPa,

$q_0 = 147.78$  kPa,  $e_c = 0.922$ ); ( $p_0 = 199.93$  kPa,  $q_0 = 183.28$  kPa,  $e_c = 0.88$ ) using the elastoplastic constitutive model and the parameters of the table 1. Onset of liquefaction is obtained by mean of the application of the critical hardening modulus. Reasonable agreement between the experiment and the simulation was reported by Ramos, Andrade and Lizcano (2011).

## RESULTS

Once verified the performance of elastoplastic constitutive model as well as the criterion for detecting the onset of static liquefaction under both, isotropic and anisotropic initial conditions by mean of comparisons with experiments under triaxial conditions of loading (Ramos, Andrade, & Lizcano, 2011), a number of simulations were carried out in order to study the influence of the void ratio, the confining pressure, and the initial stress ratio  $\eta_0 = q_0/p_0$  on the onset of static liquefaction. Figure 1a depicts the stress ratio in the onset of static liquefaction  $\eta_L = q_L/p_L$  versus the void ratio for six different mean pressures and the same initial stress ratio  $\eta_0 = q_0/p_0$ .

From figures 1a and 1b it is possible to observe that for a given  $\eta_0$ , the higher the mean pressure, the lower the stress ratio at the onset of static liquefaction  $\eta_L$ . However, it was observed that the deviatoric stress  $q_L$  is larger for larger mean pressure. If the mean pressure is kept constant, the looser the sand, the lower  $\eta_L$ . Also, it is observed that for the same void ratio, the stress ratio at the onset of static liquefaction  $\eta_L$  is different. This means that  $\eta_L$  is not a constant of the material, what was originally proposed by Vaid and Chern, (1985) in terms of a constant mobilized friction angle. Using an elastoplastic constitutive model with kinematic hardening, Andrade (2009) and Ramos, Andrade and Lizcano (2011) demonstrated that  $\eta_L$  is not constant but state variable.



**Figure 1.** Influence of the mean pressure and the initial void ratio on the onset of flow. **a)** Initial conditions  $\eta_0 = 0$ . Mean pressures of the experiments developed by Wanatowski and Chu (2007) are in the range of  $p_0 = 150$  kPa to  $p_0 = 200$  kPa. Source: Ramos, Andrade and Lizcano (2011) **b)** Initial stress conditions  $\eta_0 = 0.8$  and  $\eta_0 = 1.0$

Source: own work.

Also, an upper boundary in the figure 1a is found for  $\eta_0 = 0$ . A similar upper limit can be derived for different values of  $\eta_0$ . It is not possible to indefinitely diminish the mean pressure in order to get a higher value of  $\eta_L$  keeping constant the void ratio. Points above the upper boundary will show a strain hardening behavior. This upper boundary is given by the lower mean pressure that a sample can support in order to develop static liquefaction for a given void ratio. The curve for  $p_0 = 15$  kPa –  $\eta_0 = 0$  is also a boundary which separates the behavior between the static liquefaction and strain hardening.

An explanation for the upper boundary can be given by resorting to the critical state parameter  $\psi = e - e_c$  proposed by Been and Jefferies (1985). The state parameter is a measurement of the distance between the current void ratio and the void ratio in the critical state for a given value of mean pressure. Negative values of  $\psi$  are expected for strain hardening materials, and positive values are for strain softening. Materials with the ability to suffering static liquefaction have always  $\psi > 0$ . Let's suppose a material with a given initial

void ratio and a confining pressure such that  $\psi$  is larger than zero. As the mean pressure decreases, the state variable  $\psi$  also reduces. If the confining pressure is enough low,  $\psi$  can become negative, i.e. when the mean pressure diminishes, the material can change of having a contractive behavior to dilative one, and the material will not tend to undergo static liquefaction.

The upper boundary has values near to 1.35 for  $\eta_0$ , which is the slope of the critical state line in the  $qp$  space of stress. This means that for low values of mean pressure  $p$ , the onset of static liquefaction  $\eta_L$  is located very near to the critical state line. The critical state line departs from the origin the  $qp$  space of stress. The aspects previously mentioned could indicate that the instability line across the origin of coordinates. However, the strain softening behavior is limited by a certain value of mean pressure larger than zero for a given void ratio. In this case, the sample changes its behavior and it begins to build up negative pore pressures, indicating that the material has strain hardening behavior. Therefore, the instability line could be projected onto the origin of coordinates in the  $qp$

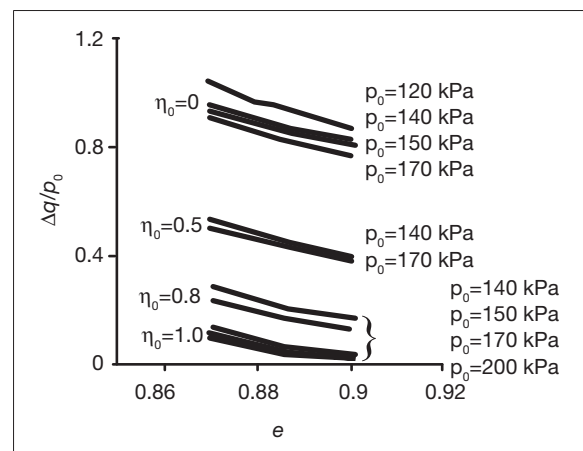
space of stress for small values of mean pressure, although it will not have a geometric place in the origin in the  $qp$  space of stress.

Wanatowski and Chu (2007) presented a line connecting the experimental points of figure 2a which relates the void ratios and the stress ratio at the onset of static liquefaction  $\eta_L$  in triaxial test for mean pressures ranging between  $p_0 = 150 - 200\text{kPa}$ . They argue in favour of that linear relation because of the narrow range of mean pressures used for the experiments. Each experiment corresponds to a different mean pressure. This means that a curve joining the experimental points would be crossed by curves of constant  $p_0$ , as shown in 2a. Consequently, there is not only one line in the  $\eta_L - e$  space, but, a family of curves for different mean pressures for each initial conditions of stress  $\eta_0$ . This hypothesis was confirmed with the experiments performed by Wanatowski and Chu (2007). On the other hand, the instability line can be easily constructed with the data from figure 1 by drawing a vertical cross section at a given void ratio (i.e.  $e = 0.89$ ). It is possible to observe that the instability line is not a straight line, as it was proposed by Vaid and Chern (1985), but it is a curve, i.e. the mobilized friction angle at the onset of static liquefaction is not a constant. This clearly means that the instability line is not an intrinsic property of the material. Andrade (2009) and Ramos, Andrade and Lizcano (2011) demonstrated that the ratio of stress at the onset of liquefaction  $\eta_L$  is not a constant for two different elastoplastic constitutive models. Figure 2 shows a plot of the ratio of deviatoric stress  $\Delta q$  normalized with initial mean pressure  $p_0$  versus initial void ratio for different initial stress ratios  $\eta_0$ .  $\Delta q$  is defined as  $q_L - q_0$ .

One might assimilate that the larger  $\eta_0$ , the steeper the slope for a given depth i.e., constant mean pressure  $p_0$ . Different values of  $\eta_0$  are representative of the anisotropy of the initial stress. By comparison of different  $\eta_0$  for a given void ratio and mean pressure in the figure 2, it is observed that

the larger  $\eta_0$ , the lower normalized increment of deviatoric stress  $\Delta q/p_0$ . This means that a steeper slope will have the chance of experimenting static liquefaction with a lower increment of deviatoric stress. It is easier that a steeper slope static liquefied experiments static liquefaction than a smooth slope. Therefore, the anisotropy of stress  $\eta_0$  plays an important role in the stability of sandy slopes. In addition, concordantly with the trend observed in the figure 1, one can see that for a given  $\eta_0$  and an initial void ratio, the larger the mean pressure, the lower the ratio of normalized deviatoric stress. This means that for an homogeneous slope (both void ratio and slope constant), the deeper, the lower the ratio of normalized deviatoric stress  $\Delta q/p_0$ . However, the deviatoric stresses  $q_L$  needed to produce static liquefaction are higher.

Conversely, the higher the initial stress  $\eta_0$ , the lower the influence of the mean pressure. This means that a slope with high inclination will suffer static liquefaction under similar values of normalized ratio of deviatoric stress  $\Delta q/p_0$  independently of the mean pressure. (Note that the curves for  $\eta_0 = 1$  are closer to each other than the curves for  $\eta_0 = 0$ ).



**Figure 2.** Influence of the anisotropy of the initial stress  $\eta_0$  and the confining pressure  $p_0$  on the onset of static liquefaction

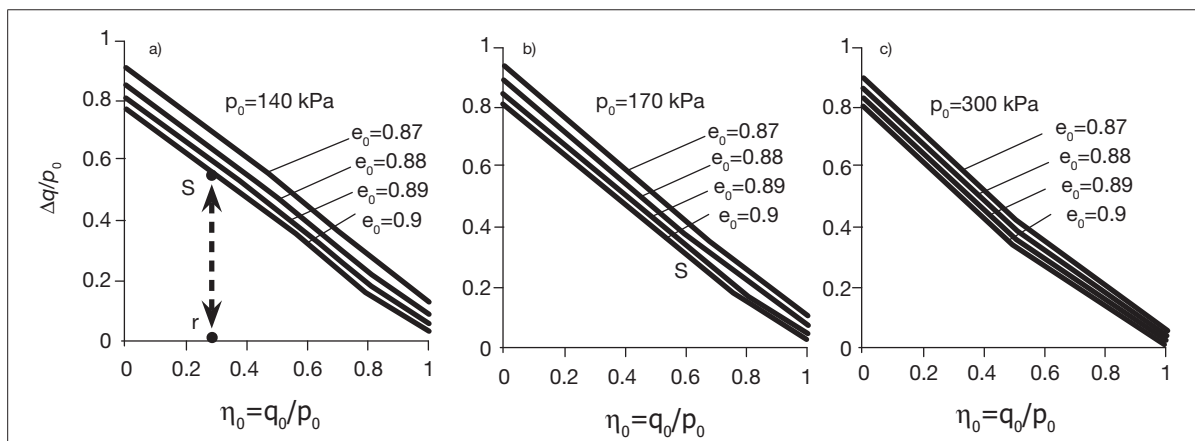
Source: own work.



By definition,  $\Delta q/p_0$  cannot reach negative values because  $q_L \geq q_0$  and  $p_0 > 0$ . When  $\Delta q/p_0 = 0$ , it means that the material is intrinsically unstable: no additional deviatoric stress needs to be applied to unleash static liquefaction. In this case, the initial stresses are located inside the zone of potential instability proposed by Lade (1994). This zone is called “potential” because the state of the material can be inside this zone under drained conditions, i.e. the formation process of the slope can lead to an in situ stress state that may lie within the potential instability zone. However, if external forces are applied fast enough and cause an undrained loading process, the material will undergo instability in the form of static liquefaction. Also, it is worth noting that due to the negative slope in the curves of figure 3, a material is intrinsically unstable if it has high values of  $\eta_0$  (slopes with high inclinations) and if it is in a very loose state. When the void ratio is increased, a lower value of normalized ratio of deviatoric stress  $\Delta q/p_0$  is necessary in order to produce static liquefaction under constant mean pressure and initial stress ratio. This would mean that it is necessary to develop lower deviatoric stresses in softer material than in a denser material under the same conditions of

both inclination of the slope and depth. Results of figure 2 previously analyzed show a general panorama of the behavior of sandy slopes with possible static liquefaction. An alternative way to represent the aforementioned analyzed data is shown in figure 3.

Figure 3 shows the normalized deviatoric stress ratio  $\Delta q/p_0$  versus the initial stress ratio  $\eta_0 = q_0/p_0$  for different void ratios and mean pressures  $p_0$ . Additional to the remarks previously made, some additional observations can be proposed. Through comparison between figures 3a and 3c, it can be seen that the higher the mean pressure, the lower the influence of the void ratio. The separation between the curves for a given void ratio is narrower in figure 3c than in figure 3a. A more important aspect can be derived from figure 3. All the variables studied in this work are included in figure 3 (anisotropy of stresses, void ratio and mean pressure) and despite the somehow intricate behavior of the samples at the onset of static liquefaction, the results might be condensed in a straight line. For this range of mean pressures and void ratios, the possibility of static liquefaction does not show great variability, because the influ-



**Figure 3.** Onset of static liquefaction in terms of normalized deviatoric stress  $q/p_0$  versus initial stress ratio  $\eta_0 = q_0/p_0$  for different void ratios. **a)**  $p_0 = 140$  kPa; **b)**  $p_0 = 170$  kPa; **c)**  $p_0 = 300$  kPa

Source: own work.

ence of the mean pressure is hidden when both, the abscissas and the ordinates are normalized. These types of graphs can be used as an indicator of the degree of safety of the slope because the abscissas are the initial conditions of stresses and the ordinates give the stresses needed to cause the instability by means of static liquefaction. Based on the knowledge of some basic characteristics of the slope, for example inclination, unit weight, ground water level and depth of the layer that one would like to analyse, it is possible to estimate the initial stress state  $h_0$ . For example, we assume that for a given slope and a given layer for analysis, a point  $r$  in the abscissa of figure 3a can be located. Then, the distance  $\Delta q/p_0$  measured along the vertical line linking the point  $r$  and the ordinate of the corresponding void ratio curve (point  $s$ ) would help to estimate the maximum overburden pressure that can be quickly and monotonically applied to the slope before reaching instability in the form of static liquefaction. Then,  $\Delta q/p_0$  could be used as an indicator of the slope's margin of safety.

## CONCLUSIONS

Once, both the constitutive model and the liquefaction criterion have been successfully tested, the following remarks can be stated:

There is no a unique relationship between the stress ratio at the onset of static liquefaction  $\eta_L = q_L/p_L$  and the void ratio. This relationship depends not only on the mean pressure but also on the initial anisotropy of stresses.

Numerical and experimental evidence show that the well known instability line proposed by Vaid and Chern (1985) and Lade (1994) is not a straight line, but a curve in the  $p - q$  stress space which can be projected from the origin for Changi sand. This means that  $\eta_L$  is not an intrinsic property of the material. Therefore, it is not possible to assume that the mobilized friction angle at the onset of static of liquefaction is constant.

A quantification of the influence of the void ratio, mean pressure and anisotropy of stresses on the onset of static liquefaction for Changi sand is presented. A similar procedure can be used for any sandy material. This methodology, based on the loss of controllability, takes into account more aspects that influence the undrained response, than simplified methods based only on phenomenology.

The most important novel aspect tackled in this paper is that despite many factors that influence the onset of static liquefaction, the behavior of the analysed sand might be condensed and described approximately by a straight line in a normalized graph of  $\Delta q/p_0$  versus  $\eta_0$ . This type of plot can be used in the practice of geotechnical engineering as an indicator of slope margin of safety against static liquefaction.

## ACKNOWLEDGMENTS

The first author acknowledges the financial support given to this work by Pontificia Universidad Javeriana- Colombia. Grant number 004709 "Instabilities in granular matter".

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